

INTERIOR BALLISTICS

Part I

PROPERTIES OF POWDERS
AND THEIR ACTION IN CLOSED CHAMBERS
AND IN CANNON



By

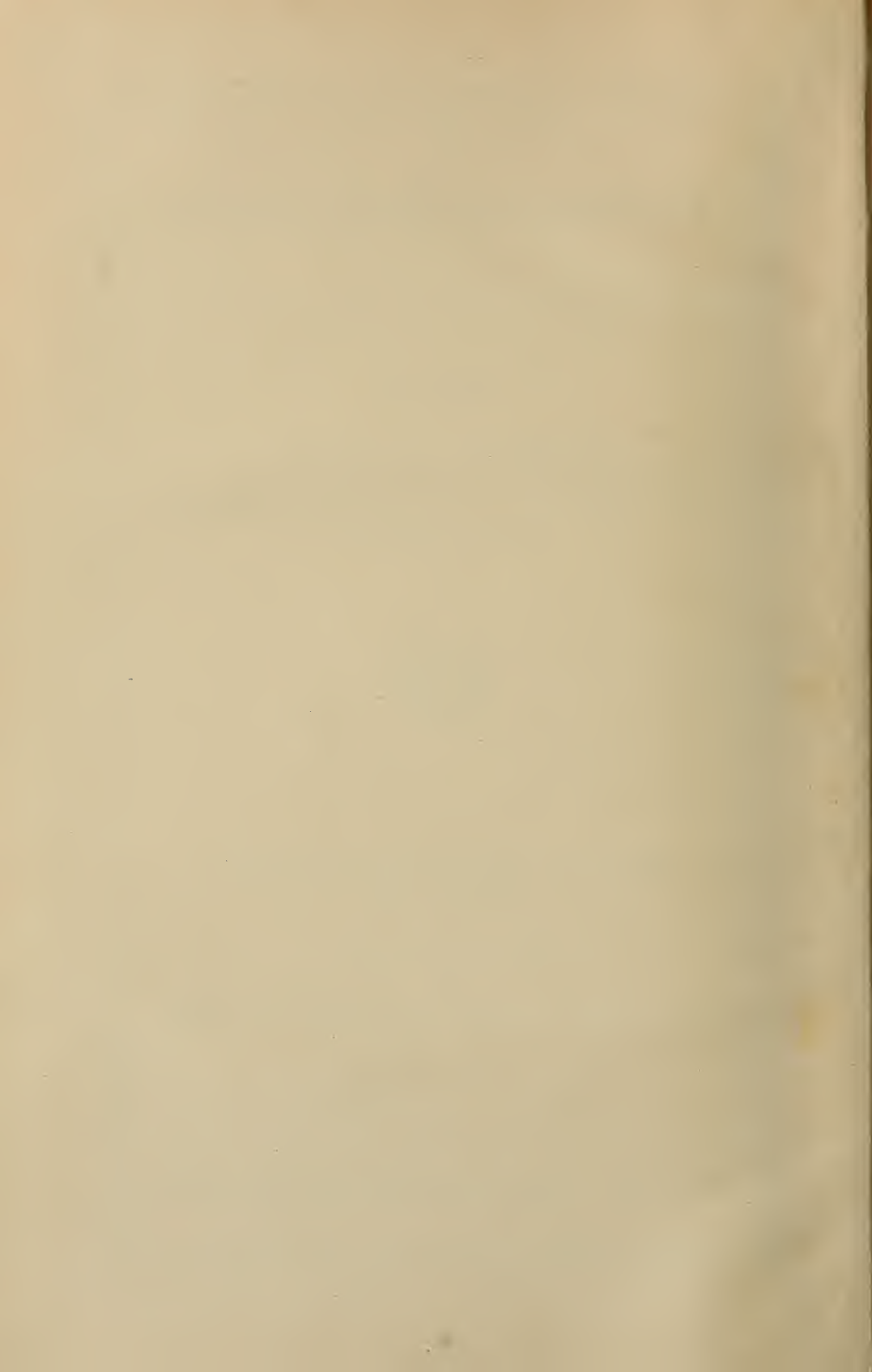
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Artillery Service

Translated from the Russian

by

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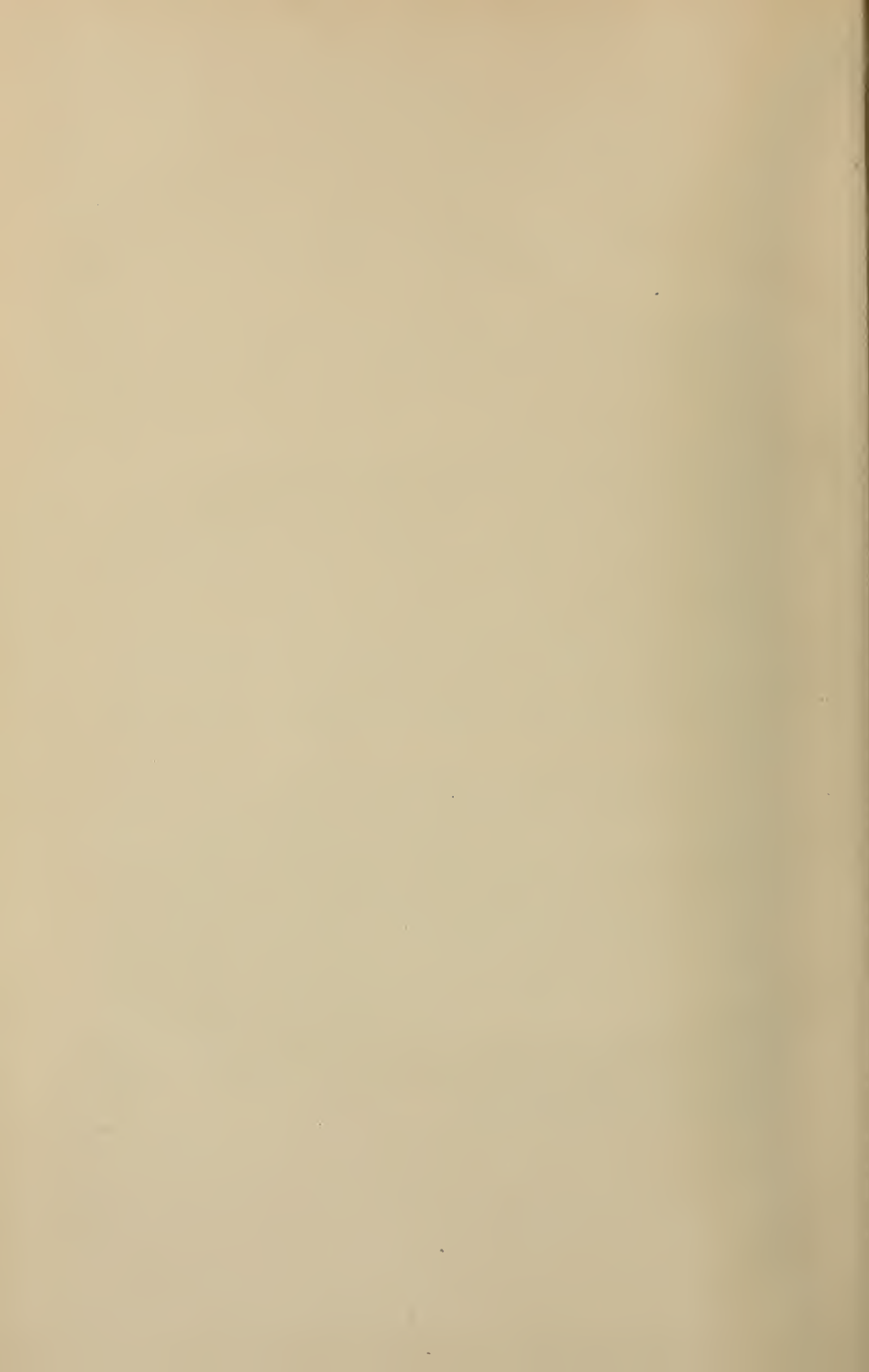
TRANSLATOR'S PREFACE.

If the whole phenomenon transpiring in the bore of the gun at the time of the explosion of the powder charge were capable of expression by a single formula, this formula would contain, among others, terms relating to the action and effects (1) of the solid residues formed from the powder; (2) of the volatile products of decomposition of the powder. Both of these classes of substances occur as decomposition products of smoking powders, while only the latter, volatile decomposition products, occur for smokeless powders. It therefore appears that the study of smoking powders calls for the consideration of more complex formulæ than those relating to smokeless powders. This fact is stated at the beginning of the present work to show why, in studying the action of smokeless powders, a certain attention must be paid to the subject of smoking powders, which, by the manner of their decomposition, represent vastly more complex conditions than those that occur in the case of smokeless powders. The study of the former serves in a measure as a proper introduction to the study of the progressive action of the latter.

The present work has been translated with the kind permission of Maj. Gen. A. Brynk, of the Russian Marine Artillery service, under the approval of the Honorable William H. Moody, Secretary of the U. S. Navy, and at the request of Rear-Admiral Charles O'Neil, Chief of the Bureau of Ordnance of the Navy, to whom the establishment of the manufacture of smokeless powder by the Government is largely due.

In conclusion, the translator desires to extend his thanks to Capt. T. L. Ames, of the U. S. Army Ordnance Corps, who has carefully read and corrected the manuscript of the present translation; also to Lieut. Volney O. Chase, U. S. Navy, Assistant Chief of the Bureau of Ordnance, who has kindly consented to read and correct the proof of the book; a work which the translator was prevented from doing by the receipt of orders to duty afloat.

WASHINGTON, *February 29, 1904.*



AUTHOR'S PREFACE.

When the author consented, in the year 1892, to deliver a course of lectures on Interior Ballistics at the Michaelovsk artillery school, he fully realized the difficulty of the task set before him. The change to smokeless powders—substances that differ markedly in their properties from smoking powders—had then just transpired, but there existed in ordnance literature at home and abroad scarcely any information relating to the new explosives and their mode of action in guns. Ordnance literature was, moreover, meager, in what related to the action of the old types of smoking powders. With the exception of Albinski's manuscript course (1880) and that of Pashkevitch (1890), the only works published were: *The Investigation of Explosive Substances*; the *Action of Inflamed Powder*—[Pashkevitch's translation of Noble's work (1878)]; *Explosive Substances*—[course for the officers of the torpedo class (2 parts, 1880 and 1883), by E. M. Tcheltsov]; *Interior Ballistics and Smokeless Powder and its Influence on Gun Construction*, by D. Longridge, [translated by Captain Nilus (1891)]; *Explosive Substances* by E. M. Tcheltsov (Brockhaus and Ephron's *Encyclopædic Dictionary*, Vol. VI, 1892), besides certain papers on the various properties of powders.

The theoretical part of the present work is based principally on the most recent works of foreign scientists—Sarrau, Vieille, Noble, Berthelot, and others. The author has taken from each such facts as seem to him the best developed and verified, and has established therefrom, as far as he could, certain deductions; while in certain other cases he has supplemented the same by further investigation.

Beginning with the presentation of the general properties of powders as sources of energy, he has considered their action in closed chambers, he has deduced the laws of their combustion and decomposition, and he has indicated the values of powders and of explosive substances in general. Finally, he gives the laws of action of powders in guns, paying heed in this regard to the determination of circumstances most favorable for the utilization of the energy of the powder and to the influence of various elements and circumstances accompanying the action of the powder in relation to the movement of the projectile in the bore of the gun.

It should be remarked that, considering the present state of the science, this work, like all other "theories of the action of powders in guns," represents only the first easy steps in this very important and exceedingly complex branch of the science of ordnance; nevertheless, results already obtained from the labors of various scientists in elucidating the laws of the action of powders in guns are of unquestionable value to the artillerist.

The action of any powder in a gun is based on its physical and chemical properties; therefore, Interior Ballistics represents the development of the application of thermodynamics, the kinetic theory of gases, thermochemistry, etc., to the investigation of such a complex phenomenon as the combustion of powder in the bore of the gun at the time of movement of the projectile therein. This emphasizes the great importance to artillerists of the development of chemical physics—a branch of science that is, moreover, exceedingly interesting in its general relations.

In his desire to make the present work accessible to as large a group of readers as possible, the author has included in the course those laws of thermochemistry that are necessary for illustrating the action of powders and explosive substances in closed chambers and in the bores of guns. The contents of this work are shown in detail in the index, while references, as complete as possible, are given in the text and show the sources from which the information was derived.

In conclusion, the author deems it a pleasant duty to thank Messrs. E. E. Kremkov, N. N. Ivanov, and N. F. Drozdov for reading this manuscript and supplying numerous useful suggestions, and also Mr. A. P. Meller, who drew the perspective view on page 257 of the bore of the gun with progressive rifling.

ST. PETERSBURG, *December 10, 1901.*

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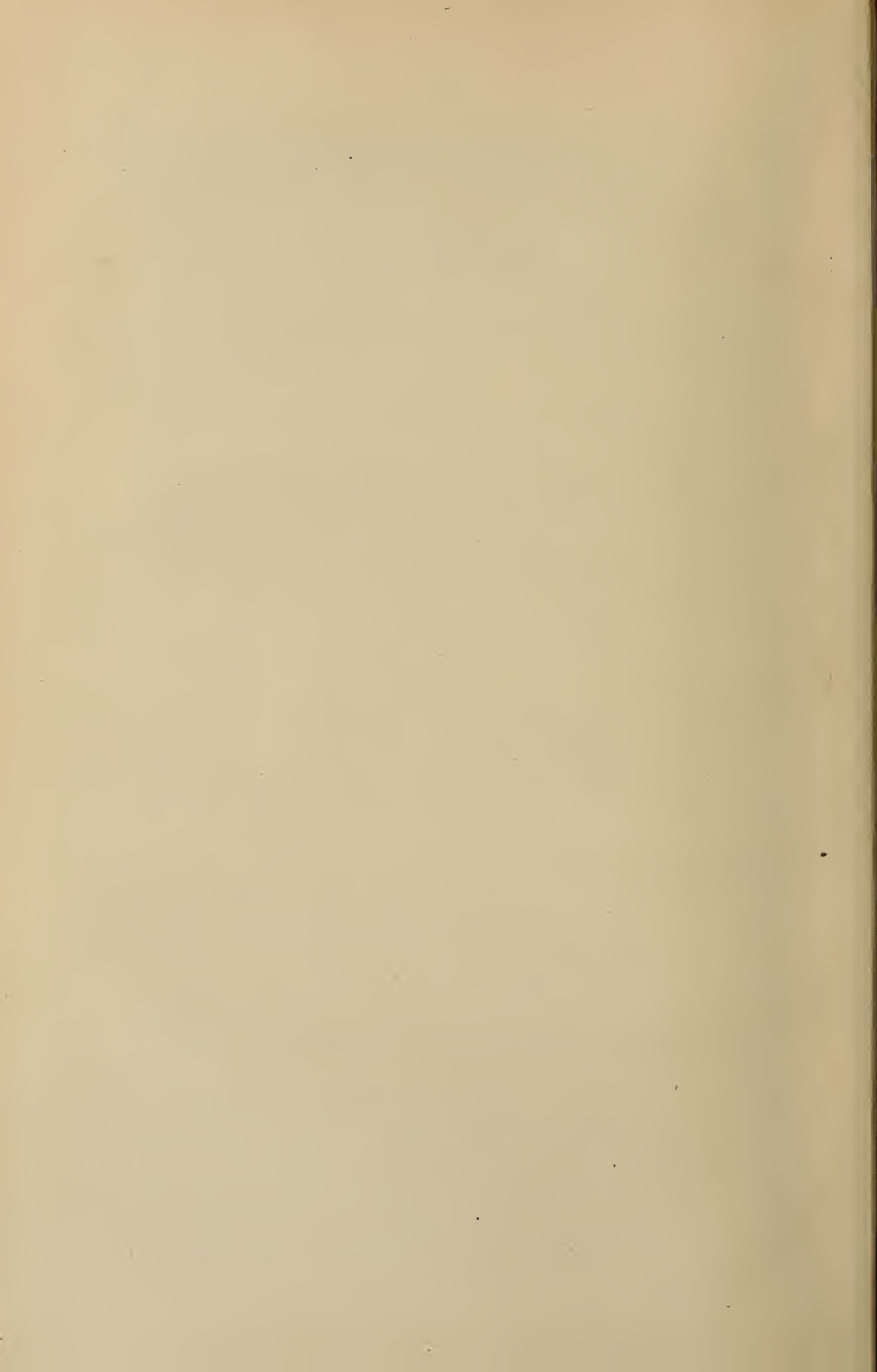
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INTERIOR BALLISTICS

(15)

Chapter I.

DEFINITION OF INTERIOR BALLISTICS.

1. Definition of interior ballistics—Its signification.

Interior Ballistics is that portion of the science of gunnery which deals with the investigation of the phenomena occurring in the gun from the moment of communicating fire to the charge until the moment of development of maximum velocity of the projectile.

The ultimate object of the investigation of these phenomena is the determination of the laws of movement of the projectile and of the distribution of pressures in the bore of the gun at the time of firing. The deep significance of these laws in their relation to ordnance matters is evident. Without a thorough acquaintance with them it is impossible to design a wholly safe gun capable of fulfilling ballistic requirements.

2. Application of the laws of thermodynamics; the kinetic theory of gases and thermochemistry—Application of the fundamental equation of thermodynamics.

The gun presents in itself a very simple form of thermodynamic engine. A portion of the energy evolved on the combustion of the charge in the bore of the gun is converted directly into vis viva, imparted to the projectile in its flight through the bore. In view of this, the laws of thermodynamics, the kinetic theory of gases and thermochemistry serve as fundamental theoretical data for the solution of various problems of interior ballistics, and the fundamental equation of thermodynamics is applicable, as Count Saint-Robert has shown, to the gun*, as follows:

$$Q=A(G+W+I)+P+N$$

where, as applied to the gun, the respective members of the equation possess, for the time t , as reckoned from the beginning of the combustion of the powder, the following significations.

Q = The quantity of heat evolved by the burning of the powder.

A = The heat equivalent of mechanical work; it equals $\frac{1}{J}$, where J is the mechanical equivalent of heat.

* Paul de Saint-Robert's *Principes de Thermodynamique*, 2d ed., 1870, 251 pp., and his *Mémoires Scientifiques*, Vol. II, 1873, p. 189 et seq.

G = The vis viva of the system—of the projectile (driving and rotating movements), of the gun, carriage, etc.

W = The external work performed by the gases in overcoming various harmful resistances, such as the resistance of the air, the friction of the projectile, the friction of products of combustion, etc.

I = The internal work performed by the gases.

P = The quantity of heat absorbed by the body of the gun.

N = The yet-unexpended quantity of heat.

If we determine the values of the separate terms as functions of time for various cases, the problem will be fully solved—that is, we will know how the pressure has acted in various sections of the bore of the gun and also the law of movement of the projectile. To accomplish this, however, it is necessary, first of all, to know the laws of combustion of powder in the bore of the gun.

3. *The complexity of the laws of combustion of powder on firing.*

The combustion of powder, as it takes place in the bore of the gun upon firing, presents an exceedingly complicated phenomenon, and the laws which relate to it are yet far from being thoroughly investigated; so that at the present time the laws of motion of the projectile and that of distribution of pressures are not *exactly* determined.

In view of this it becomes necessary, in order to solve these very important problems, to have resource to experimental data and to more or less probable hypotheses proceeding from more simple phenomena, and supported by more exact investigations.

4. *The order of investigation in interior ballistics.*

As the source of power is to be found in the powder, we shall first of all state the kinds, varieties, and compositions of the powders employed to-day. We shall also mention those qualities possessed by them which have direct influence either upon rapidity of evolution of gases or upon the magnitude of their energy.

Later we shall study products of combustion, pressures, and laws of combustion of powder in the open air and in a closed chamber; after which we shall, as far as possible, apply these data to the development of those laws of movement of the projectile in the bore of the gun which concern us, and to the distribution of resultant pressures.

This will constitute the first part of the course in Interior Ballistics.

In the second part of the course the experiments made for illustrating and verifying the various problems of Interior Ballistics will be described, and also the apparatus serving for this purpose.

Chapter II.

KINDS, VARIETIES, AND COMPOSITION OF POWDERS.

5. *Kinds of powders—Smoking, smokeless, and light-smoke.*

The following two kinds of powders are principally employed to-day for firing from cannon: Smoking and smokeless. Smokeless powders are, in turn, divisible into two principal groups: Pyroxyline and nitroglycerine powders. Powders producing little smoke, such as have been proposed for military purposes (e. g., pyroxyline powders containing barium nitrate), and also smokeless powders containing nitrobenzol $C_6H_5(NO_2)$ (e. g., the American powder of the chemist Munroe known under the name of "indurite" and consisting of pyroxyline gelatinized in nitrobenzol) have not come into general use, and therefore will not be touched upon further than to say that they do not present any specially important features in relation to ballistics.

Each powder is subdivisible into varieties according to composition, density of the grains, shapes, and dimensions.

6. *Composition and varieties of smoking powders—Constitution of the charcoal.*

The most commonly employed *smoking* powder is a mechanical mixture, in various proportions of saltpeter (KNO_3), charcoal, sulphur, and water (H_2O). In some varieties of this powder sulphur does not appear, however (e. g., the chocolate Schlüsselburg powder), while in others a part of the charcoal is replaced by paraffin (e. g., in the French powder A₁₁⁸, used in the 10 cm. gun, 1875 model). The proportions of constituent parts for some of the powders used by us are given in Table I.

The saltpeter and sulphur entering into the composition of the powders correspond almost exactly to the formulæ (KNO_3) and (S_2).

The charcoal entering into their composition varies according to the kind of wood from which it is prepared and the degree of drying to which it has been subjected. Table II furnishes the data in relation to the various kinds of charcoal.

TABLE I.—*Composition, density, and dimensions of Russian smoking powders.*

Kind of powder.	Contents in percentages.*				Limits of density.	Form and dimensions of grains.	Number of grains in 1 pound.
	Salt peter.	Charcoal.	Sulphur.	Moisture.			
(a) With black charcoal	74.33 75	14.86 15	9.91 10	0.9 0	In the form of irregular grains of size:	
Okhta—†							
1. Fine						From $1\frac{1}{2}$ to $\frac{1}{2}$ points. †	
2. Coarse					{ 1.57 1.68 }	From 3 to $1\frac{1}{2}$ points. †	
3. Old small arm †...					{ 1.56 1.63 }	From 5 to 3 points. †	308, 300.00
4. Cannon †					{ 1.55 1.62 }	From 8 to 5 points. †	95, 400.00
5. Coarse grained § ..					{ 1.67 1.71 }	From 4 to 2 lines. ‡	1, 090.00
Prismatic—							
Okhta factory—							
6. Low density †....					{ 1.67 1.71 }	Hexagonal prisms with 7 channels height, 1 in.; diameter of inscribing circle, 1.35 in.; diameter of channels, 0.2 in.	10.42
7. High density †....					{ 1.74 1.77 }		10.02
8. Schlüsselburg factory.	74.32 75	16.85 17	7.93 8	0.9 0	1.718	Hexagonal prisms with 1 channel, dimensions same as powder No. 14.	
(b) With brown charcoal ...	76.23 77	14.85 15	7.92 8	10.0 0	Irregular grains:	
Okhta—							
9. Fine **.....					{ 1.72 1.79 }	From 2 to $\frac{1}{2}$ points.	3, 376, 000.00
10. Coarse					{ 1.73 1.80 }	From 5 to 3 points.	345, 000.00
11. Small arm (new)†.					{ 1.72 1.77 }	From $3\frac{1}{2}$ to 2 points.	1, 170, 000.00
(c) With chocolate charcoal:							
12. Cubical Okhta §...	78.62 75	20.61 21	3.92 4	1.85 0	{ 1.83 1.870 }	Cubical, edge of, 0.372 in.	307.80
13. Prismatic Okhta §.		Same as cubical.				Hexagonal prisms with 1 channel height, 1 in.; diameter of inscribing circle, 1.375 in.; diameter of central orifice, 0.394 in.	9.14
14. Prismatic Schlüsselburg. ††	78.46 80	19.62 20	0	1.92 0	1.859 1.867	Also: Height, 0.997 in.; diameter of inscribing circle, 1.372 in.; diameter of central orifice, 0.394 in.	9.41

* The denominators show the composition of fully dried powders.

† These figures are taken from the Manual of Regulations Governing the Tests of Powders, prepared under the direction of Major-General Kaminsky, inspector of powder factories, ed. 1887.

‡ A small lineal unit = $\frac{1}{16}$ in. circa.

§ These data were furnished by Colonel Lipnitzky, in charge of the division of brown powder at the Okhta powder factories.

|| A small lineal unit = $\frac{1}{16}$ in. circa.

** This is also tubular.

†† Figures taken from the Report of the Naval Technical Committee on ordnance for the year 1890, p. 280 et seq.

TABLE II.—Composition of charcoals.

Kinds of charcoal.	Content, in percentages.				
	C.	O.	H.	As.h.	N.
Black:					
Okhta factory *.....	83.2	11.9	3.5	1.4
Schlüsselburg factory †.....	80.4	14.7	3.3	1.6
Brown: Okhta factory.....	74.8	20.2	4.0	1.2
Chocolate:					
Okhta factory †.....	55.0
Ditténhofer *.....	58.3	33.2	5.6	1.1	1.8
Schlüsselburg factory †.....	55.3	38.2	6.1	.4

*The figures are taken from the paper, "Brown powder," by Professor Zaboudski, in the Artillery Journal, 1885, No. 7.

†Communicated from the factory.

‡Communicated by Colonel Lipnitzky; see footnote (§) to Table I.

7. Certain properties of component parts of smokeless powders.

As various types of nitrocellulose and nitroglycerin are the principal components of smokeless powders it becomes necessary, before entering into the consideration of these powders, to bear in mind the following:

1. Cellulose belongs to the carbohydrates and on account of certain of its reactions may be expressed as a polyatomic alcohol, the simplest composition of which may be represented by the formula $C_6H_{10}O_5$. The compounds, however, which it forms with nitric acid and other bodies force us to consider the composition of cellulose as more complex, namely, as $C_{12}H_{20}O_{10}$, $C_{24}H_{40}O_{20}$, and even as proposed by Professor Mendeleef, as $C_{30}H_{50}O_{25}$; while, perhaps, it actually possesses a still more complex composition.

2. Nitrocellulose is composed of various nitric ethers of cellulose.

3. The composition of nitrocellulose according to Abel is:

Content of nitrogen.

	Per cent of N.
Trinitrocellulose $C_6H_7O_2(NO_3)_3$ insoluble	14. 14
Dinitrocellulose $C_6H_8O_3(NO_3)_2$ soluble	11. 11
Mononitrocellulose $C_6H_9O_4(NO_3)$ soluble	6. 76

which correspond to the formula for cellulose, $C_6H_{10}O_5 = C_6H_7O_2(HO)_3$.

4. According to Eder,* nitrocellulose may possess the following composition:

Content of nitrogen.

	Per cent of N.
Hexanitrocellulose $C_{12}H_{14}O_4(NO_3)_6$ insoluble	14. 14
Pentanitrocellulose $C_{12}H_{15}O_5(NO_3)_5$ soluble	12. 75
Tetranitrocellulose $C_{12}H_{16}O_6(NO_3)_4$ soluble	11. 11
Trinitrocellulose $C_{12}H_{17}O_7(NO_3)_3$ soluble	9. 15
Dinitrocellulose $C_{12}H_{18}O_8(NO_3)_2$ soluble	6. 76

which correspond to a formula for cellulose, $C_{12}H_{20}O_{10} = C_{12}H_{14}O_4(HO)_6$.

* See E. Tchelsov's "Explosive Materials." Course for the torpedo officers' class. Part II, p. 32 et seq., edition of 1883.

5. According to Vieille,* the composition of cellulose is as follows:

TABLE III.

Designation of cellulose.	Composition.	Weight of molecule.	Content of nitrogen—			
			By weight.		By volume.	
			Absolute per molecule.	In per cent.	Cubic centimeters.	Corresponding content of nitrogen.
		Grams.	Grams.			Per cent.
11-nitrocellulose, insoluble	$C_{24}H_{20}O_9(NO_3)_{11}$	1,143	154	13.47	214	13.42
10-nitrocellulose, soluble.....	$C_{24}H_{30}O_{10}(NO_3)_{10}$	1,098	140	12.75	203	12.73
9-nitrocellulose, soluble.....	$C_{24}H_{31}O_{11}(NO_3)_9$	1,053	126	11.97	190	11.92
8-nitrocellulose, soluble.....	$C_{24}H_{32}O_{12}(NO_3)_8$	1,008	112	11.11	178	11.16
7-nitrocellulose, soluble.....	$C_{24}H_{33}O_{13}(NO_3)_7$	963	98	10.20	162	10.16
6-nitrocellulose, soluble.....	$C_{24}H_{34}O_{14}(NO_3)_6$	918	84	9.15	146	9.16
5-nitrocellulose, soluble.....	$C_{24}H_{35}O_{15}(NO_3)_5$	873	70	8.02	128	8.03
4-nitrocellulose, soluble.....	$C_{24}H_{36}O_{16}(NO_3)_4$	828	56	6.76	108	6.77

which correspond to a formula for cellulose, $C_{24}H_{40}O_{20} = C_{24}H_{28}O_8(HO)_{12}$.

6. Eder's nitrocelluloses † and Abel's may be produced in an almost pure state by laboratory methods. Vieille's nitrocellulose consists of a mixture of celluloses of various degrees of nitration, such as are produced on a large scale in manufacture.

7. The solubility of nitrocellulose is usually determined in relation to a mixture of one volume of strong (95°) ethyl alcohol, C_2H_6O with 1.5 to 3 volumes of sulphuric (ethyl) ether, $C_4H_{10}O$. Generally speaking, the solubility of the various kinds of nitrocellulose in other solvents is different from what it is for the mixture above given. For example, all nitrocelluloses are soluble in acetone, C_3H_6O , and in acetic (acetate of ethyl) ether, $C_4H_8O_2$. Some nitrocelluloses (e. g., the tetra-nitro) are soluble in methyl alcohol or wood spirits, CH_4O , others (e. g., Eder's dinitro) in common spirits (ethyl alcohol).

8. The content of nitrogen is indicated either by weight (usually as a percentage), as is shown in columns 5 and 7 of Table III, or by the volume (for a temperature 0° and a barometric pressure of 760 mm. of mercury) in cubic centimeters of nitric oxide (NO), as developed by analysis, e. g., by Schloesing's method ‡ for 1 gram of the material under investigation. (See column 6, Table III.) In order to obtain the percentage composition of nitrogen from the volume content as obtained by analysis, we must multiply the number of cubic centimeters of nitric oxide by $0.062724 = \frac{14 \times 100}{22,320}$, where 14 represents the weight of nitrogen in 1 molecule of nitric oxide, and 22,320 the volume of each

* Taken from Vieille's *Récherches sur le nitrification du coton*; see *Mémorial des Poudres et Salpêtres*, Vol. II, pp. 212, 1884-89.

† With the exception of Eder's trinitrocellulose.

‡ See Vieille's paper referred to in footnote (*) above.

molecule in the gaseous state, expressed in cubic centimeters at a temperature of 0° C and a pressure of 760 mm.

9. Pyrocollodion* constitutes, they say, a special form of nitrocellulose; its composition, after removal of impurities and ash may, according to the experiments of E. and P. Tcheltsov,† be exactly expressed by the formula $C_{24}H_{30}O_{10}(NO_3)_{10}$, which corresponds (see paragraphs 4 and 5) to a content of nitrogen of 12.75 per cent; they found 12.7 per cent nitrogen in pure pyrocollodion. Pyrocollodion in an impure form contains 12.45 per cent of nitrogen and constitutes in this state a mixture of cellulose nitrates more or less rich in nitrogen. Thus, according to the investigations of A. Grigorovitch, $87\frac{1}{2}$ per cent of pyrocollodion is soluble in methylalcohol; 12.55 per cent of nitrogen was found in this part of the product, while 13 per cent of nitrogen was found in the portion insoluble in this spirit. Professor Mendeléef proposed for pyrocollodion the formula $C_{30}H_{33}O_{13}(NO_3)_{12}$ (see paragraph 1), which corresponds to a content of nitrogen of 12.44 per cent and represents only the factory product and not pure pyrocollodion.

10. Glycerin, $C_3H_8O_3 = C_3H_5(HO_3)$, is a triatomic alcohol. Nitroglycerin is the triatomic ether of glycerin, $C_3H_5(NO_3)_3$, and possesses a content of nitrogen of 18.5 per cent. It constitutes in itself a definite chemical compound. There also exists a monatomic nitric ether of glycerin, $C_3H_7O_2(NO_3)$, but it has no practical value, while the diatomic nitric ether of glycerin, $C_3H_6O(NO_3)_2$, has never been isolated. Nitroglycerin is easily soluble in alcohols, in ethers, and in acetone, and slightly so in water. It dissolves collodion cotton. On account of this property it was originally employed in the preparation of Noble's dynamite—explosive gelatine.‡

This latter material consists of from 90 to 93 per cent of nitroglycerin and from 10 to 7 per cent of collodion, and its consistency, as is stated below in section 8, is that of a nitroglycerin jelly of collodion. Finally, nitroglycerin in the presence of such solvents as acetone, ether, etc., converts pyroxyline into a nitroglycerin jelly, which retains its consistency after the expulsion of the solvent.

8. *Pyroxyline powders.*§

Pyroxyline powders consist for the greater part of a more or less homogeneous mixture of gelatinized collodion (soluble nitrocellulose) with unaltered pyroxyline (insoluble nitrocellulose), the particles of which are suspended throughout the mass of the colloid. In relation to their consistency these powders may be described as jellies|| (col-

* Proposed by Professor Mendeléef.

† See Brockhaus and Ephron's Encyclopædic Dictionary, Vol. XXI, page 180.

‡ See Brockhaus and Ephron's Encyclopædic Dictionary, Vol. X, p. 624.

§ This was the first smokeless powder in actual use. Proposed by Vieille in 1884.

|| See Brockhaus and Ephron's Encyclopædic Dictionary, Vol. VIII, p. 649.

loids) which constitute, like all colloids, indeterminate chemical combinations of solvents and nitrocellulose.

The usual pyroxyline powders are gelatinized in the ether-alcohol mixture, and therefore constitute a combination of ether-alcohol collodion with the unaltered particles of pyroxyline. There also exists powders consisting of an almost pure ether-alcohol collodion of pyrocollodion (proposed by Professor Mendeléef and E. Tcheltsov); there are also acetone colloids and colloids of nitrocellulose in acetic ether. In view of the above, pyroxyline powders contain, besides the colloided and uncolloided celluloses, a portion of the solvent employed to effect gelatinization.

These colloids attract moisture to themselves in greater or less degree, and therefore pyroxyline powders contain certain quantities of the same. A distinction is made in these powders between what is called the volatiles "expelled by drying" (heating at 100° C. for from four to six hours—the investigations of A. Sapojnikov have shown that they consist practically of moisture), and the "residual volatiles" (solvent remaining in the powders), which are determined by regelatinization. The amounts of these quantities are usually given together.

Finally, to increase the stability of the powders, small quantities of the following substances are sometimes added to the solvent: Amyl-ether, $C_5H_{12}O$; urea, $CO(NH_2)_2$; diphenylamine, $NH(C_6H_5)_2$, etc.

The approximate composition of pyroxyline powders is shown in Table IV.

9. Nitroglycerin powders.

Nitroglycerin powders represent either a nitroglycerin colloid of soluble nitrocellulose (Ballistite*; German cannon powder, Maxim & Schupphaus powder, † etc.); of pyroxyline (Cordite); ‡ or else a mixture of both nitrocelluloses together. Vaseline (a mixture of liquid and solid hydrocarbons), urea, paraffin (a mixture of solid hydrocarbons of the series C_nH_{2n+2}), graphite, barium nitrate, $Ba(NO_3)_2$, etc., are sometimes added to these powders. The quantity of moisture and volatile substances present is usually small. In view of what has been said in relation to them, they approximate in composition to explosive gelatin, but differ from the latter in possessing a larger content of nitrocellulose, in consequence of which they are of harder consistency. The composition of these powders is shown in Table V.

* Proposed by Noble.

† American.

‡ English powder "cordite," proposed by the scientists Abel and Dewar.

TABLE IV.—Composition of pyroxyline powders (approximate).

Designation of kind of powder.	Content in per cent.					Form and dimensions of grains.	Number of grains in 1 pound.	Content of nitrogen.		
	Nitrocellulose.		Moisture and solvent.	Ash.	Density of powder.			In the powder.	In nitro-cellulose.	
	Soluble.	Insoluble.							Soluble.	Insoluble.
	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	Strips, ribbons, or else in the form of cylindrical tubes of various dimensions; their thickness varies between 0.25 and 5 mm.	From $\frac{1}{8}$ to 22,000.	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>
Sample A *.....	33.88	62.40	3.02	0.70	1.600			12.26	12.23	13.01
Sample B	28.66	67.94	2.90	.50	1.610			12.37	11.60	13.31
Sample C *.....	33.36	61.94	4.00	.70	1.600			12.14	12.23	13.01
Sample D †.....	41.54	50.76	7.00	.70	1.550			11.68		
Sample E †.....	41.08	50.22	8.00	.70	1.530			11.56		
Sample F †.....	40.64	49.66	9.00	.70	1.510			11.43		
Sample G	24.78	68.11	5.85	1.26	1.590			11.97	11.85	13.24
Sample H	92.26	4.58	2.72	.41	1.619			12.15	12.49	13.72
Sample I ‡.....	89.90	5.27	4.51	.32	1.590			11.82	12.38	13.31
Sample J ‡.....	88.19	5.48	6.33			11.65	12.48	13.36
Sample K §.....	72.65	22.98	4.03	1.605			12.34	12.76	13.51
Sample L	40.86	55.65	2.69	.80	1.590			11.23	10.59	12.44
Sample M	45.23	50.23	3.90	.64	1.580	10.91	10.65	12.13		

Composition of fully dried powder, without ash, gives:

* Soluble nitrocellulose, 35 per cent; insoluble, 65 per cent.

† Soluble nitrocellulose, 45 per cent; insoluble, 55 per cent.

‡ Soluble nitrocellulose, 94 per cent to 95 per cent; insoluble, 6 to 5 per cent.

§ Soluble nitrocellulose, 74 per cent; insoluble, 26 per cent.

In consequence of the secrecy of the matter this table does not give the composition of the pyroxyline powders used by us, but the varieties contained in it are sufficient to establish a conception of powders of this class in general.*

* See Brockhaus and Ephron's Encyclopædic Dictionary, Vol. XXIV, p. 589 et seq.—the article on "Powder," by E. N. Tchelstov.

TABLE V.—*Composition of nitroglycerin powders.*

Kinds of powder.	Content.							Nitrogen content—					
	Nitro-glycerin.	Nitrocellulose.		Moisture and volatiles.	Ash.	Other ingredients.	Density.	Form and dimensions of grains in millimeters.	Number of grains per kilogram.	In the powder.	In nitro-glycerin.	In the nitrocellulose—	
		Soluble.	Insoluble.									Soluble.	Insoluble.
German, for 80-mm. gun *.	<i>P. et.</i> 42.94	<i>P. et.</i> 56.00	<i>P. et.</i> 0.40	<i>P. et.</i> 0.46	<i>P. et.</i> ‡0.20	<i>P. et.</i> 1.65	Paralleloiped, $2 \times 8 \times 8$...	4,949	<i>P. et.</i> 14.64	<i>P. et.</i> 18.12	<i>P. et.</i> 11.91
Ballistite (small arm) *.....	45.64	53.6251	.22	{ Cubical, from 1 to 1.25 mm. on edge. Different dimensions of cylinders, 5 diameters long, with 6 transverse channels from 1 to 3 mm. diameter.	{ 600,000	14.86	18.10	12.33
Maxim and Schüpphaus †..	11.00	84.00	§3.00	1.00	1.00	1.60	12.89	16.50	12.83
Cardite *.....	57.54	2.24	34.65	.18	.29	¶5.10	1.55	15.03	18.05	11.37	12.61
Nobel, ballistite *.....	48.50	50.5050	.50	1.64	{ In form of rods, (1) cylindrical and (2) of square section of different diameters.	14.76	18.07	11.27

* Based on the determination of Colonel Professor Zaboudski.

† Graphite.

§ Acetone.

|| Urea.

¶ Vasoline.

10. Density of powder.

The *density of powder* is considered, as is known, as *gravimetric* and *actual*, or simple density.

Gravimetric density is the weight in kilograms of 1 liter of loosely heaped-up powder.

Powder, generally speaking, is not homogeneous as to density of grains, and therefore the *density of powder* denotes the mean density of its grains. In corresponding metric units (ton-meter, kilogram-decimeter, and gram-centimeter) the density of substances is measured by the weight of the unit volume of the material and constitutes a special physical unit, viz., $\left[\frac{\text{unit mass}}{(\text{unit length})^3} \right]^*$.

For other units the density δ is measured by the ratio between the weights, as ascertained in the locality, p — of an arbitrary volume of the material and p_1 , of the same volume of water; i. e., $\delta = \frac{p}{p_1}$. Water approximately at 4° C. is taken as the unit of density. Tabulated densities of bodies are given for a temperature of 0° , while the density of powders is established between the limits of 15° and 25° C., in comparison with the stated unit of density.

The density of powder depends upon its kind and varies between limits of 1.5 to 1.9 for smoking and 1.5 to 1.65 for smokeless powders. Generally speaking, the density of the latter is less than the density of smoking powders, and the density of nitroglycerin powders is greater than that of pyroxyline varieties (see Tables I, IV, and V). It should be observed that the density varies with the change of the amount of moisture in the powder. The density of smoking powders containing black charcoal diminishes, while that of the chocolate powders increases in proportion to the increase of moisture, which difference is due to the peculiarities of the charcoal used, as the former expands and the latter contracts with the increase in moisture.

The density of pyroxyline powders diminishes with the increase in their content of solvent. This change proceeds in accordance with a well-established law, so that if we know the amount of solvent in the powder we may determine the density of the powder, and conversely. In the case of pyroxyline powders, gelatinized in ether alcohol, the relation between these quantities may be expressed by the following empirical formula:†

$$\delta = 1.644 - 0.012 h$$

where δ denotes the density of the powder and h the content of solvent expressed as a percentage. Knowing the density of powder, then, it

* See Chvolson: "Course in Physics," Vol. I, pp. 14, 30, etc.

† Communicated by Colonel Kisnemy, president of the Experimental Committee at the Okhta works.

is easy to ascertain its specific volume (v)—i. e., the volume occupied by the unit weight of the powder; for this we have $v = \frac{1}{\delta}$.

11. *Forms and dimensions of powder grains.*

The forms and dimensions of powder grains differ greatly. Smoking powder used at present generally possesses one of the following forms: angular and irregular (generally granular), cubical, irregular parallelopipedic, and prismatic with one or more axial channels. (See Table I.)

Pyroxyline powder (see Table IV) is usually in the form of irregular, thin flakes (for small arms); threads (for small guns); ribbons of various thicknesses, breadths, and lengths, and tubes (macaroni powder) of various dimensions (for cannon).

Nitroglycerin powders (see Table V) are prepared either in the form of parallelopipeds, cubes, flakes (German and others), or as cylindrical rods and threads of various thicknesses and lengths (cordite), or in the form of rods of square section (ballistite), or as tubes (German powder), or else in the form of small cylinders with a great number of transverse channels in the direction of the axis (multiperforated grains, such as the Maxim and Schüpphaus powder), or in the form of disks like the plates of Belleville springs.

It should be observed, however, that the forms of both of these kinds of smokeless powder may be identical.

In consequence of irregularities in manufacture the shapes of all grains vary between certain limits, so that in making various calculations in relation to the quantity of powder consumed it becomes necessary to assume a certain mean grain and substitute regular and more simple for irregular and complex shapes. Thus, spherical grains replace angular ones, regular parallelopipeds of square section replace irregular parallelopipeds, cylindrical grains with a central channel replace prismatic grains with one channel, etc. However, in making such changes the following conditions must be observed: The least dimension of the grain—i. e., its mean thickness (except for angular grains, for which this dimension is calculated), the number of grains in the unit weight, and the density of the original grains and those substituted for them must always remain the same.

If these conditions be observed the surfaces of the original grains and of the substitutes will be pretty nearly identical.

Let the number of grains in the unit weight be N , the actual density of the grains, δ and the mean thickness of the grain (i. e., the mean value of the least dimension of the grain) be a .

In view of what has already been said above, the dimensions and the form of certain mean grains will be:

(a) For angular grains, the radius r of the mean spherical grain is

$$(1) \quad r = 10^{\overline{1.792637}} (\delta N)^{-\frac{1}{3}} \cong 0.62 (\delta N)^{-\frac{1}{3}}$$

since the theoretical weight of N mean grains, $\frac{4}{3} \pi r^3 \delta N$, in metric units, should equal unity.

(b) For irregular parallelopipeds and cubical grains the edge e of the mean grain becomes

$$(2) \quad e = (\alpha \delta N)^{-\frac{1}{2}}$$

under the condition that $e > a$.

(c) For prismatic grains of height h with one channel, we have

$$(3) \quad h = 10^{\overline{1.502850}} [(r^2 - r'^2) \delta N]^{-1} \cong 0.32 [(r^2 - r'^2) \delta N]^{-1}$$

for $h > (r - r' = a)$

where r is the mean radius of the circle described in the base of the prism, and r' the mean radius of its central orifice.

If the number of grains N be determined for the kilogram, then the unit of length used in the above formula must be the decimeter, and if N be taken as the number in 1 gram, the unit must be the centimeter.

Chapter III.

THE PRODUCTS OF COMBUSTION OF POWDER; THEIR TEMPERATURE, QUANTITY OF HEAT DEVELOPED BY COMBUSTION, AND THE PRESSURE OF POWDER GASES IN A CLOSED CHAMBER.

12. Investigation of the phenomena accompanying the burning of powder.

Powders decompose on combustion, forming what may be described in general terms as highly compressed and heated gases endeavoring to expand. In addition to the gases some varieties of powder form upon combustion nonvolatile products—e. g., the smoking powders employed to-day produce at the temperature of their decomposition both gaseous and liquid products, while pyroxyline powders, under suitable conditions of decomposition, form products that are almost wholly gaseous.

The quantity of gases formed upon combustion of powder, their nature and the amount of heat evolved by them possess a vast significance in interior ballistics, as from them are chiefly determined the pressure of the gases and the amount of energy stored up in the powder. Unfortunately these elements are not yet investigated with sufficient exactness, either in a theoretical or a practical way, notwithstanding the fact that their study, which is important both in its scientific and military relations, has attracted, and still continues to attract, the attention of the most renowned scientists of the whole world. It is enough to recall the well-known names of Robins, Newton, Gay-Lussac, Bunsen, Shishkoff, and others, while at the present day Abel, Berthelot, Dewar, Noble, Sarrau, Vieille, Fedorov, Mendeléef, P. Tchelsov, and others, are occupied with the theoretical and practical study of these questions. The complete theoretical solution of the problem would appear impossible under scientific conditions that exist to-day, and we are, therefore, compelled to resort to the study of data obtained by experiment, and subsequently to endeavor to illuminate these data through theoretical investigations.

The old smoking powder has been the variety most thoroughly investigated. It was studied as far back as the beginning of the eighteenth century, but up to the year 1823 all that was known about it was that the gases formed upon the combustion of the powder occupied, at a temperature of 0° and a pressure of 760 mm., a volume from

230 to 400 times greater than that of the powder itself, assuming that each gram of the latter occupied on exploding a volume of 1 cubic centimeter. There was, however, hardly an idea existing concerning the liquid products formed at the moment of decomposition of the powder or in relation to the composition of the gases evolved.

Gay-Lussac, in the year 1823, was the first to firmly establish the fact that when powder of the usual type was burned there was formed the gaseous elements carbonic acid, carbonic oxide, nitrogen, the solid substances sulphate and carbonate of potash, and the mono and poly sulphides of potassium; also, that the volume of the gases evolved was about 250 times that of the powder. From this time on more exact studies of decomposition products of powder began to appear, but all these experiments, including those of Gay-Lussac, were conducted at a pressure about equal to that of the atmosphere—e. g., those conducted by Chevreul (1825), Bunsen and Shishkoff (the most complete) in 1857, and Link in 1858. But inasmuch as the decomposition of the powder took place in cannon at elevated temperatures and pressures the necessity of carrying on experiments under similar conditions became evident. A few experiments were undertaken by different people to determine the products of decomposition of powder at various pressures—e. g., the experiments of Karolyi (1863), in bombs, and of our own Colonel (now General) Federov (1868), who investigated the solid products of combustion collected after firing blank charges and charges with projectile from a 4-pounder gun and from a small-arm.*

In the same year the experiments of Noble and Abel were begun which still remain the most complete upon this subject.

After the publication of the first series of their results in 1875,† hitherto existing views upon these subjects changed in many respects and their experiments served as a basis for the solution of many questions of interior ballistics, and (generally) for determining the laws of formation of products of decomposition of a given powder. Some errors relating to these products crept into their data (such as that of assuming as a decomposition product the potassium thiosulphate $K_2S_2O_3$), but finally, thanks to the teachings of Berthelot, and especially of Debus, these errors were corrected. So that from the year 1880,‡ the composition of the products of combustion of smoking powders may be

* For further reference to these experiments see Colonel Federov's papers published in the Artillery Journal for the year 1868, entitled "The composition of powder residue and the products of the decomposition of powder under various conditions of pressure."

† See "Researches on Explosives—Fired Gunpowder," by Captain Noble and F. Abel, ed. 1875; and also in Russian, "Investigation of Explosive Substances, Action of Ignited Powder," by Noble and Abel, translated by V. A. Pashkevitch, 1878.

‡ See "Researches on Explosives II—Fired Gunpowder," by Captain Noble and F. Abel, ed. 1880, and also No. 1, 1880, of the journal "Philosophical Transactions of the Royal Society" for 1880.

regarded as determined with greater or less exactness, and their nature made known to all.

In addition to the above, analogous experiments were conducted by Tromenec, Roux, Sarrau, and Vieille, in France.

In order to indicate the whole importance of Noble and Abel's experiments we shall state that they illustrated more or less successfully:

(1) The composition of the products of combustion of powder; (2) the quantity of the gaseous products and their volume at 0° and 760 mm. pressure of the mercurial column; (3) the quantity of the nongaseous products at ordinary temperatures; (4) the condition at which the nongaseous products exist at the moment of decomposition; (5) the quantity of heat developed by the combustion of the unit weight of powder; (6) the mean specific heat of the products of decomposition at constant volume; (7) the temperature of the products of decomposition; (8) the relation between pressure of products of decomposition and density of loading; (9) the change produced in the products of decomposition through change of density of loading; (10) the influence of the chemical composition of powder on the products of decomposition and upon their temperature and pressure; (11) the influence upon all the above of dimensions of grains, their density, moisture, etc.

These interesting experiments were performed with closed bombs so constructed that the gases formed could be transferred to a gasometer for measuring their volume and for analysis. The bomb was provided with an apparatus for ascertaining the pressure. The experiments of Vieille and others determined not only the magnitude of the pressure but also the character of its development and the interval of time during which this development occurred, to ascertain which the stem of the pressure gauge recorded its movement upon a drum revolving at a known velocity.

The solid residues of the powder were examined and studied separately. In determining the quantity of heat evolved the powder was exploded in specially constructed calorimetric bombs placed in calorimeters so arranged that the products of combustion could be removed subsequently from the bombs for the purpose of analysis. A description of the apparatus employed in these experiments will be found in Part II of the present work.

We are able to present only very little data in relation to the newly-introduced smokeless powders on account of a certain secrecy attached to the powder question. For this reason we shall, in the investigation of smokeless powder, limit our consideration to certain data which possess a general importance and which are necessary for illustrating the differences in the character of the action of smoking and smokeless powders.

Generally speaking, all that is said in this Course in relation to the

products of decomposition of powder is based on the experiments of Noble and Abel, Sarrau, Vieille, Macnab and Ristori, our own Scientifico-Technical Laboratory of the naval service, the laboratory of the Michelsonsk Artillery Academy, and of the Okhta powder factory.

The laws of combustion of powder grains—that is, the order of combustion of their various parts under various conditions of burning—also possess a primary importance in interior ballistics. These laws have been recently determined by the French scientist, Vieille, whose labors* form the basis of Chapter V of the present work.

13. *The density of loading and decomposition products.*

By density of loading we denote the relation of the weight of the powder to the weight of water occupying the volume of the chamber in which the powder is placed.

If we denote the density of loading by Δ , the weight of the charge by ω , the volume which it occupies by V , and the weight of the unit volume of water by γ , then $\Delta = \frac{\omega}{V\gamma}$

For English units, the pound and the inch, we have $\gamma = 0.03611$ pound, whence $\Delta = \frac{\omega}{0.03611 V} = 27.7 \frac{\omega}{V}$

For a density of loading, $\Delta = 1$, $V = 27.7 \omega$, that is, each pound of powder at a density of loading 1, occupies a volume of 27.7 cubic inches. Under the metric system the density of loading is determined very simply by the relation of the weight of powder to the volume occupied by it, expressed in corresponding units.

By the *density of the decomposition products of powder* is denoted the ratio of the weight of all the products of combustion, liquid and gaseous—i. e., of the burned powder to the weight of the water occupying the volume in which the products of combustion are inclosed.

When powder is exploded in a closed chamber the density of the decomposition products, in case that the whole charge of powder is consumed, equals the density of loading. These terms differ, however, when they are applied to the gun itself, and correspond only up to the moment of movement of the shell—i. e., for cases where the powder may be said to burn instantaneously. The greatest possible (theoretical) density of loading and density of products of decomposition are evidently equal to the density of powder. (See paragraph 10.)

14. *The composition of powders investigated by Noble and Abel.*

The study of smoking powders was conducted by Noble and Abel with powders of a great many varieties, whose compositions are shown in Table VI.

* M. Vieille. "Mode de la combustion de matières explosives," published in the *Mémorial des Poudres et Salpêtres*, Vol. VI, 1893.

TABLE VI.—*Composition of powders experimented with by Noble.*

Component parts.	Powder of mark—				Chocolate powder.	Waltham Abbey—			Spanish spherical.	Curtis & Harvey's—	
	A.	B.	C.	D.		Pebble.	R. L. G.	F. G.		Hunting, No. 6.	Mining.
	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>
Saltpeter	81.30	77.83	68.74	77.24	78.83	74.76	74.36	73.91	75.59	74.68	61.92
Sulphur18	.28	14.69	6.15	2.04	10.07	10.09	10.02	12.42	10.37	15.06
Charcoal	16.71	19.72	20.18	15.43	17.50	14.22	14.29	14.59	11.34	13.78	21.41
Moisture.....	1.81	2.17	1.39	1.18	1.33	.95	1.06	1.48	.65	1.17	1.61

The properties of charcoals, which are given only for a few powders, are shown in Table VII.

TABLE VII.—*Compositions of the charcoals in certain powders experimented with by Noble.*

Component parts.	Waltham Abbey—			Spanish spherical.	Curtis & Harvey's—	
	Pebble.	R. L. G.	F. G.		Hunting, No. 6.	Mining.
	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>	<i>P. ct.</i>
Carbon.....	85.26	80.32	77.88	76.29	77.36	83.74
Hydrogen.....	2.98	3.08	3.37	3.31	3.77	3.07
Oxygen.....	10.16	14.75	17.60	14.87	16.62	10.45
Ash.....	1.60	1.85	1.15	5.53	2.25	2.74

Comparing the composition of these powders with those used by us (Tables I and II), we see that the Waltham Abbey powder alone approximates to our black powders.

The dimensions and density of grain of some of these powders is shown below. Pebble, almost cubical, edge from three-fourths to three-eighths of an inch; density, 1.75.

R. L. G., coarse grain, dimensions of grains from 0.25 of an inch to 0.125 of an inch; density, 1.7.

F. G., small grain, dimensions from one-sixteenth to one-thirty-sixth; density, about 1.6.

Chocolate prismatic powder, height of prism 24.4 mm.; diameter of circle described in its base, 34.7 mm.; diameter of the central channel, 10 mm.; density, not less than 1.8.

The data presented in relation to the kinds of powder experimented with by Noble show that the powders P., R. L. G., and F. G. differed principally in the composition of their charcoal, but that this difference might be neglected. The content of saltpeter, sulphur, and charcoal in these powders is almost the same, so that we may consider the powders identical in chemical relations, but differing in what relates to densities and dimensions of the grains, in which respect we may also consider the powders P. and R. L. G. as homogeneous in relation to density.

15. *The composition of the products of combustion of smoking powders; formation of these products—Their change with change in density of loading.*

Table VIII, as given below, illustrates* the composition of the products of combustion and their ratios by weight. In the four last columns are given the weight ratios of all decomposition products, gaseous, solid liquid, and also the volume of the gaseous products reduced to a temperature of 0° C., and to a barometric pressure of 760 mm. The data for the powders Pebble, R. L. G., and F. G. are deduced from the mean of 10 observations for each; for densities of loading ranging from 0.10 to 0.90.

*See Captain Noble, "Internal Ballistics," being a lecture in honor of the birth of James Watt, London, 1892.

TABLE VIII.—Composition of products of decomposition and their percentage composition.

Kinds of powder.	Gaseous products.						Water, H ₂ O.		Solid products.						In all.			Vol- ume of gas- eous prod- ucts in cu. cm.
	CO ₂	CO	N ₂	H ₂ S	CH ₄	H ₂	O ₂	Formed.	Occurring in the powder.	K ₂ CO ₃	K ₂ SO ₄	K ₂ S ₂ O ₃	K ₂ S	KCNS	KNO ₃	(NH ₄) ₂ CO ₃	F ₂	
	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.			P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	
	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.			P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	P. ct.	
A	22.53	5.29	11.58	0.22	2.25	1.81	54.74	0.17	0.48	0.34	0.58	241.2
B	19.15	10.14	12.00	0.19	.4226	2.17	52.96	.06	.08	.06	2.40	308.2
C	27.23	13.15	8.75	1.91	.56	.11	1.39	20.36	.42	.73	16.46	1.8781	6.27	342.2
D	24.67	5.29	11.88	.91	.06	.11	1.18	45.79	2.30	.29	3.82	.0516	3.52	278.6
P	21.98	.86	10.4904	.06	8.32	1.33	43.60	13.32	195.4
Chocolate	26.85	4.77	11.23	1.11	.06	.0695	32.58	7.10	10.42	.14	0.13	.05	4.45	275.7
R. L. G.	26.30	4.22	11.17	1.09	.08	.09	0.02	1.06	34.15	8.44	8.07	.13	.15	.04	4.90	271.3
F. G.	26.89	3.55	11.23	1.01	.04	.07	.03	1.48	28.61	12.52	9.99	.07	.09	.03	3.81	259.2
Spanish *	24.57	1.36	11.08	.9603	.0765	21.86	29.75	4.73	.03	.58	.02	4.31	232.7
Factory of Curtis & Harvey:																		
Okhta, No. 6 † ..	25.93	2.47	11.32	.83	.46	.08	1.17	34.13	12.50	7.1717	.05	3.72	238.2
Mining †	22.79	15.22	8.58	3.89	.70	.17	1.61	19.45	.28	17.45	1.39	.04	.84	6.64	354.6

* For a density of loading of 0.70.

† For a density of loading of 0.30.

As a result of the examination of data in Table VIII (p. 36), we may conclude that the course of chemical reaction of decomposition of smoking powders under pressure is as follows: The saltpeter decomposes into K_2O , N_2 , and O_2 . The free O_2 unites with S_2 and the C of the powder, forming SO_3 , CO_2 , and CO. The oxide of potash unites with the SO_3 and CO_2 , giving K_2SO_4 and K_2CO_3 . In addition to these in the case of some varieties of powder there are formed K_2S , K_2S_2 , and other compounds of potassium and sulphur. Part of the water existed originally in the powder in the form of moisture, and part is formed through action of the oxygen on the hydrogen of the charcoal.

Thus the principal products of decomposition are CO_2 , CO, N_2 , H_2O , K_2CO_3 , K_2SO_4 , and K_2S . The latter, however, was not observed in any case in chocolate powder.

Other products* of decomposition occur more or less by chance, partly as the result of secondary combinations occurring upon cooling. A certain quantity of free sulphur is found in the form of various potassium and sulphur compounds or in the form of sulphide of iron, FeS , produced by action of the sulphur on the walls of the gun. As resulting from purely accidental conditions in individual cases, undecomposed particles of KNO_3 have been found and also free C. Oxide of potassium may occur in small quantity, partly in consequence of a certain dissociation of products of decomposition, and following rapid cooling of the same which does not permit it (K_2O) to unite with CO_2 or SO_3 , and partly from saltpeter isolated by chance in the mass and decomposed at a high temperature, from which free oxygen also may result. The substances H_2 and CH_4 are formed from the component parts of the charcoal of the powder.

Cyanide of potassium, KCN, results from the action of an excess of charcoal on the saltpeter. A part of it oxidizes, and thus leads to the formation of cyanate of potash, KCNO, which is decomposed by the steam, giving rise in this manner to the formation of carbonate of ammonia $(NH_4)_2CO_3$. By the action of H_2O and CO_2 on K_2S , K_2S_2 and other sulphur compounds of K_2 sulphuretted hydrogen, H_2S may result. Finally, traces of potassium thiosulphate, $K_2S_2O_3$, have been obtained, which were doubtless formed during the cooling of the decomposition products, as this salt decomposes at a temperature of $450^\circ C$.

Beside the original composition of the powder, the dimensions and density of its grains and the density of loading exercise an influence upon the relative quantities of the various decomposition products formed.

* Berthelot: "Sur le Force," etc., Vol. II, pp. 290-293.

In Tables IX and X* are shown the results of the analysis of decomposition products of the powders P, R. L. G., and F. G., and also of other powders for different densities of loading.

Table IX gives the actual weights of decomposition products of one gram of powder, and Table X shows (*a*) the relative volumes expressed as percentages, of each of the gaseous products, and (*b*) the relative weights expressed as percentages, of each of the solid products of decomposition.

It is evident from Tables VIII, IX, and X that the composition of the products of combustion and the quantity of each of them depend primarily upon the composition of the powder itself†; only 33 per cent of the chocolate powders are really converted into gases with a volume of 198 cc.; 38 per cent of the Spanish powder with a volume of 234 cc.; 44 per cent of black P powder with a volume of 278 cc.; and 51 per cent of mining powder which evolves 360 cc. of gases. It is to be remarked that the powders P, R. L. G., and F. G., which are approximately of the same composition but which differ in density and dimension of grains, produce markedly different quantities of decomposition products; thus, regarding powder as completely dried and the water in a liquid state, P gives 278 cc. of gases, R. L. G. gives 274 cc. of gases, and F. G. gives 263 cc.—i. e., it would appear that with the diminution in the dimensions and density of grains the volume of gaseous decomposition products evolved also diminishes.

* See Tables XII and XIII in "Researches on Explosives," Part II, "Fired Gunpowder," by Captain Noble and F. Abel, 1880.

† Considering the powder as completely dried.

TABLE IX.—*Weight of decomposition products of one*

Kinds of powder.	Mean densities of loading.	Weight of products of decomposition in one gram.								Water.
		Gaseous.								
		Car-bonic acid.	Car-bonic oxide.	Nitro-gen.	Sul-phu-reted hydro-gen.	Marsh gas.	Hydro-gen.	Oxy-gen.		
		CO ₂	CO	N ₂	SH ₂	CH ₄	H ₂	O ₂	H ₂ O	
Pebble	0.10	0.2634	0.0530	0.1176	0.0137	0.0007	0.0095	
	.20	.2505	.0572	.1114	.018300090095	
	.30	.2609	.0548	.1120	.012400070095	
	.40	.2683	.0481	.1109	.0071	0.0007	.00060095	
	.50	.2768	.0472	.1137	.0084	.0012	.00050095	
	.60	.2695	.0477	.1103	.0094	.0011	.00050095	
	.70	.2748	.0438	.1124	.0135	.0007	.00050095	
	.80	.2785	.0409	.1121	.0082	.0007	.00040095	
	.90	.2743	.0367	.1103	.0087	.0014	.00050095	
Mean2685	.0477	.1123	.0111	.0006	.00060095	
Maximum2785	.0572	.1176	.0183	.0014	.00090095	
Minimum2505	.0367	.1103	.007100040095	
R. L. G.10	.2653	.0309	.1226	.0170	.0006	.00050111	
	.20	.2497	.0391	.1198	.0148	.0001	.0006	0.0022	.0111	
	.30	.2633	.0394	.1119	.012800070111	
	.40	.2702	.0371	.1172	.0080	.0006	.00060111	
	.50	.2601	.0581	.1053	.0068	.0017	.00070112	
	.60	.2480	.0495	.1101	.0177	.0007	.00050111	
	.60	.2672	.0471	.1074	.0067	.0007	.00050111	
	.70	.2655	.0454	.1035	.0118	.0011	.00040111	
	.80	.2651	.0397	.1051	.0062	.0014	.00400111	
	.90	.2760	.0358	.1089	.0077	.0015	.00030111	
Mean2630	.0422	.1117	.0109	.0008	.0009	.0002	.0111	
Maximum2760	.0581	.1226	.0177	.0017	.0040	.0022	.0112	
Minimum2480	.0309	.1051	.0062	.0001	.00030111	
F. G.10	.2512	.0580	.1163	.0098	.0004	.0010	.0006	.0148	
	.20	.2490	.0413	.1081	.015300100148	
	.30	.2621	.0374	.1065	.00890008	.0010	.0148	
	.40	.2765	.0350	.1105	.008200080148	
	.50	.2665	.0344	.1102	.0089	.0005	.00070148	
	.60	.2782	.0327	.1133	.0093	.0003	.00070148	
	.70	.2804	.0266	.1152	.0083	.0010	.0005	.0006	.0148	
	.80	.2752	.0281	.1171	.0107	.0008	.0005	.0007	.0148	
	.90	.2812	.0259	.1134	.0120	.0003	.00050148	
Mean2689	.0355	.1123	.0101	.0004	.0007	.0003	.0148	
Maximum2892	.0580	.1171	.0153	.0010	.0010	.0010	.0148	
Minimum2490	.0259	.1065	.008200050148	
R. F. G.70	.2686	.0289	.1126	.0064	.0002	.00360080	
Spanish spherical.....	.70	.2457	.0136	.1108	.00970003	.0007	.0065	
Curtis & Harvey:										
Hunting30	.2593	.0247	.1132	.0083	.0046	.00080147	
Mining30	.2279	.1522	.0858	.0389	.0070	.00170161	

gram of powder for different densities of loading.

Weight of products of decomposition in one gram.									General weight of all products.	
Solid.										
Carbonate of potash.	Sulphate of potash.	Sulphide of potassium.	Sulphocyanide of potassium.	Salt-peter.	Potash.	Carbonate of ammonia.	Sulphur.	Carbon.	Gaseous.	Solid.
K ₂ CO ₃	K ₂ SO ₄	K ₂ S	KCNS	KNO ₃	K ₂ O	(NH ₄) ₂ CO ₃	S ₂	C		
0.3174	0.0858	0.1111	0.0005	0.0027	0.0009	0.0234	0.4484	0.5418
.3203	.0765	.1127	.00030005	.04194383	.5522
.3303	.0700	.1058	.0012	.00020009	.04134408	.5497
.3176	.0760	.1028	.0014	.00040004	.0485	0.0077	.4357	.5548
.3096	.0658	.1249	.0012	.00110004	.03974478	.5427
.3257	.0763	.0976	.0020	.00180003	.04844385	.5520
.2977	.0758	.1238	.0022	.00140003	.04364457	.5448
.3428	.0601	.0926	.0003	.00180004	.05174408	.5497
.3711	.0528	.0666	.0033	.00250007	.06174318	.5587
.3258	.0710	.1042	.0014	.00130005	.0445	.0008	.4408	.5496
.3711	.0858	.1249	.0033	.00270009	.06174484	.5587
.2977	.0528	.0666	.00030003	.02344318	.5418
.3059	.1191	.092100330003	.0272	.0041	.4369	.5520
.3121	.1375	.0736	.0003	.00070003	.0380	.0001	.4263	.5626
.3037	.1403	.0772	.0003	.00020002	.03894281	.5608
.2877	.1352	.0944	.0010	.00070002	.03604337	.5552
.3601	.0276	.1083	.0017	.00300006	.05484327	.5561
.3739	.0614	.0667	.0016	.00260005	.05624265	.5629
.3629	.0624	.0778	.00150006	.05414296	.5593
.3519	.0614	.0853	.0028	.00240004	.05204327	.5562
.3802	.0504	.0620	.0014	.00100006	.07184215	.5674
.3764	.0488	.0700	.0021	.00110008	.06114302	.5603
.3415	.0844	.0807	.0013	.00150004	.0490	.0004	.4298	.5591
.3802	.1403	.1083	.0021	.00330008	.0718	.0041	.4369	.5674
.2877	.0276	.06200002	.02724215	.5520
.2872	.1042	.10030011	.0315	.0004	.02334372	.5480
.3434	.1401	.0473	.0001	.00050009	.03824147	.5705
.2683	.1321	.1093	.0003	.00050001	.05794167	.5685
.2680	.1300	.1192	.0004	.00060002	.03584310	.5542
.3269	.1210	.0708	.0004	.00050005	.04394212	.5640
.2508	.1330	.1108	.0009	.0010	.0192	.0001	.03494345	.5507
.2674	.1305	.1155	.0014	.00120002	.05644326	.5526
.2640	.1278	.1214	.0014	.00150002	.03584331	.5521
.2989	.1084	.1042	.0015	.00160002	.03714333	.5519
.2861	.1252	.0999	.0007	.0009	.0056	.0003	.03814282	.5569
.3434	.1401	.1214	.0015	.0016	.0315	.0009	.05794372	.5705
.2568	.1042	.047300050001	.02334147	.5480
.3458	.1285	.0525	.0002	.00030003	.04714173	.5747
.2186	.2975	.0473	.0002	.00580002	.04313808	.6127
.3413	.1250	.071700170005	.03724109	.5774
.1945	.0028	.1745	.0139	.00040084	.0664	.0095	.5135	.4704

TABLE X.—*Relative volumes of gaseous and weights of solid*

Kinds of powder.	Mean densities of loading.	Relative volume of each separate gaseous product of decomposition in one gram of powder.						
		Carbonic acid.	Carbonic oxide.	Nitrogen.	Sulphureted hydrogen.	Marsh gas.	Hydrogen.	Oxygen.
		CO ₂	CO	N ₂	SH ₂	CH ₄	H ₂	O ₂
	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>
Pebble.....	0.10	46.66	14.76	32.75	3.13	2.70
	.20	44.78	16.09	31.31	4.23	3.59
	.30	47.03	15.51	31.71	2.90	2.84
	.40	49.52	13.95	32.16	1.70	0.32	2.35
	.50	49.82	13.36	32.19	1.96	.58	2.08
	.60	49.48	13.75	31.83	2.24	.55	2.15
	.70	49.93	12.51	32.08	3.18	.35	1.95
	.80	51.54	11.88	32.61	1.96	.34	1.67
	.90	51.75	10.87	32.72	2.13	.68	1.85
Mean.....	48.95	13.63	32.15	2.60	.31	2.35
Maximum.....	51.75	16.09	32.75	4.23	.68	3.59
Minimum.....	44.78	10.87	31.31	1.70	1.67
R. L. G.....	.10	49.00	8.98	35.60	4.06	.29	2.07
	.20	46.56	11.47	35.13	3.58	.07	2.62	0.57
	.30	49.35	11.60	32.96	3.11	2.98
	.40	50.25	10.84	34.23	1.93	.28	2.47
	.50	47.21	17.04	30.29	1.61	.84	3.01
	.60	46.29	14.52	32.40	4.29	.36	2.14
	.60	50.22	13.95	31.74	1.62	.35	2.14
	.70	49.75	13.38	31.94	2.85	.55	1.53
	.80	51.62	12.16	32.16	1.56	.77	1.72
	.90	52.65	10.73	32.65	1.90	.80	1.27
Mean.....	49.29	12.47	32.91	2.65	.43	2.19	.06
Maximum.....	52.65	17.04	35.60	4.29	.84	3.01	.57
Minimum.....	46.29	8.98	30.29	1.56	.07	1.27
F. G.....	.10	44.76	16.25	32.57	2.26	.18	3.83	.15
	.20	47.41	12.35	32.35	3.76	4.13
	.30	50.45	11.33	32.22	2.21	3.51	.28
	.40	51.79	10.31	32.54	2.00	3.36
	.50	51.04	10.38	33.15	2.20	.27	2.96
	.60	52.00	9.60	33.28	2.26	.18	2.68
	.70	53.02	7.91	34.26	2.03	.50	2.13	.15
	.80	51.80	8.32	34.64	2.61	.41	2.04	.18
	.90	53.34	7.71	33.81	2.95	.16	2.04
Mean.....	50.63	10.47	33.21	2.48	.19	2.96	.08
Maximum.....	53.34	16.25	34.64	3.76	.50	4.13	.28
Minimum.....	44.76	7.71	32.22	2.00	2.04
R. F. G.....	.70	52.40	8.86	34.51	1.60	.12	2.51
Spanish spherical	.70	53.34	4.62	37.80	2.74	1.29	.21
Curtis & Harvey:								
Hunting.....	.30	50.22	7.52	34.46	2.08	2.46	3.26
Mining.....	.30	32.15	33.75	19.03	7.10	2.73	5.24

decomposition products for different densities of loading.

Relative weight of each separate solid product of decomposition of one gram of powder.								
Carbonate of potash.	Sulphate of potash.	Sulphate of potassium.	Sulpho-cyanide of potassium.	Saltpeter.	Potash.	Carbonate of ammonia.	Sulphur.	Carbon.
K_2CO_3	K_2SO_4	K_2S	KCNS	KNO_3	K_2O	$(NH_4)_2CO$	S_2	C
Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
58.56	13.84	20.50	0.09	0.51	0.17	4.33
58.01	13.85	20.41	.0609	7.58
60.09	12.74	19.24	.21	.0317	7.52
57.25	13.69	18.52	.25	.0807	8.74	1.40
57.04	12.12	23.02	.23	.2008	7.31
59.00	13.82	17.68	.36	.3206	8.76
54.64	13.91	22.72	.41	.2606	8.00
62.35	10.94	16.84	.06	.3308	9.40
66.43	9.45	11.92	.59	.4412	11.05
59.26	12.93	18.98	.25	.2410	8.08	.16
66.43	15.84	23.02	.59	.5117	11.05	1.40
54.64	9.45	11.92	.06	.0306	4.33
55.41	21.58	16.685906	4.93	.75
55.47	24.44	13.08	.05	.1206	6.76	.02
54.16	25.03	13.76	.05	.0304	6.93
51.82	24.35	17.00	.17	.1304	6.49
64.77	4.96	19.47	.30	.5311	9.86
66.43	10.90	11.85	.28	.4609	9.99
64.88	11.16	13.91	.2611	9.68
63.25	11.04	15.34	.51	.4408	9.34
67.00	8.88	10.92	.25	.1811	12.66
67.16	8.71	12.50	.38	.2015	10.90
61.03	13.10	14.45	.22	.2708	8.74	.08
67.16	25.03	19.47	.51	.5915	12.66	.75
51.82	4.96	10.92	.05	.0304	4.93
52.43	19.00	18.3021	5.74	.07	4.25
60.20	24.55	8.30	.02	.0815	6.70
47.17	23.24	19.23	.07	.1001	10.18
48.37	23.46	21.50	.08	.1004	6.45
57.97	21.45	12.55	.07	.0908	7.79
45.55	24.15	20.12	.17	.18	3.49	.01	6.33
48.39	23.61	20.90	.26	.2103	6.60
47.80	23.15	21.98	.26	.2804	6.49
54.17	19.64	18.88	.27	.2803	6.73
51.34	22.47	17.97	.13	.17	1.02	.05	6.83
60.20	24.55	21.98	.27	.28	5.74	.15	10.18
45.55	19.00	8.300801	4.25
60.17	22.35	9.14	.04	.0605	8.19
35.66	48.55	7.72	.04	.9504	7.04
59.10	21.65	12.422909	6.45
41.36	.59	37.10	2.95	.09	1.78	14.11	2.02

Considering the last two tables it will be noticed that the composition of the products of decomposition changes with the increase in the density of loading as follows:

(1) The quantity of carbonic acid constantly increases, while that of carbonic oxide diminishes.

(2) The quantities of carbonate of potash and of nitrate of potash also increase gradually, while the sulphate and sulphide of potassium decrease; but this does not show clearly from the decomposition of the F. G. powder.

(3) The quantities of H_2S and H_2 diminish, but those of CH_4 and S_2 increase.

(4) The quantity of undecomposed saltpeter increases.

(5) The content of nitrogen, as should be the case, does not change.

(6) Notwithstanding the changes above indicated, the relative quantities of gaseous and solid products remain pretty nearly constant for a given variety of powder for different densities of loading.

The data presented above can not be considered absolutely correct in all cases. It is evident from the tables that discrepancies occur now and then, and deviations in a direction contrary to that stated above.

16. The state of the solid products of decomposition of smoking powders.

The state of the solid products of decomposition of smoking powders at the moment of their explosion and the volume occupied by them are very difficult to determine by direct experiments. It is very important to know these facts in the solution of certain problems. Bunsen and Shishkov* showed by their experiments that the principal solid products are quite stable bodies, almost nonvolatile at very high temperatures. They exposed powder residues placed in a platinum capsule to an actual temperature of $3,200^{\circ} C.$ in the oxyhydrogen gas flame, whereupon the residue melted, but did not boil. Although there may exist doubt as to whether the solid products of decomposition are actually heated at the time of their formation to so high a temperature, yet in any case this experiment affords us full basis for assuming that the principal mass of the "solid products" exists in a liquid state at the moment of explosion. †

This question was also investigated in detail by Noble and Abel. A close examination of the inner surfaces of the walls of the bomb, shortly after the explosion, showed them ‡ that the solid products at the moment of explosion were in a liquid condition, and that afterwards on cooling they settled to the lower part of the chamber in the form of a dense, hard, difficultly separated, odoriferous substance with a

* R. Bunsen and L. Shishkov's "Théorie Chimique de la Combustion de la Poudre," Terquem's translation from the German, Paris, 1859, pp. 57-58.

† Such bodies as K_2S and S_2 doubtless occur at the time in the form of vapor, but they are neglected.

‡ See Noble's paper, "Internal ballistics," referred to above, pp. 12 to 15.

smooth dark surface, exhibiting an olive-gray tint on fracture. The walls of the chamber were covered only with a very thin layer of residue. The surface of the residue and the walls of the chamber had a greasy appearance and felt greasy to the touch. If the gases escaped rapidly from the chamber when a large charge was employed, as would happen from the great pressure to which they were subjected, the surface of the bomb near the point of escape of the gases, was covered with solid carbonic-acid gas which solidified on account of the great cooling resulting from gas expansion. When, under these conditions, the gases began to escape before the residue became solid the latter assumed the form of a sponge, as if they had escaped from its substance while it still remained in a pasty condition. The residue was far from homogeneous in its composition, which could easily be observed from the color and character of its fracture. The residue generally appeared as a noncrystalline mass, but shining crystals of bisulphide of iron (FeS_2) could be observed throughout it. The residue often emitted an odor of sulphureted hydrogen and of ammonia. It eagerly absorbed moisture, turned black from so doing, and deliquesced. This accounted for the greasiness of the surface of the walls of the bomb, as after the cooling of the products they absorbed all steam formed upon the decomposition of the powder. The residue often emitted a great amount of heat on account of the oxidation produced by oxygen absorbed from the air, whereby steam was given off and the surface assumed an orange hue. The temperature of the residue rose in the air, sometimes to as high as 315°C. , so that pieces of paper approached to it were consumed.

The following experiments were conducted in order to investigate the solid products of decomposition after the explosion:

(1) Two-thirds of the bomb was filled with F. G. powder. Thirty seconds after the explosion it was inclined to an angle of 45° , and two minutes after this the bomb was restored to its original position. When it was opened the residue was found congealed at the 45° angle, where its direct contact with the wall of the bomb was clearly and sharply defined.

(2) The bomb was filled to three-fourths of its volume with the R. L. G. powder. One minute after explosion it was rapidly inclined to an angle of 45° and three-fourths of a minute afterwards it was restored to its original position. Examination showed that at the moment of the first inclination of the bomb the residue had commenced to harden and a thin crust formed on its surface which was cut through by the more liquid part of the products after the inclination of the bomb. The residue congealed at the angle of 45° , but the crust through which the liquid products had forced their way remained like a shell of ice.

It was thus shown that for a minute after the explosion the residue remains quite liquid, but that it hardens during two minutes' interval.

The volume of the powder residues at 15° C. equals their specific volume (for the specific volume of the component parts of powder residues see Table XL). It appears that the chocolate and black powders, P., R. L. G., and F. G., afford residues of almost the same volume, namely: The decomposition of 100 grams of these powders affords, as a mean, about 24.3 c. c.; the mean weight of the residues is also about the same and equals 55.83 grams. Consequently, the mean density of fully dried powder residue is about 2.3. From the results of direct observations Noble and Abel assume that the observed volume equals 29 c. c. for black powders; this exceeds by 20 per cent their theoretical volume and may be explained to some extent by difficulties of observation and also from the fact that the residue absorbs all possible moisture and that it is always porous. In view of this the volume of the solid products at 0° may be taken as from 30 to 40 per cent less than the volume of the powder grains themselves at this temperature.

What is the volume of these residues at the temperature of decomposition?

In order to answer this question Noble and Abel placed in a Siemens-Martins furnace crucibles containing powder residues. The temperature of the furnace was about $1,700^{\circ}$ C. In all cases they observed at first a certain crackling on account of the emission of water from the mass and also of K_2S and S_2^* . However, this residue continued throughout the whole time to expand quietly and to a considerable extent. On removing the crucibles from the furnace they observed the shrinkage of the powder residues on cooling from 700° C. to 0° C. These observations showed that the expansion of the powder residues upon being raised in temperature from 0° C. to $1,700^{\circ}$ C. was more than 100 per cent, which would correspond to a mean coefficient of expansion of powder residues of 0.00059. In every case these experiments fully justified the assumption that at the moment of explosion the powder residues† existed in a liquid state, and that they are finely divided and thoroughly mixed with the gases.

17. *The decomposition of smokeless powders.*

The decomposition of smokeless powders is characterized by almost complete absence of solid residues—i. e., if the decomposition proceeds under desired conditions. In closed chambers such decomposition takes place at pressures of 150 atmospheres. In guns, in order to convert powder into gases, it is indispensable to assume that, besides developing the necessary pressure, the dimensions of the grains and their content of moisture are correspondingly suitable.

The composition of the products of decomposition and their percentage ratios for 1 gram of a pyrocollodion powder, 2 mm. in

* See first footnote on page 44.

† See second footnote on page 44.

thickness, are indicated in Table XI, which is based upon data supplied by the Scientifico-Technical Laboratory*. This powder approximates in its composition to that indicated in Table IV.

TABLE XI.—Composition of decomposition products of pyrocollodion powder.

In 1 gram of powder at 0° C. and 760 mm.	Elements of decomposition.						
	CO ₂	CO	H ₂	CH ₄	N ₂	H ₂ O	Ash.
Content by weight	0.2454	0.4685	0.0159	0.0078	0.1218	0.1309	0.0097
Percentage ratio of volume of gases	15.9000	47.7000	22.6000	1.4000	12.4000
Volume of gases per cubic cen- timeter.....	124.5000	373.5000	177.0000	10.9000	97.1000	162.3000

In 1 gram of powder at 0° C. and 760 mm.	Complete volume of gases, reck- oning H ₂ O as gaseous.	Weight of products.		
		Gaseous.	Liquid.	Solid.
Content by weight.....	0.9903	0.8594	0.1309	0.0097
Percentage ratio of volume of gases	100.0000
Volume of gases per cubic centimeter.....	945.3000	783.0000	162.3000

On comparing the composition of the gaseous products of decomposition of this variety of smokeless powder with the gases developed by smoking powders (see Tables IX and X), we observe that the composition of both is practically the same as to *quality*; but free O₂ and H₂S, which are formed upon the combustion of most smoking powders, are absent on decomposition of smokeless powders. In relation to the relative *qualities* of component parts formed, the decomposition products of pyroxyline powder contain a very large percentage of carbonic oxide (CO) (ten times more than for smoking powders with black charcoal and fifty times more than for chocolate powders), and a comparatively insignificant amount of carbonic acid (CO₂). In general carbonic oxide constitutes more than 50 per cent of the whole volume of gases, considering the water formed as being in a liquid state. In consequence of this, the gases from pyroxyline powders are very poisonous † (mixtures of CO and CO₂—charcoal gas), and are combustible (a mixture of CO and also CH₄, and H₂ with the air).

Besides the gaseous products water and ash are also formed, but the latter occurs in very insignificant amount.

The results of the investigations at the Scientifico-Technical Laboratory show that the composition of the products of combustion of foreign pyroxyline powders are practically the same as those of pyrocollodion powder; this indicates that the elementary composition of

* See Encyclopædic Dictionary by Brockhaus and Ephron, Vol. XXIV, p. 593.

† See Prof. N. B. Sokolov's paper on "Poisoning of the air by gases developed on firing smokeless powder," published in No. 13 of the Journal Vrach of the year 1894.

these powders is so much alike that they might be expressed by the same empirical chemical formula.

Finally, an analysis* was undertaken at the Scientifico-Technical Laboratory of the decomposition products of one type of nitroglycerin powder (Noble's ballistite) for a density of loading of 0.014. The composition and the quantity of its decomposition products are shown in Table XII, based upon results obtained in the laboratory.

Comparing the figures given in Tables XI and XII, we see that the composition of products of combustion of nitroglycerin and pyrocollodion powders are practically the same, in so far as relates to their quality, but there exists a marked difference in the quantitative relations, namely: The general volume of the gases evolved by nitroglycerin powder is less, while the content of carbonic-acid gas by volume is almost the same as the content of carbonic oxide.

TABLE XII.—*Composition of decomposition products of nitroglycerin powder (Noble's ballistite).*

In 1 gram of powder at 0° C. and 760 mm.	Elements of decomposition.						
	CO ₂	CO	H ₂	CH ₄	N ₂	H ₂ O	Ash.
Content by weight	0.3967	0.2459	0.0051	0.1514	0.1870	0.0139
Percentage ratio of volume of gases	35.0000	34.1000	9.9000	21.0000
Volume of gases in cubic centimeters	201.3000	196.1000	56.8000	120.8000	231.8000

In 1 gram of powder at 0° C. and 760 mm.	Complete volume of gases, reck- oning H ₂ O as gaseous.	Weight of products.		
		Gaseous.	Liquid.	Solid.
Content by weight.....	0.9861	0.7991	0.1870	0.0139
Percentage ratio of volume of gases		100.0000
Volume of gases cubic centimeter.....	807.0000	575.0000	232.0000

In addition to this the content of carbonic oxide is considerably less, while the quantities of CO₂ and water present are much greater for nitroglycerin than for pyrocollodion powders. This shows that the nitroglycerin powder is richer in oxygen, and therefore its decomposition results in a more complete combustion—i. e., a greater amount of CO₂ is formed, and there is a smaller amount of CO produced, which results in a diminution in the volume of the gases, and, as we shall see later, in an increase in the quantity of heat developed. On account of their nature the gases of nitroglycerin powder are also poisonous and burning, but in a less degree than those of pyroxyline powders.

* See Brockhaus and Ephron's Encyclopædic Dictionary, Vol. XVI, p. 219.

It therefore appears that all the decomposition products of smokeless powder, with the exception of an insignificant quantity of ash present, exist at the temperature of decomposition in a gaseous form; this is the most characteristic distinction between them and the old smoking powders.

The next question to answer is: Does the composition of the products of combustion of smokeless powder vary with the increase in density of loading? We may not be able to make direct experiments to determine this, but bearing in mind that all smokeless powders, on account of their composition, belong to the same general group of explosive substances, comprising pyroxyline, explosive gelatin, et al., we may follow the well-known investigations of Sarrau and Vieille * upon this latter class of substances and state the following:

The qualitative composition of products of combustion remains the same when the density of loading is increased, but the relative amount of the various substances formed varies, as follows: The quantities of carbonic acid and marsh gas increase, while the contents of water, carbonic oxide and sometimes of hydrogen diminish, but only under the condition that the total volume of all gases, regarding water as existing in the form of steam, remains about the same.

18. Influence of dissociation upon the character of the decomposition products of powder.

By dissociation of gases is denoted a separation of their molecules into simpler forms, accompanying which there is generally observed a change of the original volume of the gas and also an absorption or evolution of heat.† As shown in paragraphs 15 to 17, we see that the volume of gases evolved on combustion of a unit weight of powder is not only referred to 0° and to atmospheric pressure, but is determined for that temperature and stated pressure. The question naturally suggests itself therefore—Does this represent the composition of the gases under natural conditions—i. e., for the temperature of decomposition at high pressures? In relation to this the following must be noted: The measured volume of gases, reckoning water as gaseous at 0° and their composition, can differ from the actual volume and composition—i. e., from those which exist for the temperature of decomposition and for great pressures, only in the case when those bodies that occur at ordinary temperatures separate on raising their temperature into simpler substances and recombine among themselves when the temperature is lowered.

* MM. Sarrau and Vieille, "*Études sur le Mode de Décomposition de quelques Explosifs*:" *Mémorial des Poudres et Salpêtres*, 1884-1889. See also the paper of E. M. Tcheltsov, "*Explosive substances*," in Brockhaus and Ephron's *Encyclopædic Dictionary*, Vol. XI, p. 184.

† See Chvolson's "*Course in Physics*," Vol. I, p. 426, and Vol. III, p. 411.

may dissociate, including in this the separation of the diatomic elements (H_2 , O_2 , and N_2), and also of the compound molecules into simpler forms, of the same or similar compositions; but the degree of their dissociation (and the splitting up of the molecules) varies directly with the temperature of these products and inversely with the pressure under which they occur; and as the pressure of the products of combustion of powder is, at the moment of their evolution, very great, dissociation, judging from the experiments of Mallard and Le Chatelier* hardly occurs at all, or else it is so insignificant that it possesses no practical importance.

Basing our opinion upon these experiments we may state with sufficient accuracy for all practical purposes that the composition of the gaseous products of combustion of powders at the moment of explosion is the same as it is at ordinary temperatures, and therefore the magnitudes v'_0 and v''_0 may be considered as correct for the various powders.

19. Heat evolved on combustion.

Besides the quantity of gases evolved on the combustion of the powder the quantity of heat formed on the decomposition of 1 kilogram or of 1 gram of powder is of the greatest importance to consider. If we know this quantity, the composition of the products and their mean heat capacity, we may determine the temperature of explosion.

Temperatures are expressed either in great or in small calories. Under the metric system the unit is the quantity of heat which is required to raise the temperature of 1 kilogram of water in the first case and 1 gram of water in the second case from $10^\circ C.$ to $11^\circ C.$ The number of units, expressed in great or small calories, will be the same, if, on applying them to the powder, we consider the quantity of heat given off by 1 kilogram as expressed in great calories and by 1 gram as expressed in small calories.

On the basis of Rowland's experiments, we shall assume the mechanical equivalent of heat, J , corresponding to a large calory as equal to $\frac{4,200}{g} \cdot \frac{\text{kgm.}}{\text{l. c.}}$ where g denotes the acceleration of the force of gravity. In St. Petersburg at the sea level $g=9.8185$ meters, and therefore $J=427.76 \frac{\text{kgm.}}{\text{l. c.}}$.†

We must next distinguish between the two quantities of heat, Q_p for constant pressure, and Q_v for constant volume.

In the latter case heat is not expended in performing external work. In determining the quantity Q_p for constant pressure, ordinarily, at the

* See Mallard and Le Chatelier's "Recherches Expérimentales et Théoriques sur la combustion de mélanges gazeux explosifs," 1883, p. 273.

† On account of a mistake of Day this quantity, J , and the corresponding large calory relate to a temperature something under $8^\circ C.$ (See Chvolson's "Course in Physics," Vol. III, p. 234.)

Speaking strictly, all gaseous decomposition products of powder pressure of the atmosphere, it should be borne in mind that gases perform in expanding external work, and in doing this expend a corresponding quantity of heat q ; therefore the relation between the quantities Q_v and Q_p is as follows:

$$(4) \quad Q_v = Q_p + q.$$

The quantity q may be determined as follows: Let a given substance occupy a certain volume. Denote the elementary area of the surface inclosing it as s , and the constant pressure in a direction normal to this surface and exerted over unit area as P_0 . Under the action of internal forces this volume in the time dt expands uniformly in all normal directions an amount, say, dh . The elementary work of expansion of the gases under constant pressure becomes, therefore, $P_0 \Sigma s dh = P_0 dv$, where $dv = \Sigma s dh$, and represents the elementary expansion in volume. The complete work of expansion of the gases under consideration from v to v_0 , neglecting the initial volume v , equals $P_0 v_0$; and as $P_0 = 103.33 \frac{\text{kgr.}}{\text{sq. dcm.}}$ and the mechanical equivalent of heat equals $4,277.6 \frac{\text{kgr. dcm.}}{\text{l.c.}}$, therefore, the corresponding quantity of heat converted into work becomes

$$q = \frac{103.33}{4,277.6} v_0 = 10^{-2.38303} v_0 \text{ l. c.} = 0.024156 v_0$$

whence

$$(5) \quad Q_v = Q_p + 10^{-2.38303} v_0$$

where v_0 equals the number of cubic decimeters of gas evolved by 1 gram of powder.

Besides this, the amount of heat evolved depends on the condition of the water formed at the time of combustion of the powder. If, as under normal circumstances, it is found, after cooling, in a liquid state, the quantity of heat is greater than in the case where we suppose that all the water is found at 0° in a gaseous state, since, in order to convert each gram of water into steam at this temperature 0.6 of 1 large calory is expended. If, for example, there be found in 1 gram of decomposition products Ξ grams of water, then having found Q_v for water in a liquid form, we may determine Q'_v for the same quantity of water, but as vapor at 0°C by the following formula:

$$(6) \quad Q'_v = Q_v - 0.6 \Xi$$

similarly,

$$(7) \quad Q'_p = Q'_v - 10^{-2.38303} v'_0$$

where v'_0 equals the volume of gases, reckoning water as gaseous.

What is usually determined is, Q_v , i. e., the quantity of heat evolved by 1 gram of powder for constant volume and with water in the liquid state. Knowing the volume of gas at 0° and at a pressure equal to $1.0333 \frac{\text{kgr.}}{\text{cm.}^2}$, we may, by means of the above experiments, (5), (6), and (7), determine Q_p , Q'_v , and Q'_p .

HEAT EVOLVED ON THE COMBUSTION OF SMOKELESS POWDER.

Noble and Abel determined the quantity of heat Q_v for each variety of smokeless powder experimented with by them. On the basis of these experiments Table XIII is compiled, in which is shown, in parallel with the volume of the gases v_0 and v'_0 , the quantities Q_v , Q_p , Q'_v , and Q'_p . All these quantities relate to the powders containing moisture as given in Table VI.

TABLE XIII.—*Volumes of gases and quantity of heat evolved on decomposing 1 gram of certain smoking powders.*

Kinds of powder.*	Considering water—						
	As liquid.				As gaseous.		
	v_0	Q_v	$\frac{Q_v v_0}{100}$	Q_p	v'_0	Q'_v	Q'_p
	<i>Cu. cm.</i>	<i>Small cal.</i>		<i>Small cal.</i>	<i>Cu. cm.</i>	<i>Small cal.</i>	<i>Small cal.</i>
Chocolate	195.4	815.9	1,594	811.7	315.1	758.0	752.2
Spanish	232.7	762.3	1,773	757.3	240.8	758.4	753.2
Okhta, No. 6	238.2	755.5	1,800	750.4	252.7	748.5	743.5
"A"	241.2	791.3	1,909	786.1	291.5	766.9	760.6
F. G	259.2	727.2	1,885	721.6	277.6	718.3	711.7
R. L. G	271.3	718.1	1,949	712.3	284.4	711.7	705.6
P	275.7	714.5	1,970	708.6	287.5	708.8	702.6
"D"	278.6	736.2	2,048	730.2	293.2	728.1	721.8
"B"	308.2	699.5	2,156	692.9	338.3	684.9	677.6
"C"	342.2	517.7	1,772	510.4	359.4	509.4	501.7
Mining	354.6	508.8	1,804	501.2	374.6	499.1	491.1

The different forms of powder are arranged in this table in order of increasing magnitude of the volume v_0 of gases evolved by them, reckoning H_2O as liquid.

If we compare the values Q_v and v_0 together for all these forms of powders, we see that excluding the powders "A" and "D," in all the rest the greater the amount of heat evolved, the less the quantity of gas formed. It is notable that this increase progresses at such a rate that the product $Q_v v_0$ remains almost constant for powders close to one another in composition.

HEAT EVOLVED ON THE COMBUSTION OF SMOKELESS POWDERS.

Results of experiments made at the Scientifico-Technical Laboratory** by Noble and Abel,† and also by Macnab and Ristori,‡ with various kinds of smokeless powders show that pyroxyline powders evolve

*See Table VIII, "Interior Ballistic," Part I.

**Brockhaus and Ephron's Encyclopædic Dictionary, Vol. XXIV, p. 593.

†Noble: Researches on Explosives; Preliminary Note, 1894.

‡Macnab and Ristori: Researches on Modern Explosives, Vol. XXXVIII, No. 199. Journal of the Royal United Service Institution, 1894.

from 850 to 920 calories; cordite from 1,178 to 1,253 calories, and ballistite from 1,269 to 1,317 calories for constant volume and reckoning water as gaseous.

20. *The temperature of the decomposition of powders.*

In the first place we shall assume that by the temperature of the products of decomposition of powder or by the temperature of burning powder we understand that temperature which the products of decomposition would have if all the heat developed upon combustion of the powder was employed in heating the products of combustion and was not expended in performing any external work. We shall indicate the degree of heat of the products, i. e., their temperature, either by the air or by the hydrogen thermometer in accordance with the Centigrade scale—i. e., letting 0° correspond to a temperature of melting ice and 100° to the temperature of water boiling under a barometric pressure of 760 mm. of mercury. We shall indicate the whole temperature reckoning from 0°C. by the letter t , or reckoning from absolute zero by $T=t+273^{\circ}$.

Although the direct determination of the temperature of the decomposition of powder is exceedingly difficult, nevertheless, Noble and Abel made the following experiments to settle this question directly for smoking powders. In some of their experiments with the powders F. G., R. L. G., P., and Spanish powder, they introduced into the bombs platinum foil and a wire of the same material. In all cases, with the exception of experiments with the Spanish powder, the platinum foil, after the explosion, was found slightly affected, so that under the microscope both the wire and the foil showed signs of melting. For the Spanish powder the platinum melted almost completely. As the temperature of melting of platinum is reckoned at $2,000^{\circ}\text{C.}$, Noble and Abel decided as a result of these experiments that the black powder W. A. developed a temperature of about $2,100^{\circ}\text{C.}$ and the Spanish powder of over $2,200^{\circ}\text{C.}$ They concluded that the mining powder did not develop a temperature greater than $2,000^{\circ}$ – $1,800^{\circ}\text{C.}$ In this manner Noble and Abel determined that the maximum temperature of decomposition of smoking powder was not greater than $2,200^{\circ}\text{C.}^*$

On the other hand, knowing the quantity of heat developed by the products of combustion—e. g., Q_v or Q_p , and the composition of these products, we may calculate the temperature of decomposition, if we know, in addition, the specific heats of the products of combustion C_v (for a constant volume) or C_p (for constant pressure), since the temperature of the products of combustion should be

$$t = \frac{Q_v}{C_v} = \frac{Q_p}{C_p}$$

We thus see that an exact knowledge of the heat capacities (specific heats) of substances entering into the composition of the products of

*See what has been said above: "Internal ballistics", etc., p. 46.

decomposition of powder has a primary significance in the investigation of the phenomena involved in interior ballistics.

In view of the above, we must dwell with a certain amount of detail upon what is known in science in relation to the magnitude of specific heat and its change with temperature, density, composition of substances, etc.

By the specified heat of a given substance is signified the quantity of heat required to raise the temperature of a *unit weight* of that substance 1°C . Thus, if upon heating a certain weight P of a given body from the temperature t_1 , to a temperature t_2 , a quantity of heat, Q , is expended; then the specific heat of the substance is

$$c = \frac{Q}{P(t_2 - t_1)}$$

A distinction is made between the specific heat for constant pressure, c_p , and for constant volume, c_v . The former, c_p , corresponds to the quantity of heat required to raise the temperature of the body 1°C , assuming that it expands under constant pressure, usually atmospheric. The second, c_v , corresponds to the quantity of heat required to raise the body 1°C . in temperature under the condition that it does not change its volume. In addition to this, a distinction is made between the molecular and specific heats. The former represents the quantity of heat required for raising 1 gram-molecule of the substance 1°C . and the latter that required for raising one atom of it.

If we denote by M and A the weight in grams of the molecule and the atom, then the molecular specific heat will be represented by Mc_v or Mc_p ; and the atomic specific heat by Ac_v or Ac_p . In the case of complex bodies whose molecules consist of a different number ($n_1, n_2 \dots$) of atoms of various elements, the atomic weight of which are $A_1, A_2 \dots$ the weight of the molecule will be

$$M = n_1 A_1 + n_2 A_2 + n_3 A_3 + \dots$$

and the magnitude of the *mean atom* of the body is

$$A = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \dots}{n_1 + n_2 + n_3 + \dots}$$

We shall assume this quantity A , in the case of a complex body, as the mean weight of its atom.

The molecular and atomic weights will therefore be represented by MQ_v , MQ_p , AQ_v , and AQ_p , respectively.

In the case of solid and liquid bodies the specific heats for constant pressure and constant volume are identical. These quantities differ, however, for gaseous bodies. The specific heat of a gas for constant volume represents its *true specific heat*, since in this case the heat is expended only in heating the body.

On the other hand, when a gaseous body is heated at constant pressure heat is expended not only in raising the temperature of the body,

but also in performing external work in consequence of the expansion of the gases, under the assumption that in all cases the walls of the chamber in which the heated body is inclosed absorb no heat.

We shall now determine the work of expansion of a gas upon raising its temperature 1° C. under constant pressure. We shall assume that the amount of expansion of gases, due to raising their temperature 1° C., is independent of temperature, pressure, or composition of the gas unless it deviates from the laws of Boyle-Mariotte and of Gay-Lussac. The magnitude of this expansion for every gas is represented, according to Regnault's experiment, by about $\frac{1}{273}$ of its original volume of gas (v). This result was substantiated by later experiments of Daniel Berthelot.*

The work of expansion, therefore, becomes $\frac{P_0 v_0}{273} = \frac{103.33 v_0}{273}$ and the corresponding quantity of heat $\frac{103.33}{273 \times 4277.6} v_0$. We thus have

$$(8) \quad c_p = c_v + 10^{\frac{5.94687}{v_0}} v_0 = c_v + 0.00008848 v_0.$$

where v_0 corresponds to the volume of the unit weight of the gaseous body. Multiplying the terms of this equation by the weight M of the molecule and observing that Mv_0 represents the volume of the molecule of the gas for 0° and a pressure $1.0333 \frac{\text{kgr.}}{\text{cm}^2}$, which equals for all gases 22.32 cubic decimeters, we obtain, for the units employed

$$Mc_p = Mc_v + 0.00008848 \times 22,320 = Mc_v + 1.975$$

or nearly

$$Mc_p = Mc_v + 2;$$

that is, the difference between the molecular and the specific heats of gases at constant pressure and volume is a constant quantity nearly equal to 2 calories, and is independent of their pressure and temperature.

As early as 1818 Dulong and Petit established that the magnitude of the specific heat of many substances changed with their temperature, increasing with it, even in the case where there was no apparent change in the composition of the body.

As a result of this the quantity of heat Q expended on heating the unit weight of a body is represented by a certain unknown function of t , but in all cases continuous, and reducing to 0 for $t=0$. It may, therefore, be expressed in accordance with Maclaurin's theorem—i. e., we may express it in the form

$$Q = at + bt^2 + ct^3 + \dots$$

* See "Comptes Rendus," Vol. CXXVIII, p. 498-500.

If the body does not change its composition, the actual elementary specific heat C' becomes

$$C' = \frac{dQ}{dt} = a + 2bt + 3ct^2 + \dots$$

Experiment shows that we may omit all beyond the two first members in the formulæ for Q and C' . In this case the relation between the specific heat and temperature assumes the form of a lineal function. The above predicated elementary specific heat of a body for a temperature t° represents the quantity of heat which would be expended in raising the temperature of the body from t to $(t+1)^\circ$ —i. e., under the assumption that the specific heat of the body remains constant for this change of temperature.

The expression for *mean specific heat* from t_1° to t° becomes

$$C = \frac{Q}{t-t_1} = a + b(t_1+t) + c \frac{t^3-t_1^3}{t-t_1} + \dots$$

where a , b , c . . . are constants as indicated above.

The cause of increase of specific heat with temperature must be sought for in these changes which occur in the body upon heating it.

According to the conceptions of Daniel Bernouilli* and Boskovitch,† every body consists of groups of molecules, more or less complex, corresponding to the nature and condition of the body.

Every molecule constitutes, so to speak, a special system, consisting of lesser particles of the same composition as the molecule; these particles are in turn subdivisible into still smaller particles which represent the elements (atoms) of which the molecule is composed. Speaking in general terms, all of these exist in a state of constant complex motion, and all exert together a certain energy, K . On heating the body the energy of the molecule increases. In the general case it must be admitted that besides the progressive and the rotary movements of the molecule itself as a system, the component parts of it, its smaller subdivisions and particles, vibrate (revolve and oscillate) around certain axes and centers which correspond to mean positions. These vibrations of atoms and molecules are called intermolecular movements.‡ Different parts of the total energy K of the body correspond to these movements. We shall denote the energy of the progressive movement of the molecules of the body by K_n , and the rotary and intermolecular movement of its particles we shall consider together and denote as molecular energy— K_m .†

In consequence of this the more the body is heated the more the system of molecules moves§ both progressively and rotatively with great velocity, so that the particles that compose it vibrate more and

*Jamin et Bouty: "Cours de Physique," Vols. II and III, p. 140.

†Mendeléef: Brockhaus and Ephron's Encyclopædic Dictionary, Vol. VI, p. 154.

‡Chvolson: Course in Physics, Vol. I, pp. 389-399.

§Berthelot: Thermochemie, Vol. VI, chap. 2.

more rapidly, and the component parts of the molecules separate from one another. In this manner the relative position of the elementary particles changes more and more until the time when the original equilibrium is destroyed. Then occurs a rupture of the system and the particles group themselves in a different manner, although they may possess a similar but less complex composition than that of the original molecules. (This phenomenon is denoted as physical dissociation in the case of gaseous and liquid bodies and as change of physical structure in the case of solid bodies.) Finally, the molecules may undergo an actual destruction, which, in particular cases, may become dissociation (chemical).* We must also admit that the heating of the molecules in all bodies does not proceed uniformly, and that therefore the separation and displacement of the molecules is not accomplished simultaneously through the whole mass, but gradually.

The forms of internal energy considered above exist for all three states of matter, the gaseous, the liquid, and the solid; but the greater part of the energy in the first is consumed in the progressive movement of the molecules; in the second, in effecting their rotary movement; and in the third, in the oscillatory movement of their particles. The composition of the molecules also becomes more complex in a degree corresponding to the change of the body from the gaseous to the liquid, and finally to the solid state. At the same time the interaction between the molecules becomes stronger and stronger—i. e., the molecular forces (forces of cohesion), which prevent the change of the distribution of the particles, increase, but the true nature of these forces is still unknown to us.

Exact knowledge as to the magnitudes of the heat capacities of many bodies does not exist, but experiments performed up to the present time in this direction allow us to establish certain generalizations which will be of service in making approximate calculations in those cases for which exact determinations are not to be had.

(1) The specific heat of bodies in the liquid state is greater than their specific heat in the solid and gaseous states,† but the interrelations between the specific heats for these three primary conditions of a body are very varied; thus, in the case of water, H_2O , as steam possesses a molecular specific heat of 8.65 (at about $180^\circ C.$); as a liquid, 18.1; and as ice, 9. Bromine vapor possesses a specific heat of 8.8; bromine liquid, a specific heat of 18.1, while the specific heat of solid bromine is 13.3. The specific heats of metals in the solid and liquid states are, on the contrary, near to one another. Thus, solid mercury possesses a specific heat of 6.4; liquid mercury, of 6.7, and mercury vapor, of 3; solid and liquid lead of 6.63 and 7.5,

* Chvolson: Course in Physics, Chap. I, p. 435, 536, etc.

† Berthelot: Thermochemie, Vol. I, pp. 104–111.

respectively. The greatest change of specific heat in relation to temperature is observed in the case of liquid bodies and the least in the case of solid bodies.

(2) According to the laws of Dulong and Petit, the atomic heat capacity of simple solid bodies is constant under the condition that the bodies are far removed from the point of change into the liquid condition. Actual experiments of Regnault and others show that the magnitude of the atomic heats of solid bodies centers in general at about 6.4, but that certain bodies deviate wholly from this law. Thus, carbon in the form of wood charcoal possesses a specific heat of 3.13, while in the form of the diamond its specific heat is 1.35. Weber's investigations proved that the heat capacities of such bodies fulfilled the required law for certain elevated temperatures; thus for carbon at 900° and for bromine and silicon at 200° , while sulphur showed correspondence only when in a freshly melted state.

(3) The laws of Joule (1844), Woestyn (1847), and Kopp (1864) established the fact that the specific heat of a complex solid body is equal to the sum of the specific heats of the elements composing it; but in obtaining this result Kopp proposed the assumption of an atomic heat of 6.4 for all bodies, with the exception of the following eight elements, for which he assumed the specific heats indicated below, namely:

H.....	2.3	N.....	3.8
C.....	1.8	O.....	4.0
S.....	5.4	F.....	5.0
B.....	2.7	P.....	5.4

(4) Neumann's law, verified by Regnault, established that all solid bodies having a general chemical formula possessed a molecular specific heat closely agreeing to that obtained by the application of Kopp's law. We shall present the mean molecular specific heats Mc for compounds of different types.*

Type.	Mc.	Type.	Mc.	Type.	Mc.
RO	11.0	RS ₂	18.1	RCO ₃	21.0
RO ₂	14.0	RNO ₃	24.0	R ₂ CO ₃	29.1
RO ₃	18.8	RN ₂ O ₆	38.2	RSO ₄ +5H ₂ O	78.3
R ₂ O ₃	26.9	RSO ₄	26.4	RSO ₄ +7H ₂ O	97.4
RS	11.9	R ₂ SO ₄	32.9		

(5) For all gases, simple as well as complex, with the exception of monoatomic gases, it is assumed that the molecular specific heat for constant temperature equals $Mc_v = 2.4n$, where n equals the number of atoms in the molecules of a simple or complex gas. Molecular specific

* Chvolson's Course in Physics, Vol. III, p. 206.

heat for constant pressure is, therefore, approximately, $Mc_p = Mc_v + 2 = 2.4n + 2$. It must also be borne in mind that the specific heats themselves at ordinary temperatures, as also their increase with increase in temperature, are almost identical for all complex gases formed from simple gases, with identical diminution of volume. In the case of monoatomic bodies the molecular and specific heats are equal to 3 calories and do not change with change of temperature.

The experiments and investigations of Mallard and Le Chatelier* at temperatures up to $3,200^{\circ}\text{C}$. and their subsequent verification of the same in relation to the decomposition of explosive substances afford the following expressions for mean molecular specific heats at constant volume, when the temperature changes from 0° to $t^{\circ}\text{C}$.:

(1) For diatomic gases, H_2 , N_2 , O_2 , CO ,

$$(10) \quad Mc_v = 4.8 + 0.0013 t$$

(2) For steam, H_2O ,

$$(11) \quad Mc_v = 5.61 + 0.0033 t$$

(3) For carbonic acid, CO_2 ,

$$(12) \quad Mc_v = 6.26 + 0.0037 t$$

The latter expression is also applicable to CH_4 and may be applied in the case of H_2S .

Table XIV gives those specific heats adopted by Noble and Abel† for various solid bodies entering into the composition of the products of combustion of smoking powders. They relate to ordinary temperature.

TABLE XIV.—*Specific heats of solid decomposition products of powder, as assumed by Noble and Abel.*

Names of bodies.	Specific heats for constant volume.	Names of bodies.	Specific heats for constant volume.
For carbonate of potash, K_2CO_3	0.206	For saltpeter, KNO_3	0.239
For sulphate of potash, K_2SO_4190	For oxide of potassium, K_2O200
For potassium monosulphide, K_2S108	For carbonate of ammonia $(\text{NH}_4)^2\text{CO}_3$350
For sulphur, S_2171	For potassium thiosulphate, $\text{K}_2\text{S}_2\text{O}_3$197
For sulphocyanide of potassium, KCNS200		

The signification of the ratio of the specific heats of gaseous bodies—i. e., the ratio $k = \frac{c_p}{c_v}$ is also of importance in interior ballistics.

*See Mallard et Le Chatelier: *Recherches Expérimentales et Théoriques sur la Composition des Mélanges Gazeux Explosifs*, Paris, 1883, pp. 239–287; and “Rapport sur l’étude des questions relatives à l’emploi des explosifs en présence du grisou,” in *Mémorial des Poudres et Salpêtres*, Vol. II, 1884–89, pp. 392, 442, 443, and 456–460.

†See Captain Noble and F. Abel’s *Researches on Explosives*, No. 1: “Fired Gunpowder,” 1875, p. 101, and also Pashkevitch’s Russian translation of this paper, “Study of explosive materials: The action of ignited powder,” 1878, p. 57.

In view of what has been stated on page 55 we may assume approximately,

$$k = \frac{c_p}{c_v} = 1 + \frac{2}{Mc_v}$$

In the case of monoatomic bodies, $Mc_v = 3$ and, therefore, for them $k = 1.67$.

For more complex gases $Mc_v = 2.4n$, whence

$$k = 1 + \frac{2}{2.4n}$$

i. e., in their case, the value of k decreases with the increase of the number of atoms in the molecule and becomes, for diatomic gases, $k = 1.42$; for triatomic gases, $k = 1.28$; for tetratomic gases, $k = 1.20$; and for pentatomic gases, $k = 1.17$.

Another expression for the ratio of mean specific heats is derived from formula 8, on page 55, namely,

$$(9) \quad \frac{c_p}{c_v} = 1 + \frac{0.000088v_0}{c_v} = 1 + \frac{0.000088v_0}{a+bt}$$

This shows that the ratio of the mean specific heats for constant pressure and constant volume in the case of all gaseous bodies, except the monoatomic (page 59), diminishes with increase of the temperature to which they are raised. This result has been confirmed by direct experiment.*

The actual ratio $\frac{c_p}{c_v}$ may be determined independently for the quantities \dot{c}_p and c_v themselves by the velocity of transmission of sound through a gaseous medium, since $\frac{c_p}{c_v}$, for a given density and temperature of gas, is *directly proportional to the square of the velocity of sound*,† namely,

$$\frac{c_p}{c_v} = 0.00348 \frac{\delta}{T} u^2$$

where u equals the velocity of sound in $\frac{\text{cm.}}{\text{sec.}}$; δ , the density of the gas compared with that of the air, and T , the absolute temperature of the gases. Kundt and Varburg determined the velocity of sound in mercury vapor (a monoatomic body), whence they found $\frac{c_p}{c_v} = 1.67$, which agrees with the above-stated fundamental value of k for monoatomic bodies.

* Berthelot's *Thermochimie*, p. 51.

† Jamin et Bouty: *Cours de Physique*, Vol. III, p. 94.

Knowing the ratio between the specific heats we may approximately determine the magnitude of molecular heats, namely,

$$Mc_v = \frac{2}{k-1}$$

and

$$Mc_p = \frac{2}{k-1} k$$

but it must be observed in this connection that the formula above established may only be employed when there is no separation of the molecules of the body into simpler forms and when the specific heats themselves change with the temperature, but not very rapidly.*

CALCULATION OF THE TEMPERATURES OF POWDERS.

If the constituents by weight of decomposition products of any powder (see Table VIII) be multiplied by their specific heats, the sum of these products should equal the quantity (Q'_v) developed by this powder, reckoning water as gaseous. It is easy to estimate the desired temperature from this sum, since

$$(a + a_1) t_1 + (b + b_1) t_1^2 = Q'_v$$

whence

$$(13) \quad t_1 = -\frac{a+a_1}{2(b+b_1)} + \sqrt{\left(\frac{a+a_1}{2(b+b_1)}\right)^2 + \frac{Q'_v}{b+b_1}}$$

where t_1 equals the temperature of decomposition sought for.

$$a = 4.8 \left[\frac{\alpha}{28} + \frac{\beta}{28} + \frac{\gamma}{2} + \frac{\delta}{32} \right] + 5.61 \frac{\varepsilon}{18} + 6.26 \left[\frac{\eta}{44} + \frac{\mu}{34} + \frac{\nu}{16} \right]$$

and

$$b = 0.0013 \left[\frac{\alpha}{28} + \frac{\beta}{28} + \frac{\gamma}{2} + \frac{\delta}{32} \right] + 0.0033 \frac{\varepsilon}{18} + 0.0037 \left[\frac{\eta}{44} + \frac{\mu}{34} + \frac{\nu}{16} \right]$$

In which α , β , γ , δ , ε , η , μ , and ν indicate the constituents by weight (Table VIII) of the products of decomposition, CO, N₂, H₂, O₂, H₂O, CO₂, H₂S, CH₄, while the denominators of the corresponding fractions give the weight of the gram-molecules of these bodies.

a_1 = The mean specific heat capacity (for change of temperature from 1° C to 0° C) of all the solid products of decomposition,—i. e., the sum of the products of the corresponding quantities from Tables VIII and XIV.

b_1 = The increase in mean heat capacities of all the solid products of decomposition for an increase of their temperature of 1° C, whence b_1 equals the product of the numbers in the next to last column of Table VIII into b'_1 (see expression 14 farther on),—the temperature coefficient of heat capacity of nonvolatile substances.

Q'_v = The quantity of heat given in Table XIII, assuming water to exist in the form of steam.

* M. Berthelot's *Thermochimie*, Vol. I, p. 38.

We shall proceed first of all to determine the temperature of decomposition of the powder P, one of the varieties most studied. Performing the calculations above indicated and assuming, as many do, that the specific heat of the solid products does not change with temperature—i. e., that $b_1=0$, we obtain

$$a=0.0727086; b=0.0000336; a_1=0.1002336$$

and as for this powder $Q'_v=708.8$
therefore

$$t_1=2694.$$

and the mean specific heat of all products of decomposition becomes $a+a_1+bt_1=0.2631912$

Comparing this value t_1 with that determined by Noble and Abel, we are forced to the conclusion that either the experimental value ($2,100^\circ$ C., see page 53) is incorrect, or else that the specific heat of the solid products is higher than that assumed, for, as already stated (paragraph 18), the dissociation of the products of decomposition can not have direct significance in the case under consideration.

Each of these two conclusions has a certain foundation: (1) the direct observation of temperatures through a very short interval is exceedingly difficult; (2) our knowledge of specific heats is incomplete and as the specific heats of liquid bodies increase with the temperature, it is sufficient to assume simply a certain increase of specific heats of nonvolatile bodies in the products of the decomposition of powder, and the conclusion will agree with the observation.

In order to ascertain how much the mean specific heat of the products of decomposition should be changed, we must note that there is still another way of determining the temperature of decomposition based upon the relation existing between the developed pressure, volume of gases, temperature, and covolume (see farther on, paragraph 24, formula 25). Allowing that this relation is close to that which obtains under actual conditions, we may predict the value of one quantity from the data if the other quantities are known. Experiment determines with great accuracy the pressure, the volume of gases, and the covolume; therefore, formula 25 affords the possibility of computing the temperature. Knowing this, and also the quantity of heat given off, we may subsequently ascertain the magnitude of the mean specific heat C_v (formula 26) of all products at the temperature of decomposition. As we may regard the specific heat of gaseous products as known, the difference between the ascertained value of the specific heat and the specific heat of the gaseous products of decomposition $[C_v-(a+bt_1)]$ gives us the mean specific heats of the mixture of nonvolatile products at that temperature. Subtracting the quantity a_1 from this and dividing the difference by the temperature of the prod-

ucts determined by formula 25 and by the weight of the nonvolatile products, we obtain a certain mean temperature coefficient (for the powder P), $b'_1 = 0.0000714$ —i. e., the increase of the mean specific heat of 1 gram of the mixture of nonvolatile decomposition products on raising their temperature 1°C . We may assume the derived coefficient b'_1 as constant in the absence of more exact data for all mixtures of solid products of decomposition of all smoking powders, and afterwards we may calculate the actual temperature of decomposition.

Bearing in mind what was said in relation to the specific heats of the various products of decomposition; we may calculate by employing Table VIII, the quantities a , b , a_1 , and b_1 ; having these and the value Q'_v (see Table XIII), we may next find by formula 13 the temperature of decomposition t_1 of all smoking powders and finally the mean specific heat of all products of decomposition.

All these quantities, and also the volumes of the gases, including steam from water, are given in Table XV, in which smoking powders are arranged in order of their development of temperature. It is evident from the table (compare what was said about the quantity of heat Q_v , p. 52) that the higher the temperature of decomposition the less the volume of all gases, including water. Chocolate powders* give a lower temperature of decomposition than black powders, while the Spanish and Okhta powders give the highest temperatures of all smoking powders.

The temperatures of decomposition of smokeless powders may be determined in a similar manner by use of Tables XI and XII (pp. 47, 48) for decomposition products. In doing this we may assume the dust as existing as carbonate of potash. Results of calculation for pyroxyline and nitroglycerin powders (Ballistite) are given in Table XV.

Besides this, the temperature of decomposition of smokeless powders, and in general of explosive substances whose products of decom-

* Such a result for chocolate powder is contrary to the opinions of certain artillerists (see J. A. Longridge's *Interior Ballistics*, 1889, pars. 87, 89), who assume that the decomposition products of this powder develop a higher temperature than the black. Such discordance, apart from the difference in methods of calculation, is explained by the circumstance that in estimating the temperatures of powders they regard the latter as not only completely dried, but they also consider the newly formed water (Table VIII) as liquid. This is in opposition to the actual conditions of the employment of the powder as well as that of the state of the water at the temperature of decomposition, and, therefore, in our calculations we have taken powder with the moisture shown in Table VIII, where we consider water as existing in the form of steam. Besides this, we may observe that Noble and Abel (see their notes above referred to), in calculating temperatures for completely dried black powders, assume for the dry powder a less content of products of decomposition than they determined for moist powders.

position are gaseous at a known temperature, may be ascertained by utilizing experimental determinations of pressures, as is stated further on in paragraph 24, though the quantity of heat given off be not known.

In general, it may be stated that the pyroxyline powders have a temperature of decomposition quite near to that of smoking powders. Nitroglycerin powder has a markedly higher temperature of decomposition.

21. The pressure of the decomposition products of powder on explosion in a closed chamber.

Such pressures were determined by Noble and Abel for smoking powders under various conditions of density of loading, varying from 0.094 to 0.93. The actual results obtained by these investigators are given in Table XVI.

With this data they subsequently determined corresponding pressure curves, so that abnormal results could be eliminated. The pressures from these curves may be considered as observed pressures. They are given in Table XVII.

We note that the observed pressures, as the experiments show, do not depend on the quantity of burning powder, but only upon density of loading. This circumstance is true when the walls of the chambers are impenetrable to heat, and is approximately true if the absorbing power of the walls of the chamber be not especially great. Bearing the latter in mind, we may assume that up to a certain limit the increase in the weight of powder for a given density of loading may have no influence upon the temperature of the gases in the chamber, nor also, as we may see later, upon the pressure.

The number of experiments made to determine pressures developed by smokeless powder in closed vessels are very few. Some of the existing experimental data relating to them has been collected in Table XVIII. The pressures for the samples of powder from the Okhta factory represent mean values determined each from the average of 10 observations made by the Experimental Commission of the factory. They are shown in the numerators. The denominators show the pressures as calculated by this Commission in accordance with formula 20 and the data given in Table XIX.

TABLE XV.—*Temperatures of decomposition of powder and the specific heats of the decomposition products.*[For meaning of coefficients, a , b , a_1 , b_1 , see p. 61.]

Name of powder.	Coefficients.				Vol- ume of gases, includ- ing water vapor. V_0' .	Quan- ti-ty of heat given off. Q'_{τ} .	Tem- pera- ture of decom- posi- tion. t_1 .	Mean specific heat of all products of decom- posi- tion for con- stant volume.
	a .	b .	a_1 .	b_1 .				
(A) SMOKING.								
Spanish	0.0609016	0.0000289	0.1155513	0.0000438	240.8	758.4	2,237	0.3390828
Okhta No. 60694258	.0000328	.1087439	.0000412	252.7	748.5	2,197	.3407477
"P"0723086	.0000335	.1002336	.0000392	287.5	708.8	2,154	.3291380
"F. G."0719487	.0000338	.1016090	.0000398	277.6	718.3	2,151	.3318713
"D"0727605	.0000327	.1100735	.0000399	293.2	728.1	2,149	.3388514
"A"0789076	.0000357	.1164302	.0000402	291.5	766.9	2,143	.3579915
R. L. G.0716150	.0000332	.1042381	.0000399	284.5	711.7	2,141	.3345012
Chocolate0824014	.0000419	.1151240	.0000406	315.1	758.0	2,062	.3676404
"B"0836342	.0000340	.1178340	.0000397	338.3	684.9	1,974	.3469520
"C"0889633	.0000397	.0791812	.0000335	359.4	509.4	1,728	.2946340
Mining0922129	.0000401	.0766150	.0000329	374.6	499.1	1,702	.2930739
(B) SMOKELESS.								
Pyrocollodion2181175	.0000841	.0019982	.000007	945.3	880.9	2,177	.4047253
Ballistite1950697	.0000895	.0028634	.0000010	807.0	131.7	2,865	.4572156

TABLE XVI.—*Pressures actually developed in burning the powders P., R. L. G., and F. G. and Spanish powder in closed chambers.*

Density of loading.	Pressures in kilograms per sq. cm. for the powders.			Density of loading.	Pressures in kilograms per sq. cm. for the powders.		
	F. G.	R. L. G.	P.		F. G.	R. L. G.	P.
0.0940	252	0.5000	1,651	1,651
.1064	261	2195000	1,606	1,685
.1064	213	1985000	1,748
.1064	151	202	*.5300	*1,858
.1973	4205322	1,808	1,922
.2000	4256000	2,227	2,262	2,170
.2114	4626000	2,126
.2129	5836000	2,331
.2129	564	*.6100	*2,457
.2129	473	*.6200	*2,646
.2963	1,0087000	2,867	3,078	2,929
.3000	8517000	†2,678
.3171	772	*.7500	*3,449
.3193	1,0638000	3,652	3,843	4,504
.3193	9958000	4,268	3,652	3,811
*.3800	*1,3399000	4,281	5,607	5,620
.....	*1,2139000	4,977
.3860	1,2109000	4,945
.3947	1,276	*5,213
.4258	1,471	1,323	*.9000	*4,835
.4258	1,433	*5,800
.4615	1,3679150	5,433
.4893	1,5979300	5,701
.4934	1,8119300	5,355	*5,512

* These denote experiments performed by Noble alone. † Spanish powder.

TABLE XVII.—*Observed and calculated pressures for the powders P., R. L. G., and F. G.*

Density of loading.	Observed (taken from the curves) pressures for the powders (kilograms per square centimeter)—		Calculated pressures by formula 20 and by data of Table XIX for the powders (kilograms per square centimeter)—		
	P. and R. L. G.	F. G.	P.	R. L. G.	F. G.
0.1	* 232	* 232	280	280	272
.2	513	513	598	598	581
.3	839	839	963	972	936
.4	1,221	1,219	1,383	1,388	1,348
.5	1,684	1,668	1,875	1,886	1,830
.6	2,266	2,208	2,458	2,484	2,405
.7	3,007	2,884	3,160	3,196	3,099
.8	3,942	3,734	4,020	4,083	3,959
.9	5,112	4,786	5,098	5,205	5,040
1.0	6,568	6,067	6,495	6,672	6,452

22. *The relation between the pressures in closed chambers and the density of loading, assuming that the walls of the chambers have no absorptive capacity for heat.*

Upon exploding in a closed chamber gases perform no external work. The amount of internal work done by them may be neglected; therefore, in accordance with the laws of thermodynamics (Joule's experiments), the temperature of the gases remains constant. In a given case the temperature of the products of decomposition of powder will equal the temperature of the decomposition.

TABLE XVIII.—*Pressures observed on exploding various smokeless powders in a closed chamber.*

Kind of powder.	Density of loading.	Pressures observed in closed chamber. (Kilograms per sq. cm.)
Pyroxyline powders:		
Sample A	0.250	2,840 2,830
	.225	2,475 2,470
	.200	2,090 2,130
	.175	1,790 1,800
	.150	1,525 1,500
	.125	1,245 1,220
	.100	930 950
Pyrocollodion, density 1.62†200	2,270
	.150	1,630
	.100	975

* The figures in these columns are taken from Table VI of the translation into Russian of Noble and Abel's first work, published in Vol. IV of the *Mémorial de l'Artillerie de la Marine*, p. 441.

† Determined by the scientific-technical committee of the naval service at the Naval Laboratory.

TABLE XVIII.—*Pressures observed on exploding various smokeless powders in a closed chamber—Continued.*

Kind of powder.	Density of loading.	Pressures observed in closed chamber. (Kilograms per sq. cm.)
Pyroxyline powders—Continued.		
Sample B	0.2	1,823
French powder, B*05	503
	.1	1,060
	.15	1,688
French powder, sample 148*2	2,339
	.3	4,108
Nitroglycerine powders:		
Noble182	2,000
German182	1,950
Cordite*2	2,590
	.3	4,553

If the products were ideal gases, then, speaking in general terms, the relation between the pressure P , the density of the unit weight of products (in the case in question this would be the density of loading) of decomposition $\Delta = \frac{1}{v}$ and their temperature t° C. would be determined by the laws of Boyle-Mariotte and Gay-Lussac—i. e., by Clapairon's equation, to which may be given the form

$$P = R \Delta T = \frac{R}{v} (273 + t)$$

where v indicates the volume of the powder chamber and consequently of the products of decomposition; T , their absolute temperature, and R , a constant determined from the original composition of the products under a normal atmospheric pressure $P_0 = 1.0333 \frac{\text{kgm.}}{\text{cm}^2}$ a temperature $t_0 = 0$ and a density of loading $\Delta_0 = \frac{1}{v_0}$ corresponding to this pressure and temperature, namely:

$$(15) \quad R = \frac{P_0}{273 \Delta_0} = \frac{P_0 v_0}{273}.$$

As a matter of fact powder gases, as all gases in general, are not ideal and do not follow the stated law. In view of this, other equations of condition have been proposed for actual gases. The best known of

*The figures are taken from Vieille's paper, "Études des Pressions Ondulatoires;" see Mémorial des Poudres et Salpêtres, Vol. III, 1890, p. 204, and from his paper cited in par. 35.

these are the formulæ of Van der Wals and also those of Clausius and Sarrau, which may be written as follows:

$$(16) \quad \begin{cases} P = \frac{RT}{v-\alpha} - \frac{b}{v^2}, \\ P = \frac{RT}{v-\alpha} - \frac{f(T)}{(v+\beta)^2}, \\ P = \frac{RT}{v-\alpha} - \frac{D}{E^T(v+\beta)^2}, \end{cases}$$

where P , R , v , T —have the significance above given; α is a constant serving as a measure of volume of the particles of the gases, b, β, D , and E —are other constant coefficients characterizing the body in question as also R and α ; and $f(T)$ is a certain function of the absolute temperature.

To represent more clearly the above equations of condition we shall proceed to a simple deduction of them.

As has already been stated in considering the heat capacities of gases (p. 56) the full store of energy K of the latter is included in the vis viva K_n of the progressive motion of the centers of density of the particles and in the molecular energy K_m (the vis viva of the rotating movements of the particles and the oscillatory movements of the atoms constituting the particle). The direction and the velocity of the linear motions of the particles change upon passage from one particle to another, but as they are very great in number, all possible directions of movement and all possible velocities are represented. The particles impinge against one another, strike against the walls of the chamber and rebound from one another and from the walls. All these shocks proceed from the conservation of energy, which therefore remains constant for the sum of all the particles.

Although an uninterrupted transformation of vis viva of progressive motion into molecular energy commonly proceeds from the shock of the particles of the given gaseous mass against one another, yet on account of the very great number of these particles in a very small mass of the gas, we must admit that for constant temperature there is always established an equilibrium between the conversion of the two forms of energy. Therefore each of them, and consequently the relation $\frac{K_n}{K}$, may be considered as constant quantities for each particular case considered, but these values, generally speaking, change with the temperature.

Let us now assume a certain chamber (fig. 1) filled with gas, the number of particles of which we may represent by n and the mass of each particle by m . Let the chamber have a movable bottom, the areas of which is s . Let the height of the chamber be represented by h . The

particles will, among other things, impinge against the bottom and recoil from it. A certain number of them, A, will strike in a direction normal to the bottom, and the rest, A', at a certain angle thereto.

In general, the impulses of each of them will be different, but in consequence of their great number we may assume that the blows of the particles against the bottom are distributed equally over its area and follow one another continuously; therefore, in order to retain the bottom in position from the outside we must assume a certain equally distributed pressure, Ps. The influence of the attraction of gravity upon the gaseous mass may be neglected.

Although the direction of the progressive motion of the particles may be whatever we choose, yet, on account of their great number we may assume, in accordance with Kroenig's* hypothesis, that the same number of particles move in the three direct perpendicular directions, and of these one moves normal to the area in question, and the other two move parallel to it. In consequence of this, $\frac{n}{3}$ of the particles will move in a direction perpendicular to the base of the chamber, while the rest, $\frac{2n}{3}$, will move in a direction parallel to it, and will not influence the pressure.

The vis viva of the progressive motion of any gaseous particle is $\frac{m u^2}{2}$ and the whole energy of such movement is $K_n = \frac{m}{2} \Sigma u^2$.

As the velocities may be whatever we choose to assume, and the number of particles is great, we may, for simplifying the deduction, instead of the actual velocities u^2 , assume for them a mean velocity u_2^2 , so chosen that the whole energy K_n does not change—i. e.,

$$K_n = \frac{m}{2} \Sigma u^2 = \frac{m}{2} n u_2^2$$

where u_2 represents the square root of the arithmetical mean of the sum of the squares of all the velocities of the particles.

Let us consider the action of anyone particle A which at the moment of its normal impulse against the bottom possesses the mean velocity whose square root is u_2 , and after impact — u_2 . Let us denote by F the mean pressure which must be brought to bear upon the bot-

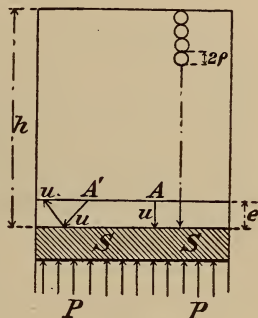


FIG. 1.

*Poggendorff's Annalen, vol. 99, p. 315. A more exact consideration of this question may be found in Chvolson's Course in Physics, vol. 1, and in Jamin et Bouty, vol. 2, Part III.

tom of the chamber in order to sustain it in equilibrium throughout a period of time θ against the action of the impulse, then,

$$F\theta = \int_{-u_2}^{+u_2} m du_2 = 2 m u_2$$

whence

$$F = \frac{2 m u_2}{\theta}$$

We ascertain the *laws of ideal gases* under the assumption that the particles of the gases possess no volume and do not attract one another. Under these conditions the progressive velocity u_2 of the particles is the same throughout the whole volume of the chamber. In consequence of this any particle having recoiled from the bottom and proceeded through a distance $2h$ again recoils from it; to do this a time $\theta_1 = \frac{2h}{u_2}$ is required. The number of impulses in the unit of time will consequently be $\frac{u_2}{2h}$. Denoting the corresponding pressure throughout the unit of time as F_1 , and multiplying the second member of this expression for F by the number of impacts we obtain $F_1 = \frac{m u_2^2}{h}$

Multiplying the second member of this equation by $\frac{n}{3}$ we obtain an expression for the whole pressure, P_s —i. e.,

$$1 = \frac{n}{3} \frac{m u_2^2}{h}$$

or

$$P_s v = \frac{n m u_2^2}{3} = \frac{2}{3} K_n,$$

where $v = sh$ = the volume of the chamber and $K_n = n \frac{m u_2^2}{2}$, the full energy of the progressive motion of the particles of the gas.

If we include the same quantity of gases at the same temperature in another chamber, the volume of which is v_1 , we obtain in a similar manner

$$P_1 v_1 = \frac{n m u_2^2}{3} = \frac{2}{3} K_n$$

since in the second case n , m , u_2 , and consequently K_n remain the same.

In consequence of this $Pv = P_1 v_1 = R' = \frac{2}{3} K_n$, where R' represents a

certain constant. The equation as written represents the law of Boyle-Mariotte and shows that the product Pv for a given temperature is a constant quantity R' and equals two-thirds of the whole energy of the progressive movement of the gaseous particles.

If a mixture of gases be inclosed in the same volume, the masses of which are $n m, n' m', n'' m'', \dots$ and the mean square roots of the velocities of the particles are u_2, u'_2, u''_2, \dots then, reasoning as above, it is not difficult to obtain

$$Pv = \frac{n m u_2^2}{3} + \frac{n' m' u'^2_2}{3} + \frac{n'' m'' u''^2_2}{3} + \dots$$

$$= p v + p' v + p'' v \dots$$

and consequently

$$P = p + p' + p'' \dots$$

which represents Dalton's law—i. e., that the pressure of a mixture of gases is equal to the sum of their partial pressures.

In order to establish an equilibrium in the mixture of gases, it is necessary to assume that the energies of the progressive movements of the particles of each gas are equal among themselves, that is

$$\frac{m u_2^2}{2} = \frac{m' u'^2_2}{2} = \frac{m'' u''^2_2}{2} = \dots$$

If this equation were not established, we would be compelled to assume a change in the temperature of the particles, which could not take place in the case of an established gaseous mixture.

If we next include such a volume of these gases in a chamber v , at a given temperature t° , that the pressures are identical, then as found,

$$Pv = \frac{n m u_2^2}{3} = \frac{n' m' u'^2_2}{3} = \frac{n'' m'' u''^2_2}{3} = \dots$$

and as we have demonstrated that for a given temperature the vires vivæ of the separate particles of all gases are the same, then from this last equation follows the law of Avogadro, which is that all gases possess the same number of particles in the same volume for the same temperature and pressure. Calculation shows that at 0° and at atmospheric pressure there exists in 1-cm^3 of each gas about 20 trillions of molecules, possessing a mean distance between molecules of from three to four millionths of a millimeter.

Finally, if we multiply both members of the expression $Pv_0 = R'$ for a temperature $t = 0^\circ$ by the quantity $(1 + \alpha t)$ we obtain

$$Pv_0 (1 + \alpha t) = Pv = R' (1 + \alpha t)$$

where $v = v_0 (1 + \alpha t)$

As, on the other hand,

$$Pv_0 = \frac{n m u_{2.0}^2}{3}, \quad Pv = \frac{n m u_2^2}{3}$$

therefore

$$\frac{u_2^2}{u_{2.0}^2} = 1 + \alpha t$$

For any other gas at the same temperature we ascertain

$$\frac{u_2'^2}{u_2'^2} = 1 + \alpha' t$$

but in this case

$$\frac{m u_2'^2}{2} = \frac{m' u_2'^2}{2}$$

and

$$\frac{m u_2'^2}{2} = \frac{m' u_2'^2}{2}$$

whence

$$\frac{u_2'^2}{u_2'^2} = \frac{u_2'^2}{u_2'^2}$$

it therefore follows that

$$1 + \alpha t = 1 + \alpha' t$$

and consequently $\alpha = \alpha'$, i. e., all ideal gases possess the same coefficient of expansion, which establishes Gay-Lussac's law.

Regnault's experiments show that $\alpha = \frac{1}{273} = 0.003663$, whence

$$P = R' \frac{273 + t}{273} = R T = \frac{2}{3} K_n$$

where $R = \frac{R'}{273}$, and $T = 273 + t$. This expression represents Clapairon's equation, which is connected with the laws of Boyle-Marriotte and Gay-Lussac. It also shows that the full energy of progressive motion of the particles of a gas is proportional to its absolute temperature, T .

The law of Avogadro shows that the weight of a molecule of a gas is proportional to its density. As the weight of a molecule of hydrogen is assumed as 2 and the density of each gas in relation to hydrogen = 14.44δ , where δ represents the density of gas in relation to the air, the molecular weight M of each gas becomes

$$M = 28.88\delta.$$

We shall determine the constant $R = \frac{Pv}{T}$ in Clapairon's equation (1) for one kilogram of gases, assuming the kilogram and the decimeter as units and reckoning $T = 273$; for which units

$$P = 103.33; \frac{\text{k-gr}}{\text{sq dm}} \quad w = \frac{1}{\delta_1} = \frac{1}{0.000131844\delta}$$

where δ_1 represents the density of the gas in relation to water and 0.000131844g the weight of 1 cubic decimeter of dry air in kilograms for an acceleration of gravity, g ; (2) for a gram-molecule of gas, the volume of which is 22.32 cubic decimeters at atmospheric pressure and for $T = 273^\circ$; for a unit of length, the decimeter.

In the first case, for St. Petersburg,

$$R = \frac{103.33}{273 \times 0.012945148} = 29.239\delta^{-1}$$

and in the second

$$R = \frac{22.32}{273} = 0.081758$$

Clapairon's equation therefore assumes the form for a kilogram of gas with the units kilogram and decimeter

$$Pv = 29.239 \delta^{-1} T$$

and for the gram-molecule for the units atmospheric pressure and decimeter

$$Pv = 0.081758 T$$

Finally, on the basis of the above deductions

$$R = \frac{n m u_{2.0}^2}{3 \times 273}$$

Equating this with the corresponding expression for Pv we obtain

$$u_2^2 = \frac{u_{2.0}^2}{3} \frac{T}{273}$$

whence

$$u_2 = u_{2.0} \sqrt{\frac{T}{273}}$$

i. e., the square root of the arithmetical mean of the sum of the squares of the velocities of the particles is directly proportional to the square root of the absolute temperature.

From the expression Pv we obtain

$$\frac{n m u_2^2}{3} = RT$$

Multiplying both members by g , the acceleration of the force of gravity, assuming the weight of the gas, $n m g$, as equal to 1 kilogram, and substituting for R its value as found for 1 kilogram of gas, we obtain

$$u_2 = \sqrt{29.239 \cdot 3g \frac{T}{\delta}} = 293.49 \sqrt{\frac{T}{\delta}}$$

For $T = 273^\circ$,

$$u_{2.0} = 4,849 \sqrt{\frac{1}{\delta}}$$

where the coefficient 4,849 represents the mean square root of the velocity of the particles of air in $\frac{dcm}{sec}$ for 0° C. and at the pressure of the atmosphere, since for air, $\delta = 1$. For other gases at different temperatures, we obtain the following values for the mean square roots of the velocities of their particles in $\frac{m}{sec}$:

For temperatures.	Meters per second.				
	0°	100°	1,000°	2,000°	3,000°
Velocity of molecules of hydrogen	1,843	2,153	3,978	5,254	6,378
Velocity of molecules of marsh gas	651	758	1,402	1,874	2,248
Velocity of molecules of water vapor	614	718	1,326	1,772	2,127
Velocity of molecules of carbonic oxide	493	576	1,064	1,422	1,707
Velocity of molecules of nitrogen	492	575	1,062	1,419	1,703
Velocity of molecules of air	485	567	1,047	1,399	1,679
Velocity of molecules of oxygen	461	539	996	1,331	1,597
Velocity of molecules of carbonic acid	392	458	847	1,131	1,358

Each increase, dK , of the full energy of the unit volume of gases is proportional to the increase dT of the temperature for constant volume, i. e.,

$$dK = Jc_v dT$$

As $K=0$, for $T=0$, therefore,

$$K = Jc_v T$$

On the other hand, from expression 8, page 55,

$$c_p - c_v = \frac{P_0 v_0}{273 \cdot J}$$

Therefore, bearing in mind the expression for K deduced above and for Pv (p. 72) we obtain

$$K(c_p - c_v) = \frac{P_0 v_0}{273} c_v T = c_v RT = c_v \frac{2}{3} K_n$$

whence

$$\frac{K_n}{K} = \frac{3}{2} \left(\frac{c_p}{c_v} - 1 \right) = \frac{3}{2} (k - 1)$$

This expression shows that the ratio of the energy of progressive motion of the molecules of a gas to its full energy is a constant independent of temperature.

Employing this, we also find

$$\begin{aligned} \frac{K_m}{K} &= \frac{3}{2} \left(\frac{5}{3} - k \right) \\ \frac{K_m}{K_n} &= \frac{\frac{5}{3} - k}{k - 1} \end{aligned}$$

which enables us to calculate the molecular energy of the gaseous particles and its ratio to the energy of progressive motion.

As the molecular energy possesses as its limits 0 and K , the ratio of the specific heats, k , may be included between 1 and $k = \frac{5}{3}$.

For the latter value for k the molecular energy of the gas equals 0, and as such a ratio of specific heats (see p. 60) belongs to the mono-atomic gases, the full energy of the latter is included altogether in the progressive energy of the movements of their molecules.

The laws of *actual gases* are deduced upon the assumption that each particle occupies a certain volume, e. g., a sphere of radius ρ , and that there exists between the particles a reciprocal reaction in the form of an attractive force. In consequence of the latter, the gaseous particle

preserves its velocity, u_2 , only as long as the mean effect of the medium remains the same upon it on all sides; therefore, it follows that the particles which are found at a certain distance, e (fig. 1, p. 69), from the walls, e. g., from the bottom of the chamber, will receive less effect from the particles in the direction of the bottom than from those in the middle of the chamber, hence the progressive motion of each particle in a direction normal to the bottom begins to diminish as soon as the particle passes the limit e , from which it follows that it impinges against the bottom with a certain diminished velocity u_2' . Having rebounded with this latter velocity, the particle moves upward and its movement will be accelerated, since the greater of the attractive forces acting upon it is exerted in the direction of its movement. Such an accelerated movement will continue as long as the particle does not pass the limit e ; upon arriving there, however, its velocity again becomes equal to u_2 , and the energy of the particle consequently becomes uniform.

Reasoning as in the deduction of the laws of ideal gases we obtain

$$F = \frac{2 m u_2'}{b}$$

After the particle under consideration has recoiled from the bottom it strikes again, evidently, after it has traversed the distance $2(h - 2\nu\rho)$, where ν represents the number of particles placed ("lined up") in the same normal line of movement. If the difference in the velocities of the particle during its movement through the distance $\pm e$ be neglected, the elapsed time after which the particle will again strike the bottom becomes $\frac{2(h - 2\nu\rho)}{u_2}$, and the number of impacts in the unit of time, — $\frac{u_2}{2(h - 2\nu\rho)}$. Consequently, for the unit of time

$$F = \frac{m u_2' u_2}{h - 2\nu\rho}$$

and the full pressure

$$Ps = \frac{n}{3} \frac{m u_2' u_2}{h - 2\nu\rho},$$

or

$$P(v - \alpha) = \frac{n m u_2' u_2}{3} = \text{Const.},$$

where $\alpha = \nu \cdot 2\rho s$ represents the volume in which the molecules are packed together, and therefore α is proportional to the volume of the molecules themselves and is called the *covolume* of the gas in question.

On the other hand, each energy is proportional to the absolute temperature T , and, therefore,

$$\frac{n m u_2'^2}{3} = R T$$

or

$$\frac{n m u_2' u_2}{3} = R T \frac{u_2}{u_2'}$$

Substituting this in the previous equation we obtain

$$P = \frac{R T}{v - \alpha} \frac{u'_2}{u_2}$$

In order to deduce the ratio $\frac{u'_2}{u_2}$ which, generally speaking, is close to unity, we denote by f the mean value of the force altering the velocity of the particle in its passage through the distance e ; then

$$fe = \frac{1}{2} m (u_2^2 - u_2'^2) = \frac{1}{2} m u_2^2 \left[1 - \left(\frac{u_2'}{u_2} \right)^2 \right]$$

whence

$$\left(\frac{u_2'}{u_2} \right)^2 = 1 - \frac{2fe}{m u_2^2}$$

and, on account of the small value of the ratio $\frac{2fe}{m u_2^2}$

$$\frac{u_2'}{u_2} = 1 - \frac{fe}{m u_2^2}$$

Finally, as $\frac{m n u_2^2}{3} = R T$ we obtain

$$\frac{u_2'}{u_2} = 1 - \frac{n}{3} \frac{fe}{R T}$$

and

$$P = \frac{R T}{v - \alpha} - \frac{n}{3} \frac{fe}{v - \alpha}$$

In relation to the quantities f and e it is necessary to observe the following:

The quantity e represents the sphere of action of the particles upon one another. It does not depend on the number of molecules in the volume, i. e., on the density of the gas, but may vary with change in temperature; whence we have, in general terms,

$$e = F(T)$$

The magnitude of the mean force f acting upon the particle depends evidently on the number of particles and upon the distance between them, i. e., upon the density of the gas. It may also depend upon the temperature of the gas, and therefore we may write in general terms

$$\frac{n}{3} f = \Phi(Tv)$$

Substituting this in the preceding expression we finally obtain

$$(17) \quad P = \frac{R T}{v - \alpha} - \frac{F(T) \Phi(Tv)}{v - \alpha}$$

From this general equation it is not difficult to derive the partial values of the expressions of Van der Wals, Clausius, and Sarrau as given in formula (16), page 68.

According to the definitions of Amagat,* “the covolume represents the smallest volume which the gaseous mass is able to occupy under

* Comptes Rendus de l'Academie des Sciences, Vol. XC, p. 995, and XCI, p. 428, 1880.

great pressure" (compare what is stated on p. 76), in establishing which he assumes that when a gas is compressed the phenomenon proceeds as if there existed between the particles of the gaseous substances a mobile liquid (more or less compressible than the ether), which follows rigidly the laws of Boyle-Mariott and Gay-Lussac; the particles themselves exert no other influence than that of occupying a certain volume, and are, as it were, hard bodies floating in the liquid.

On the basis of Amagat's experiments Sarrau* found by calculation that the relation of the covolume of a given gaseous body to the volume occupied by it at 0°C and atmospheric pressure $\left(1.0333 \frac{\text{kg.}}{\text{cm}^3}\right)$ varied for certain gases between narrow limits, namely,

Nitrogen	0.001359	Oxygen	0.000890
Marsh gas001091	Hydrogen000887
Ethylene000997	Carbonic acid000886

In view of this, Mallard and Le Chatelier assume, in application to explosive substances,† what Bouty allows for all gases in general—i. e., that the desired ratio is uniform and equal to 0.001.

In our case the covolume becomes, evidently,

$$\alpha = 0.001 v'_0 = 0.001 \frac{V'_0}{\omega}$$

where v'_0 represents the volume of the gases evolved on burning a unit weight of explosive substance, reckoning water as existing in the form of steam (at a temperature 0°C. and a pressure $1.0333 \frac{\text{kg.}}{\text{cm}^3}$); and

where V'_0 represents the same thing for a weight of charge ω .

In application to explosive substances whose action is considered at high temperatures, when the gases are far from a state of compression, $F(T)$ is less than unity and $\Phi(Tv)$ —i. e., the force of reciprocal action of the molecules upon one another, approaches 0. In view of this we may, with sufficient accuracy for practical purposes, neglect the second member of the expression 17 on page 76.

The maximum pressure of gases in a closed chamber—i. e., the pressure at which their temperature is equal to that of decomposition—then becomes

$$(18) \quad P = \frac{R T_1}{v - \alpha} = \frac{R \Delta T_1}{1 - \alpha \Delta}$$

where T_1 is the absolute temperature of decomposition and R equals

$$\frac{P_0 (v_0 - \alpha)}{273} = \frac{P_0 (1 - \Delta_0 \alpha)}{273 \Delta_0}$$

* See Comptes Rendus, Vol. XCIV, pp. 639, 718, 845.

† See Memorial des Poudres, etc., Vol. II, p. 444.

but since α is very small in comparison with v_0 , and bearing in mind the degree of accuracy of our observations, we may neglect the quantity α in the expression and then R assumes the form 15—i. e., the same as for perfect gases.

Denoting the pressure of the powder gases for a density of loading $\Delta=1$ by F , we obtain from equation 18

$$R=F \frac{1-\alpha}{T_1}$$

and consequently

$$(19) \quad P=F \frac{1-\alpha}{v-\alpha}=F \frac{(1-\alpha) \Delta}{1-\alpha \Delta}$$

The quantities F and $(1-\alpha)$ are constant for a given variety of powder, and therefore, denoting their product by f we obtain

$$(20) \quad P=f \frac{\Delta}{1-\alpha \Delta}=\frac{f}{v-\alpha}$$

Equating expressions 15, 18, and 20 together, for conditions of explosion in a closed chamber, we find also that

$$(21) \quad f=F(1-\alpha)=R T_1=P_0 \frac{v_0' T_1}{273}=0.0038 v_0' T_1$$

The quantity f is called the *force of the powder* and it serves as a characteristic of the substance just as α does. It is evident from expression 21 that f , the force of the powder, represents the work of expansion of the gases under a constant pressure P_0 , on heating the gases to a temperature T_1 . In actual fact the force of the powder f is measured in units of pressure.

Substituting in equation 20 $P=f$ we find that a certain density of loading Δ_r corresponds to such a condition, the said density of loading being determined from the expression

$$\frac{\Delta_r}{1-\alpha \Delta_r}=1$$

i. e.,

$$\Delta_r=\frac{1}{1+\alpha}$$

Consequently the volume v_r in which the unit weight of powder should burn in order to develop a pressure measured by the value f , will be $v_r=1+\alpha$, and as α represents the covolume we also have that on burning a unit weight of powder in the volume v_r the *compressed ether* occupies a volume equal to the unit of the volume. In this manner we may also state that the *force of the powder* (and of all explosive substances in general) denotes that pressure which would be developed upon burning a unit weight of powder (or of an explosive substance) in a closed chamber the volume of which is equal to the assumed unit of volume plus the volume of the covolume of the given body; the density of loading corresponding to the unit volume (v_r) will be, therefore, less than unity.

It may also be stated that *the force of the powder equals the pressure developed on burning a unit weight of powder in a closed chamber of such a capacity (v_v) that the "compressed ether" formed upon the combustion of the gases occupies a volume ($v_f - \alpha$) equal to the assumed unit of volume.*

From expression 21 we see that the force of the powder, f is always less than the pressure F , corresponding to a density of loading equal to unity.

As $T_1 = 273^\circ + t_1 = 273^\circ + \frac{Q'_v}{C_v}$ where Q'_v and C_v have the significations above given; then, upon substituting for T_1 in equation 21 we obtain

$$(22) \quad f = P_0 v_0' + P_0 \frac{v_0'}{273} \frac{Q'_v}{C_v} = P_0 v_0' [1 + \alpha t_1]$$

Bearing in mind that in comparison with the second member, the quantity $P_0 v_0'$ is usually small, the expression as written shows that the force of powders is almost proportional to the "characteristic expression" $\frac{v_0' Q'_v}{C_v}$ (produit caractéristique) of Berthelot.* If we

assume the mean specific heats to be equal for powders that approximate to each other in composition, the forces of the powders become almost proportional to the product $v_0' Q'_v$ (compare paragraph 18).

Formula 22 may be employed for calculating the force of a powder when the composition of its products of combustion is known.

23. *The investigation of the expressions for pressure and the signification of covolume.*

We shall continue the investigation of expression 20. The density of loading Δ may, generally speaking, vary between the limits $\Delta = 0$ and $\Delta = \delta$ where δ equals the density of the powder; therefore, there exists evidently for each powder a certain density of loading Δ_∞ , for which the denominator of the expression 20 reduces to 0, and consequently the pressure P becomes infinitely great. This value of the density of loading will be

$$23. \quad \Delta_\infty = \frac{1}{\alpha},$$

i. e., it equals the reciprocal of the value of the covolume, and the volume of the chamber consequently equals the volume of the covolume.

For $\delta \geq \Delta > \Delta_\infty$ the formula under consideration 20 gives a negative value for P , but under actual conditions and for such densities of loading, the pressure would be infinitely great. In general terms, it may be said that the pressure becomes infinitely great when the minimum volume of the products of decomposition (i. e., their covolume) equals to or is greater than the volume of the chamber in which the combustion of the powder occurs. But will all the material included in the chamber explode under such conditions? To this question there

* See M. Berthelot's *Sur la force des Matières Explosifs d'après la thermochimie*, Vol. I, p. 64, 3d ed., 1893.

is no direct, definite answer. In conducting the corresponding experiments we conclude that for densities of loading not less than Δ_∞ , **23**, although the substance may not wholly explode, the developed pressure is greater than any measurable magnitude, i. e., we assume that although the covolume under certain conditions would be greater than the volume of the chamber in which the explosion occurs, the whole material decomposes and develops a pressure greater than any measurable quantity. If we assume under these conditions that the pressure is not infinitely great then we are forced to assume in lieu thereof a radical change in the nature of the decomposition products, namely, that their composition changes in such a manner that the covolume diminishes. It follows from this that formula **20** may be employed for all densities of loading $\Delta \leq \Delta_\infty$ (**23**).

Let us now assume that we have two powders or two explosive materials, one of which possesses the characteristics f and α and the other f' and α' ; it is evident from formula **20** that the pressures developed by these powders for the same density of loading will, speaking in general terms, differ from one another. Beginning with lesser densities, let us say that one of the pressures P will be greater than the other, P' ; subsequently, for a certain density Δ_p obtained by equating the expressions (**20**) for the two powders under consideration, i. e.,

$$\Delta_p = \frac{f-f'}{\alpha'f-\alpha f'} = \frac{1}{\alpha} \frac{\frac{f}{f'}-1}{\frac{\alpha'}{\alpha} \frac{f}{f'}-1}$$

or, see expression **23**,

$$\Delta_p = \Delta_\infty \frac{\frac{f}{f'}-1}{\frac{\alpha'}{\alpha} \frac{f}{f'}-1} \quad \text{24.}$$

we find that the pressure P becomes equal to P' ; while for densities of loading greater than Δ_p , the pressure P will be less than P' . Such is the result of the mathematical investigation of equation **20** for the powders under consideration. In practice it may happen that the equality of pressures will be found only for a value of Δ_p greater than the lesser of the densities Δ_∞ and Δ'_∞ (**23**) for which the pressure P or P' , as above stated, becomes infinitely great, which is equivalent to stating that the equality of the pressures under consideration can not occur at all. On this account we shall determine the conditions under which the equality $P=P'$ may exist in practice. Expression **24** shows

that $\Delta_p > \Delta_\infty$:

(1) For $f > f'$, if $\alpha = \alpha'$, or what is the same, if $\Delta'_\infty > \Delta_\infty$; and (2)

for $f < f'$, if $\alpha = \alpha'$, or what is the same, if $\Delta'_\infty < \Delta_\infty$.

Examining these conditions we see that an equality of pressures may be obtained only for $f \geq f'$ and $\alpha \leq \alpha'$ with a density of loading Δ_p (24) which is positive and less than the values Δ_∞ and Δ'_∞ , for which the given powders develop infinitely great pressures.

It is, consequently, possible in these cases only to obtain a great pressure by employing a powder whose force is smaller but whose covolume is greater. This remarkable property has a practical significance in mining where great pressures are employed. This conception is chiefly based upon the fact that the greater the covolume of a given material the more readily may we obtain an infinite pressure with a smaller weight of it (i. e., for a smaller density of loading Δ_∞), although the force f of this substance may be very small. This explains why efforts to obtain such explosive materials have been made in recent times. In cannon comparatively small pressures are required, which correspond, for strong powders, such as the smokeless, to a small density of loading almost always less than Δ_p , (24). In view of this, better results are obtained in ordnance with these powders now existing, whose force f is greater (see Table XIX further on), irrespective of the magnitude of their covolumes, under the condition, however, that the quantity of heat given off be not less.

Let us take the first derivatives of expressions 19 and 20 for α ; we obtain

$$\frac{dP}{d\alpha} = F \Delta \frac{\Delta - 1}{(1 - \alpha \Delta)^2}$$

and

$$\frac{dP}{d\alpha} = f \frac{\Delta^2}{(1 - \alpha \Delta)^2}.$$

The first of these derivatives shows that it is negative for densities of loading less than unity, so that the value of P for the same values of F and Δ increases as α diminishes; while it follows from the second expression that on diminishing α the pressure also diminishes. This seeming contradiction is fully explained, if we remember that in this case the quantity under consideration is f and not F , and that this, itself, diminishes as α increases. (See formula 21.)

24. *The determination of the temperature of decomposition, covolume, and the force of the powder.*

As the determination of the pressures, volumes of gases (including water) v_0' , and the quantities of heat Q_0 , is least difficult of all, we may, under the assumption that the magnitude of the covolume and expression 18 are applicable to powders, determine the temperature of decomposition, by employing expressions 21 and 19, namely,

$$25. \quad T_1 = \frac{273^\circ}{P_0} \frac{f}{v_0'} = 264, 208 \frac{F(1 - \alpha)}{v_0'} = 10 \frac{2.42195}{v_0' \Delta} \frac{P(1 - \alpha \Delta)}{v_0' \Delta}.$$

All the quantities given in the second members of these expressions may be comparatively easily determined by experiment.

Bearing in mind, subsequently, that $t_1 = T_1 - 273 = -\frac{Q'_v}{C_v}$, we may, knowing the temperature T_1 , (25) and the quantity of heat Q'_v evolved, ascertain the mean specific heat of the products of decomposition of a given powder, namely,

$$26. \quad C_v = \frac{Q'_v}{t_1^0} = \frac{Q'_v}{T_1 - 273}$$

where C_v represents the magnitude of the mean specific heat of the products of decomposition at constant volume, the temperature of which changes from t_1^0 to 0°C .

Let us apply this to the pressures evolved by smoking powders. The varieties most studied are the powders P., R. L. G., and F. G., the experimental data relative to which are given in Tables XVI and XVII. As we know (section 15), the products of combustion of smoking powders at ordinary temperatures become gaseous, liquid, and solid. We assume that the last, at the moment of decomposition (paragraph 16), occur in a minutely subdivided state, completely mixed with the gases. In the given case the coefficient α in expression 18 should, therefore, be equal to the covolume of the gases added to the volume of nonvolatile products at the temperature of decomposition (pages 46 and 77), i. e., $\alpha = 0.28 + 0.55 = 0.83$. Direct calculation, however, convinces us that this affords pressures considerably less than those actually observed. The question therefore arises whether the particles of a gaseous body upon mixing with the particles of a liquid body undergo any contraction in their general volume. In order to establish the conception of covolume, first Bunsen and Shishkoff,* and subsequently Noble and Abel,† proposed for calculating the pressures what was practically the same thing, namely, that the powder gases follow the laws of Boyle-Mariotte and Gay-Lussac, and that the solid products are nonvolatile at the temperature of decomposition, and consequently only diminish the volume of the chamber in which the products of decomposition are inclosed.

According to the investigations of the former, $\alpha = 0.454$. The latter assumed at first $\alpha = 0.6$, and subsequently, for fully dried powder $\alpha = 0.57$, which almost equals the specific volume of the powder P, the density of which is 1.75 (page 34). Starting from these experiments of Noble and Abel, Sarrau determined ‡ $\alpha = 0.6833$. The smaller α is, the more the pressures obtained by formula 19, or what is the same, by formula 20, exceed the observed pressures. (See what is stated at the end of paragraph 22.) As Table XVII shows, the

* See their *Théorie Chimique de la Combustion de la Poudre*, Terquem's translation, p. 59, ed. 1859.

† See the *Researches*, etc., above referred to; ed. 1876.

‡ See E. Sarrau, "*Nouvelles Recherches sur les effets de la Poudre dans les Armes*," p. 9, ed. 1876.

pressures (mean and small) calculated by these formulæ for the powders P., R. L. G., and F. G. for a value of α of about 0.6, somewhat exceed the observed pressures. Until new and more exact experiments are made we shall assume this value for α as a fundamental datum; it is the more probable, since the observed pressures (see Part II, Interior Ballistics) for the methods of measurement employed are always something less than the actual pressures. Subsequently we shall assume also as a fundamental datum the mean pressure value for the powders P. and R. L. G. for $\Delta=1$ as equal to $F=6.568 \frac{\text{k gr}}{\text{c m}^2}$. In their latest experiments Noble and Abel assumed $F=6.772$

kilograms,* but as this quantity relates to perfectly dried powder, which is never used in practice, we shall rest upon figures previously deduced for experiments with damp powders.

Finally, on the basis of the experiments with the powders P. and R. L. G., we shall assume as fundamental data—

$$\begin{aligned}\alpha &= 0.6 \\ F &= 6,568 \frac{\text{kgr.}}{\text{cm}^2} = 6,356 \text{ atm.} = 41.7 \frac{\text{English tons}}{\text{dc m}^2} \\ t_0 &= 2154^\circ \text{†} & (\text{by formula 25}) \\ C_v &= 0.3298050^\circ \text{†} & (\text{by formula 26})\end{aligned}$$

In the absence of more exact data we shall assume, in calculating the covolumes of other smoking powders, that they (the covolumes) equal the volumes of the solid products at the temperature of decomposition, assuming that the specific volumes of the solid products at 0°C . and their coefficients of expansion are the same as those obtained for the powders P., R. L. G., and F. G., as based upon experimental results of Noble and Abel. Assuming then for the powders P. and R. L. G. a volume of residues at the temperature of decomposition equal to 0.6, and their mean weight (Table VIII) as 0.55375, the density of the products of decomposition at the temperature of explosion becomes 0.9229, and at a temperature of 0°C . becomes 1.91 (compare paragraph 16). In consequence of this the “covolume” of any smoking powder will be—

$$27. \quad a = \left[\frac{0.29}{0.55375} + \frac{0.31 t_1}{0.55375 \cdot 2154} \right] b_{\text{II}} = \left[10^{1.71909} + 10^{4.41480} t_1 \right] b_{\text{II}},$$

where b_{II} represents the weight of the solid products of decomposition as given in the next to last column of Table VIII.

The force of any powder or explosive material may be calculated by formula 21 if we know the volume of the gases v'_0 and the temperature $T_1 = t_1 + 273^\circ$.

* See their Researches, etc., No. II, ed. 1880.

† Calculated from the mean values for v'_0 and $Q'v$, taken from Table XIII for these powders.

The calculation of the covolume and the force of both smoking and smokeless powders is possible also from the results of the determination of pressures obtained in a closed chamber. In this case it is necessary to have at least two pressures, P and P' , corresponding to two arbitrary densities of loading, Δ , Δ' . Writing the two expressions (20) for these data, we obtain two equations with two unknown quantities, α and f . Solving them we obtain:

$$28. \quad \left\{ \begin{array}{l} \alpha = \frac{P}{\Delta} \frac{P'}{\Delta'} \\ f = \frac{P}{\Delta} \frac{P'}{\Delta'} \frac{\Delta - \Delta'}{P - P'} \end{array} \right.$$

If from other experiments we know either the covolume or the force of the powder, the other of these quantities may be obtained from equation 20, knowing one pressure for any corresponding density, as follows:

$$29. \quad \left\{ \begin{array}{l} f = P \left(\frac{1}{\Delta} - \alpha \right) \\ \text{and} \\ \alpha = \frac{1}{\Delta} - \frac{f}{P} \end{array} \right.$$

Finally, if we know the covolume of smokeless powders, and in general, of any materials, the products of combustion of which are gaseous at the temperature of decomposition, we may obtain the volume of the gases at 0°C and 760 c. c. of the mercurial column, as the covolume constitutes 0.001 of this volume; and consequently having determined one pressure we may calculate by formula 25 the temperature of decomposition.

The values of the forces for various powders, their covolumes and their other elements, as calculated from data given above, are collected in Table XIX, in which is also shown, for purposes of comparison, similar data for other explosive materials.

We consider it useful and make the following observations in general in relation to the elements presented in this table:

(1) Columns 2, 3, and 4 are taken from previous tables. We must again remember that smoking powders evolve the greater quantity of heat and possess the higher temperature, the less the volume occupied at 0° and atmospheric pressure, by their gaseous products of decomposition, including water (see page 52). The same may be said in general of the explosive materials in each group.

(2) Column 5 presents the covolumes (α), the values of which for smoking powders are calculated according to formula 27, page 83. In this latter case the covolume is not the least volume of the molecules of the gaseous products but rather the volume of solid products at the temperature of decomposition; the designation is preserved only for general purposes of comparison. For smoking powders the value of the "covolume" is greater, the less the volume of gaseous decomposition products, and almost the greater the greater the weight of the solid products, (Table VIII) which constitutes their difference from smokeless powders and other explosive substances for which the covolume equals 0.001 of the volume of the gases.

TABLE XIX.—*Forces of powders and explosive substances; their volumes and other elements.*

1	2 v_0'	3 Q_v	4 t_0°	5 α	6 f	7 Δf	8 F	9 Δ_∞	10 Δ_p	11 Pressure corresponding to a density Δ_p	12 Ratio of the force of the powder to the pressure F for the powder P .	13 Ratio of the pressure F to the pressure P .	14 Value of proportionality to Berthelot's characteristic expression, $v_0'Q_v' : 100$	15 Ratio of the characteristic expression to the same for powder P .
Designation of powders and explosive substances.	Volume of gaseous decomposition products and water vapor at a temperature of 0° and a pressure of 760 mm. (cu. cm.)	Quantity of heat evolved by 1 gram of powder, for water in the gaseous state (small cal.)	Temperature of decomposition (Degrees C.)	Covolume (cu. cm.)	Force of the powder or explosive substance (k. gr. per sq. cm.)	Density of loading corresponding to the force of the powder.	Pressure corresponding to a density of loading of unity (k. gr. per sq. cm.)	Density of loading, for which the pressure is infinitely great.	Density of loading, for which the pressure of the powder P and of the powder compared with it are the same.					
SMOKING POWDERS.														
Spanish	240.8	758.4	2,237	0.679	2,297	0.595	7,156	1.473	$P_p > P_p$ beginning with $\Delta_p = 0.8156$.	4,165	0.890	1.096	5,387	0.870
Okhta, No. 6	252.7	748.5	2,197	.633	2,372	.612	6,463	1.579	$P_{aa} > P_p$ beginning with $\Delta_p = 1.0584$.	7,829	.899	.990	5,552	.896
F. G.	277.6	718.3	2,151	.604	2,557	.623	6,490	1.655	$P_p > P_p$ beginning with $\Delta_p = 1.1357$.	9,219	.969	.994	5,971	.964
R. L. G.	284.4	711.7	2,141	.606	2,609	.623	6,622	1.650	$P_p > P_p$ beginning with $\Delta_p = 0.7816$.	3,874	.997	1.014	6,089	.983
P.	287.5	708.8	2,154	.594	2,651	.627	6,530	1.683			1.000	1.000	6,193	1.000
"D"	293.2	728.1	2,149	.605	2,701	.623	6,838	1.652	$P_{p,p} > P_p$ always $> P_p$	0	1.019	1.047	6,301	1.017
"A"	291.5	766.9	2,143	.608	2,676	.622	6,827	1.644	$P_{p,p} > P_p$ always $> P_p$	0	.969	1.045	6,247	1.009
Chocolate	315.1	758.0	2,062	.602	2,793	.624	7,018	1.662	$P_{p,p} > P_p$ always $> P_p$	0	1.052	1.075	6,497	1.049
"B"	338.3	684.9	1,974	.575	2,889	.635	6,798	1.739	$P_{p,p} < P_p$ beginning with $\Delta_p = 1.2421$.	12,556	1.070	1.041	6,678	1.078
"C"	359.4	509.4	1,728	.454	2,738	.688	5,005	2.202	$P_{p,p} < P_p$ beginning with $\Delta_p = 0.1958$.	587	1.022	.766	6,240	1.009
Mining	374.6	499.1	1,682	.443	2,772	.693	5,047	2.258	$P_m < P_p$ beginning with $\Delta_p = 0.3218$.	1,060	1.000	.773	6,376	1.080

SMOKELESS POWDER.														
Pyrocollodion cannon	945.3	880.9	2,177	.945	8,800	.512	160,000	1.018	$P_{\text{inc}} \text{ always} > P_p$	0	3,320	21,500	20,579	3,323
Sample <i>a</i> of pyroxylyne powder.	986.0	2,090	.986	8,820	.503	630,000	1.014	$P_{\text{pyrex}} \text{ always} > P_p$ $P_{\text{pyrex}} \text{ always} > P_{\text{pyr col}}$	0	3,327	96,480	20,607	3,327
Nitroglycerine, Noble.....	807.0	1,317.0	2,865	.807	9,375	.550	49,706	1.239	$P_N \text{ always} > P_p$	0	3,630	7,640	23,121	3,733
EXPLOSIVE SUBSTANCES. (*)									$P_{\text{pyr col}} > P_N \text{ beginning with } \Delta = 0.4114$	5,911	(II)			
Picric acid (†).....	877.0	759.0	2,629	.877	9,671	.533	78,626	1.140	$P_{\text{pic ae}} \text{ always} > P_p \text{ (II)}$	0	3,648	12,040	23,560	3,804
Endeka-nitrocellulose (†) ..	860.0	1,026.0	2,650	.860	9,552	.538	67,784	1.163	$P_{\text{pyr col}} > P_{\text{pic ae}} \text{ beginning with } \Delta_p = 0.6100$	12,420	(II)			
									$P_{\text{in}} \text{ always} > P_p \text{ (II)}$	0	3,603	10,450	22,790	3,680
									$P_{\text{pyr col}} > P_{\text{in}} \text{ beginning with } \Delta_p = 0.5149$	8,827	(II)			
Octo-nitrocellulose (†).....	974.0	722.0	2,064	.974	8,650	.507	331,920	1.027	$P_{\text{in}} \text{ always} > P_p \text{ (II)}$	0	3,263	50,950	20,103	3,246
									$P_{\text{pyr col}} < P_{\text{in}} \text{ beginning with } \Delta_p = 0.3796$	5,201	(II)			
Explosive gelatine (§)	709.0	1,545.0	3,224	.709	9,422	.585	32,165	1.411	$P_{\text{gel}} \text{ always} > P_p \text{ (II)}$	0	3,554	4,960	22,858	3,691
									$P_{\text{pyr col}} > P_{\text{gel}} \text{ beginning with } \Delta_p = 0.2341$	2,645	(II)			

(*) The figures in the first 5 columns are taken from Mallard and Le Chatelier; see "Mémorial des poudres et salpêtres," Vol. II, pp. 451-454.

(†) $C_{24}H_{32}N_8O_{38} + 51 C_6H_6N_2O_5$; (§) $C_{24}H_{32}N_8O_{38} + 51 C_6H_6N_2O_5$; (II) The denominators give comparative data with pyro-collodion powders taken as a unit, 9 nitrocellulose, Nitroglycerine.

(3) In columns 6, 7, and 12 are given the "forces" of powders and explosive substances (f), their corresponding densities (Δ_i) of loading and the ratios of these "forces" to the force of Pebble powder which is assumed as a unit for comparison. The forces of smoking powders are, in general, the less the higher their temperatures of decomposition and consequently the less the volume of the gases produced by them. The force of chocolate powder is higher than that of all other smoking powders with the exception of powder B and mining powder. This accounts for the ballistic superiority of chocolate powder.* The densities Δ_i , at which the pressures equal the forces of the corresponding powders are the less, the greater the covolumes. The values of the forces of smokeless powders are 3.5 times greater than the forces of smoking powders. The most powerful of the explosive substances cited (picric acid) is almost 3.65 times as powerful as Pebble powder.

(4) In columns 8 and 13 are given the value of the pressures F developed for a density of loading $\Delta = 1$ in comparison with the same pressure for Pebble powder. As the greater the pressure F the greater the covolume, powders and explosive materials are, consequently, when arranged in order of these pressures, in an entirely different order than they are when arranged in accordance with the magnitude of their forces, as for certain substances the pressures F assume enormous values: e. g., for pyroxyline powder, $F = 630,000 \frac{\text{kgr.}}{\text{cm.}^2}$, and for octo-nitrocellulose (according to Vieille), $F = 332,692 \frac{\text{kgr.}}{\text{cm.}^2}$, i. e., for them the pressures F are 96.5 and 51 times greater, respectively, than the value of F for Pebble powder.

(5) In column 9 are given the values of density of loading Δ_∞ , for which the given substance develops an infinitely great pressure. Pyroxyline smokeless powder ($\Delta_\infty = 1.014$), octo-nitrocelluloses ($\Delta_\infty = 1.027$), and pyro-collodion powder ($\Delta_\infty = 1.048$) occupy the first positions in this relation. Explosive gelatin is almost comparable with Spanish powder. These deductions, it must be remembered, are theoretical and only could be considered true if the chambers in which the explosions took place were infinitely strong.

Actually and in practice, explosive gelatin, the decomposition of which proceeds with extreme speed, proves the most powerful. The powders "C" and mining powder can never develop infinitely great pressures, since the density of loading required for this exceeds their actual density.

(6) In columns 10 and 11 are given the values of the densities Δ_p (24), at which the pressures for Pebble powder and the substances compared therewith are equal and also the values of these pressures.

* We note here that Longridge determined the force of this powder as considerably less than the force of black powders, only because he did not take into account water vapor, as is explained in the footnote to page 63.

The figures here presented and the considerations above given (par. 5) show at a glance that in application to guns the value of the covolume possesses no direct significance. In application to mines and explosive shell the value of the covolume is almost the predominant factor, and, when detonation does not occur, the substances nearest approaching this condition appear, theoretically, to be pyroxyline smokeless powder and the octo-nitro cellulose of Vieille.

(7) In columns 14 and 15 are given the values and ratios of the characteristic expressions of Berthelot. The figures show that in this case powders and explosive materials are arranged in almost the same order as they are when compared in respect to the values of their "forces."

(8) In order to decide how nearly the results of calculation for smoking powders by the formulæ above given and the data in this table differ from actual observations, the calculated pressures are presented in Table XVII. A comparison with the corresponding columns of the latter table shows that the calculated pressures are higher than the observed, but not as much higher as Noble and Abel assumed in their memoirs. For smokeless pyroxyline powders, comparisons of calculations and observations are presented in Table XVIII, from which it is evident that there is an almost complete agreement of these quantities. In relation to other powders, whose characteristic values are presented in Table XIX, they must be considered only as approximate. Such as they are, however, they are sufficient to enable one to judge as to the relation that smokeless powders occupy to other explosive materials.

(9) The pressures of powders, smoking and smokeless, in closed chambers, must be considered in reference to the laws formulated in paragraphs 21-23, only under the assumption of absence of detonation and under the condition that the charge in the chamber (if the chamber is long) is equally distributed throughout it and that inflammation is accomplished simultaneously everywhere. If the substance happens to be at the end of a long closed chamber, then, for sufficient rapidity of combustion, there will occur local centers of evolution of gases which develop abnormal pressures. Pressures of this latter kind were carefully investigated by Vieille,* and are explained in Part II of this course. Detonation itself is a special variety of explosion for which the reaction proceeds with such rapidity that the whole charge burns almost in a space equal to its volume; and, as explained above in relation to the covolume, this leads to an infinitely great pressure. Therefore, there can then exist no mean pressure throughout the chamber. Evidently detonation can only be employed for bursting shells and in mines. The conditions indispensable for this and the phenomena relating thereto are considered in courses upon explosive materials.†

*See his work cited on page 115.

†Reference may be had upon this question to the works of Berthelot and Chvolson previously cited.

Chapter IV.

INFLAMMATION AND COMBUSTION OF POWDER IN THE OPEN AIR.

25. *The inflammation of powders—The temperature of inflammation.*

In order to inflame any powder it is necessary, speaking in general terms, to heat a small part of it to a certain temperature called *the temperature of inflammation*. The heat required to produce this result should be produced quite rapidly, for slow heating may occasion in many kinds of powder changes which prevent complete explosion. The temperature of inflammation depends upon the chemical and physical properties of the powder. The varieties most experimented with in this relation are smoking powders. According to Violett,* smoking powders explode when heated to a temperature of from 320° to 270° C. To determine the temperature of inflammation of smoking powders Violett threw powder grains and dust formed from them upon the surface of an oil bath heated to a determined temperature. He assumed as the temperature of inflammation the lowest temperature for which powder thrown upon a vessel immediately exploded.

It follows from these and from other experiments that the temperature of inflammation of powder is the higher the more thoroughly the ingredients composing it are mixed together, the denser the powder is, and the less charred the charcoal entering into its composition. Powder crushed into dust burns more rapidly than powder of the same composition and the same method of fabrication in the form of grains. The temperature of inflammation of powder dust depends but little upon the composition of the powder and varies from 270° to 265° C.

The influence of the quality of charcoal entering into the composition of the powder is exceedingly great. Thus, according to Berthelot,† some kinds of charcoal taken from the hearts of trees possess the capability of evolving carbonic acid gas at 100° C. If for any such powder, especially when it is in the form of cake, the particles of the charcoal are insufficiently blended with those of sulphur and saltpeter, the evolution of carbonic acid is easily effected, whereby, in the case of large masses of powder, its temperature is so raised that the powder explodes,

* See Désortiaux *Traité sur la poudre*, etc., ed. 1878, p. 447.

† M. Berthelot, *Sur la force des Matières Explosives*, 3d ed., Vol. II, p. 287.

although the external temperature may be considerably less than that required for explosion.

If smoking powder be heated slowly, the sulphur begins to melt at 120° C. and the grains fuse. Upon further heating the sulphur may wholly volatilize and the powder will not ignite at all. This experiment may be performed with special ease in a vacuum. Upon heating powder slowly it may happen that the sulphur melts, envelops the charcoal and saltpeter, and subsequently ignites and burns completely at a temperature of 250° , while the charcoal and sulphur remain untouched, as a much higher temperature (about 400°) is required for their decomposition. However, if the phenomenon described (combustion of the sulphur) be too prolonged, the temperature may rise so under the influence of the air, that the charcoal will ignite and occasion the decomposition of the saltpeter; in other words, the powder explodes. The inflammation of powder is most satisfactorily effected by touching it with a red-hot body. Flames of various kinds produce but a small effect upon it, especially the flame of spirits.

Besides this, powder may be inflamed by the electric spark (produced in a specially constructed apparatus). The direct action of the sparking is not always sufficient. Powder will also inflame by friction, by shock, and in general by all means capable of raising it to the temperature of inflammation.

In relation to the sensitiveness of smoking powders to shock, it is found that they explode from the impact of iron against iron or brass, brass against brass, iron against marble, quartz against quartz, and of a lead bullet against a tree, etc.

Generally speaking, the temperature of inflammation of smokeless powders is less than that of smoking powders and varies between the limits of from 170° and 190° C., according to the composition and purity of the nitrocelluloses and nitroglycerin employed in the manufacture of the powder. The temperature of inflammation of these powders is determined in the following manner: A small quantity of powder is placed in a capsule and the latter is immersed in a paraffin bath previously heated to 100° C.; further heating is so regulated that the temperature of the bath rises from 5° to 6° C. a minute, and at the moment of explosion the temperature of the bath is noted.

Smokeless powders inflame from the same causes as smoking powders, and while the former inflame evidently more easily from shock and friction than do the smoking powders, experiments conducted up to this time are insufficient for establishing definite conclusions.*

26. The velocity of inflammation of smoking powders.

The combustion of smoking powders in the air proceeds very rapidly. It is difficult to establish the laws controlling this phenomenon; nevertheless, a series of systematic experiments will show, in general, the

*TRANSLATOR'S NOTE.—Our own experiments tend to show that smoking powders explode far more easily from friction than do smokeless powders.

following: The combustion of the powder begins when any point whatsoever of the surface of the powder is heated to the temperature of inflammation. The inflammation subsequently extends over the surface in all directions uniformly, beginning with the point where the combustion started, after which the grain continues to burn (in the air) uniformly in all directions—i. e., it burns in concentric layers, the centers of which are at the center of the grain. It is therefore necessary, in investigating the combustion of the grain, to distinguish between the speed of inflammation of the grain—i. e., the speed of the propagation of the flame over the surface of the grain, and its speed of combustion—i. e., the speed of penetration of the flame within the grain. The speed of inflammation is the most difficult to determine by experiment. Piobert's* experiments, conducted principally in the open air, with smoking powders of usual composition, show that:

(1) The propagation of inflammation throughout the powder cake is very slow.

(2) In the open air the velocities of propagation of inflammation, in the case of trains of powder grains distributed along iron troughs of different diameters, appear to be almost inversely proportional to the fourth roots of the mean radii of the grains (paragraph 11). With similar grains the velocity was the less (*a*) the less the cross section of the powder train; (*b*) the more carefully the grains were polished; (*c*) the greater the density of the powder; and (*d*) the less completely the charcoal employed in the manufacture of the powder was charred. The composition of the powder, the method of its fabrication, and the time that it had been kept on hand, all other conditions being the same, had almost no influence on the speed of inflammation. In general, the speed of inflammation in troughs in the open air for different powders varied between 1 and 3 meters per second on account of various circumstances.

(3) If the gases meet any resistance to free expansion in a lateral direction, away from the powder troughs, the speed of inflammation increases. When a sheet of paper was placed over the trough filled with powder, the speed of inflammation increased from 50 to 100 per cent and more when compared with velocities obtained for the open troughs.

The speed of inflammation is very low for chocolate powder, the flame spreading very slowly over the surface of the prism, and the combustion of the grain proceeding very quietly in the open air.

27. *The speed of combustion of smoking powders.*

The speed of combustion of smoking powders in the open air was determined by Piobert. To establish this he prepared prisms of different lengths from powder cake; he placed these vertically, and to

* For particulars see G. Piobert's *Traité d'Artillerie*, Part I, *Propriétés et effets de la poudre*, 2d ed., pp. 185-239.

insure combustion by horizontal layers he covered the sides of the prisms with tallow. The time was observed by a chronograph, registering to tenths of a second. Piobert came to the following conclusions from his observations of different powders:

(1) For all powder masses the speed of combustion or the rapidity of the passage of the flame from layer to layer proceeds uniformly over the whole extent of the cross section of the homogeneous grain, and does not depend upon the magnitude of this cross section.

(2) The speed of combustion of powder cake of any one kind of powder for different densities is inversely proportional to these densities; hence it follows that almost the same quantity of substance burns away in the unit time, whatever be the density, the surfaces being equal.

(3) Brown charcoal, all other conditions being equal, lessens the speed of combustion.

(4) Prolonged admixture generally increases, but probably only to a certain degree, the speed of combustion of powders, for black charcoal as well as brown.

(5) Powders preserved for a considerable time in the form of cake, and also those preserved many centuries in the form of grains, do not apparently change their rates of combustion.

(6) Powder dried in the sun increases its rate of combustion in comparison with its rate as developed by artificial drying.

(7) The rate of combustion diminishes upon increase of moisture in the powder.

Certain numerical values relating to speeds of combustion as obtained by Piobert (see pages 130 to 157 in the work above cited) are presented in Table XX.

28. *The combustion of smokeless powders.*

There exists no complete data in relation to the inflammation and combustion of smokeless powders in the open air. We may remark, however, that the dry strips of smokeless powder burn very quietly and very slowly, and that the rate of the passage of the flame over the surface of the grain proceeds very gradually, so that speeds of combustion and of inflammation of smokeless powder in the open air are practically identical.

29. *The relative quantities of powder burned in the open air in a given time for various shapes of grain.*

If we know the speed of combustion of a given powder, smoking or smokeless, in the open air, we may deduce what quantity of it burns in a given time, t , in relation to the form and dimensions of its grains, assuming that the composition of the powder, the density of the grain, methods of manufacture, etc., remain the same.

If we denote the speed of combustion and the ratio of the weight of the burned part of the grain to its original weight by the letters u , and Ψ , we may determine this ratio as a function of the elapsed

time. All further investigation may be easily extended to any quantity of powder desired; to accomplish this, all that is necessary to do is to multiply the formulæ obtained for the unit grain by the weight of the total amount of powder. The value of Ψ thus determined represents the quantity of burned powder sought for.

(1) *Spherical grain of mean radius r .*—When a certain time, t , has elapsed, the radius of the grain becomes $r-u_0t$.

The weight of the consumed portion of the grain is—

$$\frac{4}{3}\pi[r^3-(r-u_0t)^3]\delta$$

and consequently the desired relation becomes—

$$\Psi=1-\left(1-\frac{u_0t}{r}\right)^3$$

Denoting the time of combustion of the complete grain in the open air by H_0 and observing that in the given case $H_0=\frac{r}{u_0}$, the expression for Ψ may be written as—

$$\Psi=1-\left(1-\frac{t}{H_0}\right)^3=3\frac{t}{H_0}\left[1-\frac{t}{H_0}+\frac{1}{3}\left(\frac{t}{H_0}\right)^2\right]$$

TABLE XX.—*Velocities of combustion of smoking powders in the open air.*

Method of fabrication of powder.	Composition of powder.			Density.	Speed of combustion (in mm. per second).	Quantity of burned powder in 1 sq. cm. of area of cross section of pellet (grams).
	Salt-peter.	Sul-phur.	Char-coal.			
Pulverized a second time and the cake subsequently formed under a press	75	12.5	12.5	1.600-1.740	12.8-11.60	2.038
	75	10.0	15.0	1.625-1.770	11.4-11.20	2.003
	76	11.0	13.0	1.565-1.760	12.8-11.30	2.013
Mixing conducted:						
In barrels—						
6 hours	75	12.5	12.5	11.65	1.750
9 hours					12.80	1.920
Under stamps, 11 hours					12.33	1.850
Mixed under stamps:						
20 hours	78	10.0	12.0	1.460-1.490	7.62-8.00	1.150
12 hours				1.333-1.423	8.15-8.36	1.137
Manufactured with stamps	78	10.0	12.0	* 8.86 † 8.20	1.330 1.230
Manufactured 1724-1726.†	75	12.5	12.5	1.540-1.610	12.0-11.30	1.825
Newly made				1.610-1.640	§ 12.2	1.930
Subsequently pulverized again and pressed in to pellets.						
Long preserved in pellets:						
Dried in the sun	75	12.5	12.5	1.674-1.790	11.7-11.00	1.970
Dried in the usual way				1.620-1.645	§ 11.80	1.900
With moisture—						
1.5 per cent				1.680	10.00	1.680
2.5 per cent				1.728	8.40	1.450

* Black powder.
† Brown powder.

‡ Experiments made in the sixties, nineteenth century.
§ Mean.

(2) *Cylindrical solid grain* whose height is h and the radius of whose base is r .

In the same manner as above we ascertain

$$\Psi = 1 - \left(1 - \frac{u_0 t}{r}\right)^2 \left(1 - \frac{2u_0 t}{h}\right)$$

The time of complete combustion of this grain will be either $\frac{h}{2u_0}$ or $\frac{r}{u_0}$, according to which dimension of the grain is greater.

(a) Let us assume that $h < 2r$; then $H_0 = \frac{h}{2u_0}$ and let, besides this, $h = \alpha r$; the desired function then becomes

$$\Psi = 1 - \left(1 - \frac{\alpha t}{2H_0}\right)^2 \left(1 - \frac{t}{H_0}\right) = (\alpha + 1) \frac{t}{H_0} \left[1 - \frac{\alpha(\alpha + 4)}{4(\alpha + 1)} \frac{t}{H_0} + \frac{\alpha^2}{4(\alpha + 1)} \left(\frac{t}{H_0}\right)^2\right]$$

(b) Let us now assume $h > 2r$; then $H_0 = \frac{r}{u_0}$ and the desired function becomes

$$\Psi = 1 - \left(1 - \frac{t}{H_0}\right)^2 \left(1 - \frac{2}{\alpha} \frac{t}{H_0}\right) = 2 \frac{\alpha + 1}{\alpha} \frac{t}{H_0} \left[1 - \frac{\alpha + 4}{2(\alpha + 1)} \frac{t}{H_0} + \frac{1}{\alpha + 1} \left(\frac{t}{H_0}\right)^2\right]$$

(c) Assuming that $h = 2r$, then $H_0 = \frac{h}{2u_0} = \frac{r}{u_0}$ and the desired function becomes

$$\Psi = 3 \frac{t}{H_0} \left[1 - \frac{t}{H_0} + \frac{1}{3} \left(\frac{t}{H_0}\right)^2\right]$$

i. e., the cylindrical solid grain whose height equals its diameter burns in relatively the same manner as the spherical grain inscribed within it, but the absolute quantity of the powder burnt is evidently different for these grains.

(3) *Cylindrical grains of dimensions above defined with a central channel of radius r' for which $h = \alpha(r - r')$.*

In this case

$$\Psi = 1 - \left(1 - \frac{2u_0 t}{r - r'}\right) \left(1 - \frac{2u_0 t}{h}\right)$$

(a) Assume $h < (r - r')$, then

$$\Psi = (\alpha + 1) \frac{t}{H_0} \left[1 - \frac{\alpha}{\alpha + 1} \frac{t}{H_0}\right]$$

(b) If $h > (r - r')$, then

$$\Psi = \frac{\alpha + 1}{\alpha} \frac{t}{H_0} \left[1 - \frac{1}{\alpha + 1} \frac{t}{H_0}\right]$$

(c) Finally, if $h = (r - r')$, then

$$\Psi = 2 \frac{t}{H_0} \left[1 - \frac{1}{2} \frac{t}{H_0}\right]$$

(4) *The prismatic grain with a single channel of dimensions adopted by us* may, for simplicity, as stated in page 29, be compared with the cylindrical grain with a central channel, whose dimensions are the following: The external diameter equals the diameter of the circle inscribed

in the base of the prism; the diameter of the channel equals the diameter of that for the prism, and the height is determined from equation 3.

The exact expression of the value Ψ for the prismatic grain with a central channel, under the assumption that $2(r-r') < h$, is obtained, after reduction, in the following form:

$$\Psi = 2 \frac{r-r'}{h} \frac{2\sqrt{3}r(h+r) + \pi r'(h-r')}{2\sqrt{3}r^2 - \pi r'^2} \frac{t}{H_0} \left[1 - \frac{r-r'}{4} \frac{2\sqrt{3}(h+4r) - \pi(h-4r')}{2\sqrt{3}(h+r) + \pi r'(h-r')} \frac{t}{H_0} + \right. \\ \left. + \frac{(r-r')^2}{4} \frac{2\sqrt{3} - \pi}{2\sqrt{3}(h+r) + \pi r'(h-r')} \left(\frac{t}{H_0} \right)^2 \right]$$

where h is the height of the prism, r is the radius of the circle described in the base, and r' is the radius of the central channel.

(5) *A grain possessing the form of a rectangular parallelepiped, whose dimensions are, say, a , b , and c , of which a is the least dimension. In this case*

$$\Psi = 1 - \left(1 - \frac{2u_0 t}{a} \right) \left(1 - \frac{2u_0 t}{b} \right) \left(1 - \frac{2u_0 t}{c} \right)$$

and

$$H_0 = \frac{a}{2u_0}$$

(a) Let us consider the general case when all the sides are unequal. Assume that $a = \alpha b = \beta c$, then

$$\Psi = 1 - \left(1 - \frac{t}{H_0} \right) \left(1 - \alpha \frac{t}{H_0} \right) \left(1 - \beta \frac{t}{H_0} \right) \\ = (\beta + \alpha + 1) \frac{t}{H_0} \left[1 - \frac{\beta\alpha + \beta + \alpha}{\beta + \alpha + 1} \frac{t}{H_0} + \frac{\beta\alpha}{\beta + \alpha + 1} \left(\frac{t}{H_0} \right)^2 \right].$$

(b) Let two sides be equal, i. e., the powder becomes a disk powder of square section. Putting $\alpha = \beta$ we obtain

$$\Psi = (2\alpha + 1) \frac{t}{H_0} \left[1 - \alpha \frac{2 + \alpha}{2\alpha + 1} \frac{t}{H_0} + \frac{\alpha^2}{2\alpha + 1} \left(\frac{t}{H_0} \right)^2 \right]$$

(c) Finally, in the case of cubical grains $\alpha = \beta = 1$, and therefore,

$$\Psi = 3 \frac{t}{H_0} \left[1 - \frac{t}{H_0} + \frac{1}{3} \left(\frac{t}{H_0} \right)^2 \right]$$

i. e., the form of Ψ for the cubical grain is the same as for the spherical grain inscribed within it and for the solid cylindrical grain.

Comparing together the values obtained for Ψ , for the different types of grains considered, we see that on burning powder at atmospheric pressure the function may be given the following general form:

$$30 \quad \Psi = \kappa \frac{t}{H_0} \left[1 - \lambda \frac{t}{H_0} + \mu \left(\frac{t}{H_0} \right)^2 \right]$$

in which the coefficients κ , λ , μ possess values determined from comparing the expressions corresponding to the given forms of grain and collected in Table XXI.

30. *The influence of form of the grain upon uniformity of combustion—The Extent of grain.*

As the more or less uniform combustion of the powder is a consideration of the greatest importance, let us consider in what degree it occurs when powder is burned in the open air. Formula 30 shows that the uniform development of gases (i. e., that equal quantities of powder burn in the same intervals of time) for combustion of powder in the open air is accomplished only in the case when the coefficients λ and μ equal 0. This can never occur in practice, and as λ is included in a separate term of the equation and is generally much larger than μ and as the ratio $\frac{t}{\Pi_0}$ is less than unity, it follows that, generally speak-

ing, a much larger quantity of powder burns in the first portion of the elapsed time than in the last. For convenience in inspection, the relative quantities of powder burned in the open air for various forms and dimension of grain are presented in Table XXII. It is assumed that the speed of combustion is the same in all cases, and that the least dimension of the grain, viz, that by which is determined the duration of time Π_0 of combustion of the grain, is the same in all cases.

An inspection of the figures in this table shows:

(1) The form and relative dimensions of the grain exercise a great influence over the uniformity of combustion of the powder.

(2) The spherical grain (1), the cubical grain (8), and the solid cylindrical grain, whose height equals its diameter, appear uniformly unsuitable in this respect; in the first tenth of a second of the whole duration of combustion more than 0.27 of the whole quantity of the powder is burned and in the last tenth only 0.001.

TABLE XXI.—Signification of the coefficients κ , λ , μ , entering into formula 30.

Form of grain.	Value of the coefficients.		
	κ	λ	μ
<i>Spherical grain</i> of radius r	3	1	$\frac{1}{3}$
<i>Solid cylindrical grain</i> , of height h , for diam. of base $2r$ and $h=ar$: $h < 2r$	$a+1$	$\frac{a(a+1)}{4(a+1)}$	$\frac{a^2}{4(a+1)}$
$h > 2r$	$\frac{2a+1}{a}$	$\frac{a+1}{2(a+1)}$	$\frac{1}{a+1}$
$h=2r$	3	1	$\frac{1}{3}$
<i>Cylindrical or prismatic grain</i> , with one channel, of height h , for a diameter of base $2r$, of channel, $2r'$ and $h=a(r-r')$: $h < (r-r')$	$a+1$	$\frac{a}{a+1}$	0
$h > (r-r')$	$\frac{a+1}{a}$	$\frac{1}{a+1}$	0
$h=(r-r')$	2	$\frac{1}{2}$	0
Exact expression for <i>prismatic grain</i> with channel, of height h , radius of inscribed circle r , radius of channel, r' , if $h < r-r'$	$\frac{2r-r'}{h} \cdot \frac{2\sqrt{3}r(h+r)+\pi r'(h-r')}{2\sqrt{3}r^2-\pi r'^2}$	$\frac{r-r'}{4} \cdot \frac{2\sqrt{3}(h+4r)-\pi(h-4r')}{2\sqrt{3}(h+r)+\pi r'(h-r')}$	$\frac{(r-r')^2}{4} \cdot \frac{2\sqrt{3}-\pi}{2\sqrt{3}(h+r)+\pi r'(h-r')}$
<i>Parallelepipedic grain</i> , whose edges are a, b, c , of which a is the least dimension and $a=a, b=\beta, c$, the coefficients a and β being not greater than unity and less than 0: General case ($a \geq \beta$).....	$\beta+a+1$	$\frac{\beta a + \beta + a}{\beta + a + 1}$	$\frac{\beta a}{\beta + a + 1}$
Square ($a=\beta$) of small height.....	$2a+1$	$\frac{2+a}{2a+1}$	$\frac{a^2}{2a+1}$
Cubic ($a=\beta=1$).....	3	1	$\frac{1}{3}$

NOTE.—For irregular forms the dimensions of the mean grains are determined as stated in section 11.

TABLE XXII.—Relative quantities of powder burnt in the open air for various forms and densities of grain.

Form and relative dimensions of grains.		Values of coefficients.			Ratio $\frac{t}{P_0}$.				Maximum extent of grain.
		κ	λ	μ	0.1	0.3	0.5	0.7	0.9
1	Spherical grain, radius, $r=1$	3.000	1.0000	$\frac{1}{3}$	0.27100000	0.65700000	0.87500000	0.97300000	0.99900000
2	$\left\{ \begin{array}{l} h < 2r \\ a=0. \\ a=0.1. \\ a=0.5. \\ a=1. \end{array} \right.$	1.000	0	0	.10000000	.30000000	.50000000	.70000000	.90000000
3		1.100	.0093	.0002	.10897750	.32081250	.52468750	.72063250	.90879750
4		1.500	.3750	.0417	.14437750	.40106250	.61718750	.79581250	.93993750
5		2.000	.6250	.1250	.18775000	.49425000	.71875000	.89250000	.94275000
6	Solid cylindrical grain, with diameter of base $2r$, and height $h=\alpha r$.	3.000	1.0000	$\frac{1}{3}$.27100000	.65700000	.87500000	.97300000	1.000
7	$\left\{ \begin{array}{l} a=5. \\ a=100. \\ h > 2r \end{array} \right.$	2.400	.7500	$\frac{2}{3}$.22240000	.56880000	.80000000	.93520000	2.500
8		2.020	.5149	.0099	.19162000	.51291000	.75050000	.91126000	.99018000
9		2.000	.5000	0	.19000000	.51000000	.75000000	.91000000	.99000000
10		1.000	0	0	.10000000	.30000000	.50000000	.70000000	.90000000
11	Cylindrical grain with one channel and diameters, — of base, $2r$; of channel, $2r'$; with height, $h=\alpha(r-r')$.	1.100	.0909	0	.10900000	.32100000	.52500000	.72100000	10.000
12		1.500	$\frac{1}{3}$	0	.14500000	.40500000	.62500000	.86500000	2.000
13		1.900	.4737	0	.18100000	.48700000	.72500000	.88900000	1.100
14		2.000	.5000	0	.19000000	.51000000	.75000000	.91000000	1.000
15	$\left\{ \begin{array}{l} a=2. \\ a=100. \\ h > r-r' \end{array} \right.$	1.500	$\frac{1}{3}$	0	.14500000	.40500000	.62500000	.86500000	2.000
16		1.010	.0909	0	.10900000	.30210000	.50250000	.70210000	100.000
17		1.000	0	0	.10000000	.30000000	.50000000	.70000000	.90000000
18		3.000	1.0000	$\frac{1}{3}$.27100000	.65700000	.87500000	.97300000	1.000
19	Parallelepipedic grains whose edges are a, b, c .	2.000	.5000	0	.19000000	.51000000	.75000000	.91000000	.99000000
20	General case of which a is the least and $a \geq \beta$.	1.110	1.000	.0009	.10989100	.32301300	.52737500	.72253300	100.000
21		1.105	.0955	.0005	.10945450	.32208150	.52681250	.72198650	200.000
22		1.100	.0909	0	.10900010	.32100090	.52500250	.72100430	.90900090
23		1.015	.0148	0	.10134955	.30314885	.50374875	.70311265	200.000
24	coefficients a and β are not greater than 1 nor less than 0.	1.010	.0099	0	.10090000	.30910000	.50250000	.70210000	.90090000
25	Square section of height $\alpha=\beta$	1.000	0	0	.10000000	.30000000	.50000000	.70000000	.90000000
26		1.002	.0020	0	.10001910	.30041370	.50048750	.70040530	1,000.000
27		1.200	.1750	.0083	.11791000	.34137000	.54875000	.74053000	10.000
		2.600	.8615	.1969	.23824000	.59468000	.82000000	.94192000	1.250

(3) The most suitable forms of solid cylindrical grains are those whose height is very small. As their height increases (to $h=2r$) the inequality of rate of combustion increases; when $h=2r$, this form of grain becomes one of the most unsuitable in this respect (see paragraph 2); when the length of the grain is further increased it begins to burn more and more uniformly, but this improvement increases very slowly, so that for even an infinitely great length of grain (9) much remains to be desired in the way of uniformity. In fact, for such a form about 0.2 of the whole burns in the first tenth of a second and 0.01 burns in the last tenth. It is only when the thickness of the grain is infinitely small that it burns quite uniformly.

(4) Cylindrical grains with one channel are very suitable in relation to uniformity of combustion when the grains possess very small heights (10) and when they are very long (17). Between these extreme limits the grains are less suitable and the disadvantage becomes a maximum when their height approximately equals the thickness of the wall of the grain. When $h=r-r'$, grains of this form (14) are very unsuitable, but nevertheless they are better than spherical grains and solid cylindrical grains of height greater than the radius of their base (5-9).

(5) The most unsuitable parallelepipedic grains are the cubical (18) (compare paragraph 2). According as two dimensions of the grain increase in comparison with the third the uniformity of combustion improves, so that grains in the shape of large plates and broad strips develop a very uniform combustion (20-21). In particular, in relation to grains of square section (19) we may say the same as has been said in paragraph (3) in relation to solid cylindrical grains.

Therefore, for powders burned in the open air in concentric layers (from the experiments of Piobert such conditions of burning may be fully assumed for smoking powders made from black charcoal) the speed of whose inflammation is considerably greater than their speed of combustion (paragraphs 26-27), the most suitable forms of grains are:

(1) The cylindrical grain with a central channel with the thinnest possible walls and the greatest possible length, and,

(2) The flat grain of the smallest possible height and the greatest area, of any form whatever. In other words the greater the *extent* the grains possess in two directions, the more suitable they are; understanding by the use of the word "extent" in a given direction the ratio of the dimension of the grain in this direction to the least dimension of the grain. Evidently the extent of the grain may vary between 1 and ∞ and the greater its value the larger the number expressing this amount. In the last column of Table XXII are presented the *extent* of various grains in relation to their least dimension.

Chapter V.

SPEEDS OF COMBUSTION OF POWDERS FOR PRESSURES DIFFERING FROM THAT OF THE ATMOSPHERE.

31. *The combustion of powder in a vacuum.**

As early as the middle of the seventeenth century it had been observed that powder burned with greater difficulty the rarer the atmosphere surrounding it. Thus, Count Joseph Ange Soluces and Papacino d'Antoni had observed these circumstances; the treatise of the latter (*Uso delle Armi da Fuoco*) shows that during the war (1743-1747) in the Alps, powder burned more slowly upon the summit of the mountains than at the foot. He and many others (Boyle, Huyghens, Munke, Heerder, Bianchi) had remarked that smoking powder fuses but does not explode in a vacuum. The more detailed experiments of Heeren and Abel confirmed previous observations. Thus, if a platinum capsule placed under a bell glass in a vacuum be heated red-hot, powder placed in it begins, after a certain time, to burn, but burns slowly and does not explode; if the capsule be not heated and the powder be touched by a platinum wire heated to redness the grain in contact with it begins to melt and disappear, but combustion does not begin at once; if the heating be prolonged, the grain in contact with the wire begins to burn only after the lapse of a certain time, but the rest of the powder remains unaltered. If, however, the powder be put under a pressure of one-tenth of an atmosphere, which corresponds to a height of the mercurial column of 76 mm., general combustion of its substance ensues easily; and in effecting this, it is immaterial whether the gas in contact with the powder be nitrogen, carbonic acid, or oxygen, which shows that the powder burns on account of the oxygen of the saltpeter. Hence it follows that it is necessary, in order to effect combustion, both to heat the powder and surround it with an atmosphere of some kind, producing a certain pressure (not less than one-tenth of an atmosphere). In general, as the latter increases the combustion becomes more energetic.

32. *General conceptions upon combustion of powder under various pressures.*

What has been said above shows directly that the speed of combustion of powder depends actually upon the pressure of the surrounding

*See Paul de Saint Robert's *Mémoires Scientifiques*, Vol. II, p. 167, and (2) *Traité sur la Poudre, les corps explosifs et la pyrotechnie*, par J. Upmann et Meyer, traduit de l'allemand et augmenté par E. Désortiaux, ed. 1878, p. 457 et seq. (3) Berthelot's *Sur la Force*, etc., Vol. I, 3d ed., 1883, p. 83 et seq.

atmosphere and, as we have seen, this had been observed long ago; nevertheless, experiments for measuring the dependence of the phenomenon under consideration upon pressure were not made until long afterwards. On account of the difficulties of observation, Piobert* came to the conclusion that pressure had no practical influence upon speed of combustion, which he assumes to be the same both for the combustion of powder in the bore of the gun and in the open air. Besides the fact relating to firing upon mountains, already alluded to, there may be presented in opposition to this the considerations of Count Saint-Robert† and of Captain Castan.‡ Both show that if the speed of combustion of powder fired from cannon and small arms be considered as identical with the speed of combustion in the open air, the powder could not burn completely and that a great part of the charge would be expelled in the form of partly burnt grains. Thus, according to Saint-Robert's conclusions: (1) The duration of the movement of the ball in a musket is about one two-hundred-and-fiftieth of a second; consequently, if the speed of combustion of powder be 12.5 mm., about one-twentieth of the radius of the grain is burned away and its thickness becomes 0.9 mm. instead of 1 mm., i. e., about three-fourths of the weight of the charge is not burned at all; (2) if the duration of combustion of the charge in the bore of the gun be one two-hundredth of a second, then only one-fifth of the whole of a charge of cannon powder, the mean diameter of whose grains is 2 mm., would burn away, and the remaining four-fifths would be expelled. Castan states that the powder A_e , employed in the French naval artillery, possessing a mean diameter of grain of 8 mm., burns in the open air at the rate of 10 mm. and is almost completely consumed in the 24 cm. gun, in which the movement of the shell in the bore occupies one-eightieth of a second. If the rate of combustion in the bore be taken as 10 mm., then only 3 kilograms of the whole 24-kilogram charge would be consumed and more than 21 kilograms of the powder would be expelled from the gun. Assuming complete combustion, we must allow that the mean velocity of combustion of this powder in the gun for variable pressure, is not less than 320 mm. per second, and as the initial speed of combustion at the moment of inflammation is 10 mm., therefore, for the greatest pressure, the speed of combustion must reach as high as 1,500 mm., or 2,000 mm. per second--i. e., at least 150 times greater than the speed of combustion in the open air.

Finally, Captain Rovel§ ascertained experimentally that at a pressure of 16 atmospheres the speed of combustion is almost double what it is in the open air.

*See p. 158 of his work, and the parts previously referred to.

†See his work, previously referred to.

‡See his paper, "Étude des Poudres," in "Revue d'Artillerie," 1872, Vol. I, p. 107.

§See his work previously referred to, pp. 104-105.

The relation between the speed of combustion and pressure, generally speaking, is very complex, and, through lack of exact data, may for simplicity, be expressed as a function of the form

$$31 \quad u = AP^\nu,$$

where u is the speed of combustion for the pressure P ; A a constant coefficient equal to $\frac{u_0}{P_0^\nu}$; (u_0 is the speed of combustion at atmospheric pressure P_0); and ν is a positive exponent determined by experiment, of which more will be said later.

A second assumption usually made in considering the combustion of powder at various pressures is that the powder grains burn in concentric layers, just as they do in the open air (paragraphs 27-29).

In support of this Castan adduces the following facts for smoking powders: In firing large-grain charges from guns some of the grains are found outside the gun and burnt over on all faces; this is evident

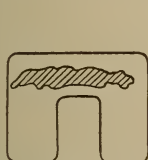


FIG. 2.



FIG. 3.



FIG. 4.



FIG. 5.

from the fact that their dimensions are reduced and their surfaces are more or less covered with pittings. The form of the remaining portions of the grains, for a given original form, depends evidently upon the original density of the grain and upon the pressures developed upon firing. In figures 2 and 3 are shown the powder grains, of density 1.77, of pellet (Spanish) and Belgian powders; the hatched area shows the magnitude and probable position of the unburned portions in relation to the form of the full grain.

For powders of density higher than 1.8 the outlines of the unburned portions of the grain are shown by their form to resemble more closely the original form of the grain, as the pittings from irregular combustion are considerably less; while for very great densities as high as 1.87 pitting hardly occurs and the residual forms closely resemble the original, as shown in figures 4 and 5.

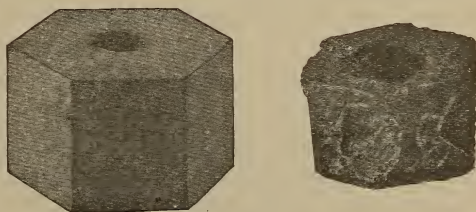


FIG. 6.

On the right side of figure 6 is shown a partly burned grain of chocolate powder, such as is sometimes found in front of the muzzle of the gun, even when high pressures are developed upon firing.

Apart from all these facts, the most careful investigations show, as

we shall see in paragraphs 43-47, that the combustion of smoking powders at great pressures can only occur in concentric layers under exceptional circumstances.

As far as relates to practically employed varieties of well-gelatinized smokeless powders, their combustion for pressures developed in cannon is accomplished completely in concentric layers, as is proven by experiments cited farther on in paragraph 50, and also by the unconsumed remnants of powder grains sometimes found outside of the gun.

*33. Combustion of powders at pressures less than atmospheric.**

In 1849 the English artillerist Mitchell determined in India the duration of combustion of tubes of powder 3 inches long at various altitudes. The results of these experiments show at a glance that the duration of combustion of the tubes increased in accordance with the diminution of pressure. Francland repeated the experiments in 1861. The latter were conducted at the Woolwich Arsenal. Tubes 6 inches long were burned in a closed chamber, the atmosphere of which was reduced artificially, and the pressure maintained uniform throughout the whole period of experiment by removal of the air. These and Mitchell's experiments show that upon reducing the pressure of the atmosphere one seven-hundred-and-sixtieth the prolongation of the combustion is increased by 0.0013 of its whole amount at atmospheric pressure—i. e., increase in prolongation of combustion is proportional to the diminution of pressure.

M. L. Dufour, professor of physics at the Academy of Lausanne, observed in 1862-1863 the combustion of powder tubes on the Alps in the open air at various heights, and also found that the increase in the duration of combustion was proportional to the diminution of pressure and amounted to from 0.0011 to 0.0010 for one seven hundred and sixtieth of an atmosphere.

In 1864-1865 Count Saint-Robert experimented upon the combustion of powder at various heights. For this purpose leaden tubes (external diameter 25 mm. and internal diameter 17 mm.) were filled with powder and subsequently drawn out until their diameters were reduced five times. By this means maximum uniformity of distribution of the powder mass throughout the whole length of the tube was secured. The drawn tubes were divided into sections about $\frac{1}{4}$ d. m. long, and their combustion at different altitudes was observed. The variation in speed of combustion of these tubes did not exceed one-fifth of a second, and therefore the determinations were considered as very exact. They show (Table XXIII) that the retardation of combustion does not fall as rapidly as the pressure diminishes.

On the basis of these experiments the exponent ν in formula 31 is taken as equal to two-thirds, i. e.,

$$u = AP^{\frac{2}{3}}.$$

*See the works referred to in paragraph 31.

TABLE XXIII.—*Durations of combustion of powder for pressures less than that of the atmosphere as found by Count St. Robert. [Experiments in the year 1865.]*

Pressure in mm. of the mercurial column.	Duration of combus- tion in seconds.	Pressure in mm. of the mercurial column.	Duration of combus- tion in seconds.	Pressure in mm. of the mercurial column.	Duration of combus- tion in seconds.
740.0	35.0	694.1	36.1	559.4	41.5
724.3	35.2	618.7	38.8	529.4	44.0
726.0	35.1	610.4	39.0	-----	-----

The results of these experiments are illustrated graphically on page 106.

34. *Existing hypotheses relating to combustion of powder at pressures greater than that of the atmosphere.*

In view of the absolute inaccuracy of Piobert's assumption (32) that the speed of combustion does not change with the pressures, and on account of the absence of any experimental data upon this subject, it was necessary, up to very recent times, to remain content with more or less probable hypotheses when considering the combustion of smoking powders. On account of the importance of this question we shall present here both existing hypotheses:

First hypothesis.—Sarrau* in 1876, starting from the hypothesis proposed by Boussinesq for deducing the laws of diffusion of gases through porous bodies, assumed that the speed of combustion of powder depends upon the rapidity of penetration of the inflamed gases through the powder mass—i. e., he assumed that the mechanism of combustion of the grain consists in the diffusion of highly heated gases. For this it must be admitted that the penetration of the gases through the powder mass proceeds under the action of two opposing forces; the external pressure P and the resistance of the substance to the passage of the gases. The latter manifests itself in consequence of the movement of the gases, and, therefore, it will be a certain function ($f(u)$) of the speed of combustion. As the movement of the gases is uniform for constant pressure, then, when movement ceases, the two forces under consideration should be in equilibrium, i. e.,

$$P=f(u).$$

The resistance $f(u)$ in relation to the unit of mass of the gases may be assumed as proportional: (1) to the velocity u i. e., to the number of particles of the resisting medium which oppose the movement of the gas in a unit of time, and (2) to a certain function of the velocity u reducing to 0 for $u=0$, and representing the resistance corresponding to each particle of the medium. It depends upon the nature of the substance and its structure.

*See Sarrau's "Nouvelles recherches sur les effets de la poudre dans les armes," 1876, p 46.

Assuming next that this latter function is also proportional to u , Sarrau obtains

$$P=A' u^2, \text{ or}$$

$$u=A P^{\frac{1}{2}}$$

i. e., when smoking powder burns at pressures greater than that of the atmosphere the degree of the exponent ν in expression 31 equals $\frac{1}{2}$. The latest experiments of Vieille have justified the assumption of this value of the exponent (52.)

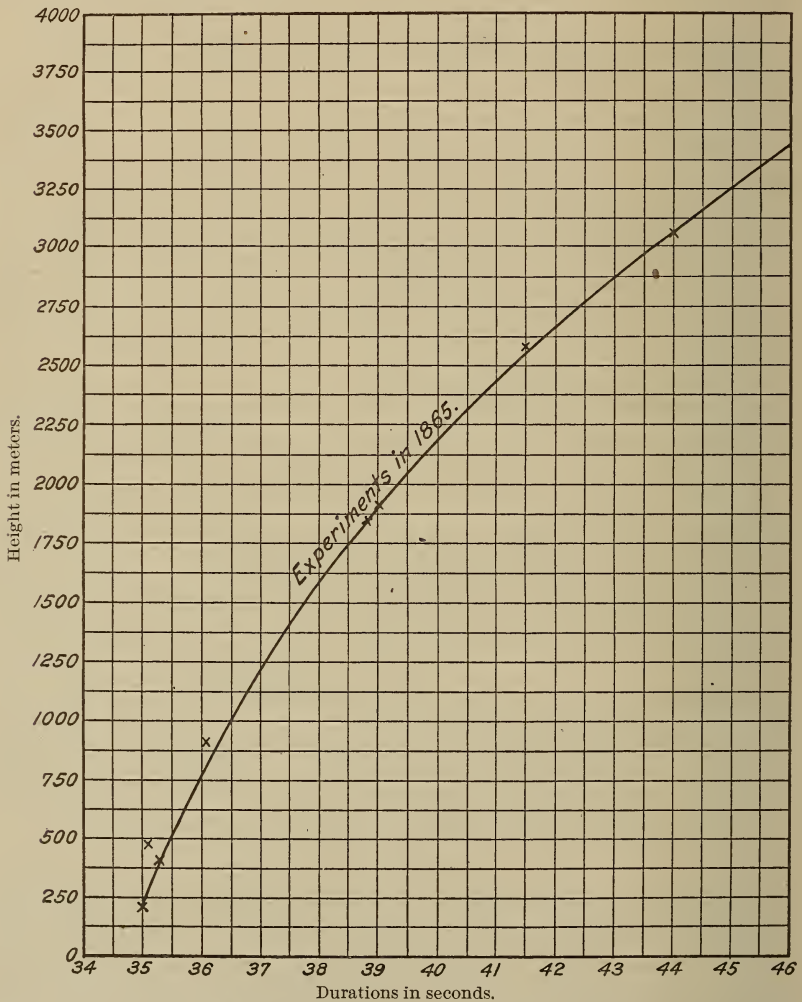


Fig. 7.—Curve representing the relationship between height above the level of the sea and duration of combustion of tubes of powder. The points \times indicate the points of the curve actually found.

Second hypothesis.—As a result of experiments with the velocimeter in connection with the 10-cm. gun Sebert and Hugoniot * came to

* H. Sebert et Hugoniot—"Études des effets de la poudre dans un canon de 10 centimètres," 1882, p. 74 and 111.

the conclusion that the speed of combustion of powder in the gun is proportional to the pressure i. e., in Formula 31, $\nu=1$. Captain Moisson* explains this assumption by the following theoretical considerations relating to the mechanism of combustion. The powder mass can only burn when it is heated to the temperature of inflammation; this heating proceeds on account of the heat developed by the layers first burned. On the basis of the general laws of the theory of heat, the heating is proportional to the difference between the temperatures of the substance of the powder and the heated gases in contact with it. The more rapidly the powder mass is heated, the more quickly it burns; therefore, it may be admitted that the speed of combustion is proportional to the stated difference of temperature. On the other hand, it is evident from thermodynamics that the pressures of gases are directly proportional to their temperatures (paragraph 18, p. 77) and consequently we may assume, in view of what has been said above, that the speed of combustion is directly proportional to the pressure. The fact that, in accordance with the experiments of Count Saint-Robert, the speed of combustion for pressures less than that of the atmosphere was proportional to two-thirds power of the pressure of the surrounding atmosphere may be explained under the present theory by the assumption that the pressure is actually greater than the atmosphere at the actual surface of combustion. We shall present here an approximate method of determining this pressure and speed of combustion for atmospheric pressure, under the assumption of the accuracy of the hypothesis under consideration ($\nu=1$). Let the corresponding quantity of powder burned in the interval of time t be Ψ , as before. Let the original weight of the grain or of a number of similar grains be ω . As combustion proceeds under our assumption in concentric layers, the surface of combustion s , after an interval of time t has elapsed, will be

$$s=h'\omega^{\frac{2}{3}}(1-\Psi)^{\frac{2}{3}}.$$

The speed of combustion u is proportional, by assumption, to the pressure P_t at the surface of the grain. In consequence of this the quantity of powder burned in the element of time dt will be

$$\omega d\Psi = h_1 u s dt = h_2 P_t s dt = h' P_t \omega^{\frac{2}{3}} (1-\Psi)^{\frac{2}{3}} dt, \quad \text{or}$$

32.

$$\omega^{\frac{1}{3}} d\Psi = h P_t (1-\Psi)^{\frac{2}{3}} dt,$$

where h, h_1, h_2 , and h' are certain constant coefficients. If we assume that P_t is constant and equal to P , the deduced expression may be easily integrated between the limits $t=0$ and $t=II$; and from $\Psi=0$, to $\Psi=1$, namely,

$$3 \omega^{\frac{1}{3}} = h P II$$

whence,

$$P = \frac{3 \omega^{\frac{1}{3}}}{h II}.$$

* A Moisson—"Pyrodynamique" Théorie des explosions dans les canons et les torpilles," 1886, Vol. XIV, "Memorial d'Artillerie de la Marine," p. 487.

But ω^3 is proportional to the linear dimensions of the grain, i. e., $\omega^3 = A_1 a$, where a is the least dimension of the grain and A_1 depends on its form, density, and dimension; and as besides $a = 2ut$, the previously deduced expression assumes in consequence the following form:

$$P = 3 \frac{A_1 a}{h \Pi} = 6 \frac{A_1}{h} u,$$

$$u = \frac{Ph}{6 A_1}$$

where u is the speed of combustion corresponding to the pressure P .

For the Wetteren coarse-grain powder the speed of combustion in the open air is $u = 0.01$ m. per second. Sebert and Hugoniot, on the basis of results obtained from firing this powder in the 10-centimeter gun, assumed for the coefficient $\frac{h}{6A_1}$, the value 0.00124. Substituting this value in the expression found, we obtain

$$u = 0.00124 P$$

$$P \approx 8 \text{ atm.},$$

i. e., upon burning the variety of powder under consideration in the open air the pressure at the surface of the grain is almost eight times greater than the atmospheric pressure.

If the combustion be so ordered that the pressure at the surface of the grain be not greater than one atmosphere, then the speed of combustion corresponding to one atmosphere as obtained from the previous expression would be $u = 0.00124$ m. per second instead of 0.01.

The hypothesis above considered in relation to the speed of combustion of powder forms the basis of the work of Sebert and of Hugoniot and of Moisson, before mentioned, and also of the Spanish treatise, "Tratado da Balistica Interior," par Onufre Mata, ed. 1890. The same hypothesis is applied at the present time to smokeless powders by Colonel Moisson in his recent work, "Étude sur le tir des poudres B," 1898.

35. Experiments to determine the mode of combustion of various powders at pressures higher than that of the atmosphere.

At the close of 1893 the commencement of the memorable work of Vieille on the determination of the mode of decomposition of various powders at great pressures appeared in the third part of the sixth volume of the journal "Mémorial des poudres des Salpêtres," and in 1894 there was published the conclusion of this interesting study under the title "Étude sur le mode de combustion des matières explosives." As far as is known to us the above constitutes the first serious work yet issued upon this very important question in relation to the application of powders to ordnance use.

Vieille's experiments began in 1884 and were made with various powders, both smoking and smokeless, yet it should be remarked that

comparatively few experiments were cited in relation to the latter. In view of this we shall also cite, for pyroxyline powders, the experiments of the experimental committee of the Okhta factory, conducted in 1897. The method employed by Vieille is very instructive, and results obtained from it allow us to-day, upon the basis of theoretical considerations, to predict the action of a desired material in a gun with sufficient accuracy for practical purposes.

In paragraph 13 the means employed by Sarrau and Vieille for determining pressures in a closed chamber were explained in general terms, and in the second part of the present work they are given in detail. From them curves were obtained, such as are shown in fig. 8, page 111. They express the movement of the piston of the crusher as a function of elapsed time. The complete movement of the piston corresponds to the complete compression of the copper disk, and the duration of movement corresponds almost to the duration of the pressure action on the piston rod. As the movement of the piston is rectilinear, and as in the experiments considered (pressures less than $4,500 \frac{\text{kgr.}}{\text{cm}^2}$), the force of its inertia may be overlooked, the resistance of

the piston to movement and the pressure acting on the piston rod may be equated for each instant of motion. Bearing subsequently in mind that resistance to movement is taken up by a copper disk, and that the ratio of the resistance to the magnitude of compression is known with sufficient accuracy, we may for each moment determine the pressure from the amount of compression of the disk. Knowing the pressure, the force f of the powder under consideration, and its covolume, α , we may by employing equation 20, page 78, determine the density of the products of decomposition for the corresponding moment, and may therefore ascertain the weight of the consumed portion of the charge. The laws in which we are interested may subsequently be deduced from a series of similar experiments.

36. *Determination of the relative weight Ψ of the burned portion of the charge corresponding to a pressure P_t .*

When combustion occurs in a closed chamber, the pressure at each moment is the result of the action of the products of decomposition of the burned portion of the charge in a volume less than that of the chamber in which the charge was placed, and is determined by expression 20,

$$P_t = f \frac{\Delta_t}{1 - \alpha \Delta_t}$$

where Δ_t represents the density corresponding to a pressure P_t .

The volume of the chamber in the bomb is

$$W = \frac{\omega}{\Delta},$$

where ω , the weight of the charge and Δ , the density of loading, are constant quantities for the given experiment.

The density of the products of decomposition in the given time will be

$$\Delta_t = \frac{\omega \Psi}{W - \omega \frac{1-\Psi}{\delta}} = \frac{\Delta \Psi}{1 - \frac{\Delta}{\delta} (1-\Psi)}$$

and consequently

$$P_t = \frac{f \Delta \Psi}{1 - \frac{\Delta}{\delta} (1-\Psi) - \alpha \Delta \Psi}$$

Bearing in mind that, at the end of combustion $P = \frac{f \Delta}{1 - \alpha \Delta}$, we obtain

$$P_t = \frac{P (1 - \alpha \Delta) \Psi}{1 - \frac{\Delta}{\delta} (1 - \Psi) - \alpha \Delta \Psi}$$

whence

$$33. \quad \Psi = \frac{P_t \left(1 - \frac{\Delta}{\delta}\right)}{P(1 - \alpha \Delta) + P_t \Delta \left(\alpha - \frac{1}{\delta}\right)} = \frac{1}{1 + \frac{1 - \alpha \Delta}{1 - \frac{\Delta}{\delta}} \frac{P - P_t}{P_t}} = F(P_t).$$

For smoking powders the covolume varies between limits 0.4 and 0.7, the density δ , between 1.6 to 1.9, and therefore δ almost equals $\frac{1}{\alpha}$.

Consequently, the expression $\frac{1 - \alpha \Delta}{1 - \frac{\Delta}{\delta}}$ is very close to unity. In conse-

quence of this formula 33 for smoking powders will be almost exactly equal to the ratio of the corresponding pressure P_t to the full pressure P —i. e., $F(P_t) = \frac{P_t}{P}$.

In the case of smokeless powders the covolume approaches unity and the density to the value 1.6, so that the expression under consideration may differ considerably from unity. If these powders be compared for such densities of loading Δ as develop the same final pressures P , as in the case of smoking powders, then, in this case, the values Ψ are expressed almost by the same function P_t .

In any case formula 33 permits us to calculate the quantity of powder Ψ burned up to the time when the pressure P_t is developed in the bomb and determines the law of combustion of the charge, on the basis of the law of pressures as a function of time.

37. The characteristic elements of the curve of the law of combustion.

It follows from the above that the law of development of pressures, or the law of evolution of gases by a given substance may be obtained from the experimental curve developed for definite conditions of loading; and this law will serve as a basis for the comparison of different powders.

Vieille found it more convenient, however, to determine only certain elements of each experimental curve and to characterize the law of combustion of the substance by them.

The law of development of pressures from burning powder in a closed chamber is represented by a curve like that shown in fig. 8, in which the abscissæ represent times, and the ordinates, pressures.

In fig. 8 A_0 corresponds to the beginning of inflammation, A to the commencement of the actual compression of the disk, whence, with the methods of determining pressures usually employed, the graphic law of pressure takes its origin. J is the point of inflection, the tangent of whose angle of inclination at this point $\left(\frac{dP_t}{dt}\right)_{\max.}$, may be obtained by direct observation with a protractor; its value measures the maximum rate of increase of developed pressure; the point B corresponds to the end of combustion.

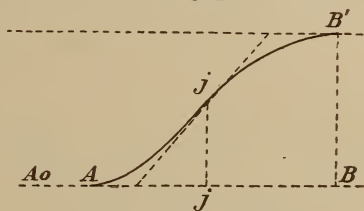


FIG. 8.

The most characteristic elements of the curve are:

$AB = \Pi_a$, the complete duration of the combustion, starting from the pressure corresponding to the first compression of the copper disk.

$\left(\frac{dP_t}{dt}\right)_{\max.}$ —the tangent of the angle of inclination at the point of inflection, which measures the greatest rate of development of pressure, and P , the final pressure in the closed chamber.

33. General procedure in determining the speed of combustion of a substance.

Generally speaking, the law of development of pressure depends upon two unknown functions: the law of change of surface on combustion of the charge, and the law of change of rate of combustion. In fact, the elementary quantity of the burned portion of the powder will be $\omega d\Psi = h_1 \delta s u dt$, whence,

$$\Psi = \frac{\delta h_1}{\omega} \int_0^u s u dt = \frac{h_1}{v} \int_0^a \frac{1}{2} s da,$$

where u is the speed of combustion, $da = udt$, the elementary thickness of the consumed layer, reckoning the thickness as normal to the surface; s , the extent of surface of the partly burned grain at the time t ; δ , the density of the powder, and h_1 , the coefficient of proportionality. Hence, it is evident that, having calculated the quantity Ψ by formula 33, page 110, and knowing only how the *amount* of powder burned changes, we can not determine the law of change in rate of combustion. To do this we must know the law of change of the surface on combustion—e. g., as a function of the thickness of the consumed layer. The latter is known to us only when the composition proceeds in concen-

tric layers. In such a case, the determination of the surface of combustion as a function of the change in thickness of the consumed layer requires but a simple geometrical calculation. In this manner we may, by a series of simple calculations, ascertain from the curve developed experimentally, the law of change of thickness of the consumed layer as a function of time—i. e.,

$$a=f(t)$$

where

$$\frac{da}{dt}=u=f'(t)=\varphi(P_t)$$

since, as previously stated, P_t is a definite function of the time as obtained from the same curve.

39. *Conditions of combustion of similar grains in concentric layers in closed chambers for the same densities of loading, i. e., for the same final pressure.*

The weights of the charges and the volumes of the powder masses are evidently the same in this case, the difference being only in the number of grains. The pressures P_t and P'_t are the same for the same quantity of burned powder, Ψ . The rates of combustion, u and u' , whatever their dependence upon pressures, are consequently also the same in both cases for the same pressures, as the nature of the substances is the same. Moreover, the mean velocities of combustion for a change of pressure from 0 to P_t i. e., $u_m=u'_m$ are also equal. It remains to determine in this case how times of combustion, corresponding to the development of the same pressure, vary. Although these times are different for similar grains of different sizes, equal pressures correspond in either case to the same quantity Ψ of burnt powder (33), whatever the dimensions of the grain may be and whatever the difference between the times Π_L and Π_s , which elapse in each case up to the development of this pressure. It follows from the above that

$$\Psi = \frac{\text{Volume L} - \text{Volume L}'}{\text{Volume L}} = \frac{\text{Volume S} - \text{Volume S}'}{\text{Volume S}}$$

where L and S denote the initial dimensions of the large and small grains, and L' and S', the dimensions of the same grains when they have burned to such an extent that the same quantity of gases is developed in each case for both of them.

The expression as written gives

$$\frac{\text{Volume L}}{\text{Volume L}'} = \frac{\text{Volume S}}{\text{Volume S}'}$$

or

$$q^3 = \frac{\text{Volume L}}{\text{Volume S}} = \frac{\text{Volume L}'}{\text{Volume S}'}$$

where q denotes the ratio of the corresponding dimensions of similar grains.

Denoting the smallest initial dimension of grain;—for the large grain by a_L , and for the small one by a_s ; and denoting the corresponding dimensions for the moment in question by a'_L and a'_s , we may, consequently, write,

$$q = \frac{a_L}{a_s} = \frac{a'_L}{a'_s}$$

whence,

$$\frac{a_L - a'_L}{a_s - a'_s} = \frac{a_L}{a_s} = q$$

i. e., the thicknesses of the burned layers for both the large and small grains when the same pressure, P_t , is developed, will also be similar. Moreover, as

$$\frac{a_L - a'_L}{a_s - a'_s} = 2u_m \frac{\Pi_L}{\Pi_s}$$

then,

34.

$$\Pi_L = q \Pi_s$$

i. e., the duration of combustion corresponding to the same development of pressure P_t for different yet similar grains of the same material bears to one another the ratio q . It follows from the above that when similar grains are burned in concentric layers the curves of the developed pressures as a function of time will differ only in the scales of time, which bear to one another the constant relation q . This ratio is evidently borne for the full durations of combustion Π_a and the inverse for the point of inflection.

$$\left(\frac{dP_t}{dt} \right)_{\max.}$$

Consequently, it is necessary to have two experiments with similar grains in order to determine whether the combustion of these grains at great pressures proceeds in concentric layers. For, when similar grains of any substance are consumed, it is found that the times of combustion corresponding to the development of the same pressure bear to one another the relation q . This will serve as a proof that the grains of the substance under consideration burn in concentric layers. If such a relation be not established then the grains do not burn in concentric layers.

40. *The combustion of dissimilar grains in concentric layers.*

Let us consider e. g., the conditions of combustion of two charges composed, the one of large and the other of small cylindrical grains, of the same density and height, but of different diameters, assuming that the densities of loading and, consequently, the weights ω and the volumes V of the substance are the same in both cases. The initial surface of combustion S_i is less for the larger grains than for the smaller S'_i . On the contrary, at the end of combustion the larger grains possess a greater surface, S_f , than the smaller grains, S'_f . If we lay off the thickness of the burned layers along the axis of

abscissæ and the corresponding pressures as ordinates we shall obtain two curves (fig. 9), of which, as we shall see beyond, the one for the larger grains lies constantly below that for the smaller grains, but the extremities of the two curves coincide. The corresponding burned portions of the parts of the charges of these two kinds of powder will be $d\Psi_1$ and $d\Psi'_1$ respectively, when the combustion is completed from the beginning through an infinitely small thickness of grain, da .

And as

$$\frac{S_1 da}{V} < \frac{S'_1 da}{V}$$

then

$$d\Psi_1 < d\Psi'_1$$

The quantity Ψ is the same function, P_t (33, p. 110) for both charges and $d\Psi = F'(P_t) dP_t$, in which $F'(P_t)$, obtained from 33 is also the same for both charges; whence $d\Psi$ is proportional to dP_t , and consequently,

$$dP_1 < dP'_1$$

or

$$\frac{dP}{da} < \frac{dP'_1}{da}$$

Reasoning in a similar manner for the end of combustion, we obtain

$$\frac{dP_F}{da} > \frac{dP'_F}{da}$$

The last two equations give the values of the tangents of inclination for the curves in question at their beginning and at their termination, and, therefore, they show that the tangents at the initial and final elements of the curve for the large grains lie below the corresponding tangents for the corresponding elements of the curve for the small grains, and consequently the whole former curve lies below the latter. Hence it follows that for the same thickness of consumed powder the pressures are less for the charge made up of the large grains, and the corresponding speeds of combustion will be less in this case.

As the speed of combustion is $u = \frac{da}{dt}$, then

$$dt = \frac{da}{u}$$

and consequently the full time of combustion of the grain will be

$$\Pi = \int_0^{\frac{a}{2}} \frac{da}{u}$$

This integral may be integrated for both charges for the same number of elements, the numerators of which, from the equality of thickness of both grains, will be the same. The denominators will always be less for the larger grains than for the smaller. This shows that the duration of combustion increases, for equal thicknesses, along with the *extent of surface* of grain. (See the end of paragraph 30.)

The difference between the two curves and consequently the difference in the duration of combustion becomes considerable only in the case when $\frac{S_I'}{S_I}$ and $\frac{S_F'}{S_F}$, which are the measures of the ratio of the tangents of the angles of inclination at the beginning and end of the curves, are considerably greater than unity.

In order to show how these relations change with extent of the grains in question, we shall write the expressions for the said relations for two cylindrical grains, assuming that the radius of the larger grain is R and of the smaller r ; that the heights of both grains are the same and equal to a , and that the charges are composed of one large grain and n small ones. Then

$$\frac{S_I'}{S_I} = \frac{n(r^2 + ra)}{R^2 + Ra} = \frac{1 + \frac{a}{r}}{1 + \frac{a}{R}}$$

since $n\pi r^2 a = \pi R^2 a$, whence $n = \frac{R^2}{r^2}$.

The written ratio shows that, apart from the consideration of the absolute dimensions of the grains, it is the nearer to unity, the smaller the ratios $\frac{a}{r}$ and $\frac{a}{R}$ —i. e., the greater that the extents $\frac{R}{a}$ and $\frac{r}{a}$ of the grains are. Thus, for $\frac{R}{a} = 20$ and $\frac{r}{a} = 10$, $\frac{S_I'}{S_I} = 1.05$; and for $\frac{R}{a} = 2$ and $\frac{r}{a} = 1$, $\frac{S_I'}{S_I} = 1.33$.

The same may be shown not only from comparison of grains of any other form, but also from the comparison of grains of the same thickness and of different forms. Thus, if we consider the smaller grains as cylindrical and the larger grains as parallelopipeds, with the dimensions $a \times h \times h$, we have

$$\frac{S_I'}{S_I} = \frac{\pi n(r^2 + ra)}{h^2 + 2ha} = \frac{1 + \frac{a}{r}}{1 + 2\frac{a}{h}}$$

In this way it may be stated, in general terms, that if experiments be performed upon grains whose extent is considerable, and whose combustion is accomplished in concentric layers, the duration of combustion depends on the thickness of the grain; the form of the grain possesses no special influence.

41. *Conditions under which it is possible to neglect mistakes in determining the beginning and end of combustion.*

The copper disk of the pressure gauge affords a measurable residual compression for a certain initial pressure (about 100 kilograms), which, it should be remarked, is a constant quantity for all experiments. In view of what is said, the beginning of the reduction in its length, as shown upon the curve obtained by experiment, does not correspond to the beginning of combustion. Under the conditions considered, dissociation of the products of combustion does not occur (par. 18, p. 49), and therefore the end of combustion should correspond to the full pressure; and as the piston of the crusher does not develop any sensible energy, the amount of the full (final) pressure should correspond to the end of compression of the copper disk. It is, therefore, necessary in estimating the speeds of combustion to determine the point where the curve unites with the tangent corresponding to the final position of the stylus of the pressure piston. With existing means of measurement Vieille assumes that in determining this point there may arise an error to be deducted from the magnitude of compression of the disk of one-twentieth of a millimeter. In consequence of this error the latter part of the combustion, for which the pressure increases approximately 26 kilograms, is neglected. In order to determine the amount of powder substance whose combustion corresponds to the initial pressure of 100 kilograms, and to a change of final pressure of 26 kilograms, we shall differentiate the expression 33, page 110.

$$d\Psi = \frac{\left(1 - \frac{\Delta}{\delta}\right)(1 - \alpha \Delta) P dP_t}{\left[P(1 - \alpha \Delta) + P_t \Delta \left(\alpha - \frac{1}{\delta}\right)\right]^2}$$

At the beginning of combustion for those pressures for which the piston begins to move, we may neglect the term containing P_t in the denominator of the complete expression for $d\Psi$, and then multiplying the numerator and denominator by the area s , the cross section of the piston, we obtain

$$d\Psi_1 = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha \Delta} \frac{s dP_t}{s P}$$

At the end of the experiment $P_t \approx P$, and therefore

$$d\Psi_2 = \frac{(1 - \alpha \Delta) dP_r}{\left(1 - \frac{\Delta}{\delta}\right) P}$$

Finally, denoting the magnitude of compression of the copper disk by ε and observing that $Ps = k_0 + k_1 \varepsilon$, where k_0 and k_1 are constant coefficients determined by experiment, we obtain

$$dP_r = \frac{k_1 d\varepsilon}{s}$$

consequently

$$d\Psi_2 = \frac{1 - \alpha \Delta}{1 - \frac{\Delta}{\delta}} \frac{\kappa_1 d\varepsilon}{s P}$$

Under the conditions of Vieille's experiments, as is stated in paragraph 36, the quantity $\frac{1 - \frac{\Delta}{\delta}}{1\alpha - \Delta}$ and its reciprocal may be considered as

constant for all substances experimented with: $s\delta P_1$ —i. e., the force necessary to effect the initial movement of the piston and $\delta\varepsilon$ are, from what has been said in relation to determining the point of the beginning of the curve and the union of its extremity with the tangent, also constant quantities.

In view of this the quantities $\delta\Psi_1$ and $\delta\Psi_F$ will not depend upon the circumstances of the experiments, if sP is constant—i. e., if in experiments with different densities of loading Δ , the areas of the cross sections of the pressure pistons be so chosen that they are inversely proportional to the full pressure P . In other words, the areas of the pressure pistons must be so chosen that the magnitudes of compression of the copper disks are equal. Under these conditions errors in determining the beginning and end of combustion will be fully compensated. This deduction is especially important, because the first phase of the combustion preceding the beginning of compression of the disk is a comparatively long one and is generally accomplished irregularly, so that it serves rather as a measure of the rate of inflammation of the powder than of the actual speed of combustion. Vieille determined the latter fact by a series of special experiments in which he left a windage between the piston and the copper disk. He ignited the charge in the bomb with a train of powder that burned so quickly that its speed of combustion might be neglected, while at the same time the train was so chosen that the pressure developed by it was insufficient to compress the disk but enough to displace the piston with its obturator—i. e., to press it up against the disk. In the case in question there occurred a retardation in the displacement of the piston in relation to the instant of the initial inflammation of the charge, for the free piston with the obturator, required a pressure of 40 kilograms to set it in motion; but the duration of the latter retardation did not exceed one one-thousandth of a second.

With the existing windage the experimental curve took the form shown in fig. 10, and, therefore differs from the usual form AB, by the addition of the portion AD, in which may be distinguished the part DC, corresponding to the movement of the free piston with obturator; and the part AC, corresponding to the duration of combustion of that first part of the charge, for which the evolved gases produced no sensible compression

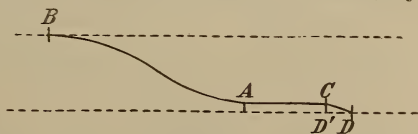


FIG. 10.

of the disk. Throughout the extent CA the piston remained at rest and therefore this part is parallel to the initial tangent. The quantity D'C measures the magnitude of the windage existing between the crusher and the disk.

42. *Combustion of varieties of smoking powder.*

A great number of Vieille's experiments were performed with the powders:

(1) Mark 30/40 and 26/34 differing from one another only in what relates to dimensions of grain. Their shape was a parallelepiped of square section with corners rounded on account of the grains being polished. The dimensions in millimeters were for the first, 40 by 40 by 30 and for the second 34 by 34 by 26; the composition was salt-peter 75 per cent. sulphur 10 per cent., and black charcoal 15 per cent. (75-10-15); the density about 1.8. They were manufactured out of pellets from the hydraulic press consisting of a mixture of small grains and mill-cake.

(2) Prismatic powder of the mark PRH of the composition 78-3-19, from the Dittenhoffer factory, at Rottweil, near Stuttgart, and Rhenish Westphalian powder of the mark PRW. The prisms of these powders were each prepared separately under the press from small grains, without any mixture of dust.

All experiments were made with one or two pairs of specimens for one or two densities of loading varying from 0.6 to 0.3, and in each case with two charges of the same weight, but made up from grains of two different thicknesses. They were cut with a saw from whole grains.

As the powders taken were made up of mixtures of different lots, in order to secure uniformity the experimental specimens were cut from the same grain, in doing which the outer polished surfaces of the powders (mark 30/40 and 26/34) were first removed and subsequently the samples were taken from the heart of the grain. The order of cutting the samples is shown for powders 30/40 and 26/34 in fig. 11, and for prismatic powders in fig. 12. Some pairs of samples were exactly alike; others were not.

Duration of combustion was determined sometimes from the beginning of inflammation of the grain and sometimes from the beginning of compression of the disks. Pressures corresponding to a density of loading 0.3 were about $1,200 \frac{\text{kg.}}{\text{cm}^2}$ and those corresponding to a density of 0.6 about $3,000 \frac{\text{kg.}}{\text{cm}^2}$. Some of the results of the experiments are assembled in Table XXIV.

From these it becomes evident that smoking powders *do not burn in concentric layers* as had been assumed up to this time on the basis of Piobert's experiments.

An examination of the figures of this table shows, in fact, that when the thicknesses of the samples from the substance of powders 30.40 and 26.34 were reduced as much as from 3.5 to 5 times the duration of combustion changed only by an amount varying between one-sixth and one-ninth of its value. The powder PRH, 4 mm. in thickness, gave a duration of combustion only 0.0072 less than for a thickness of 13 mm.; the powder PRW affords almost the same speed of combustion for thicknesses of grain of 13 mm. and 2 mm., but if these powders burned in concentric layers the duration

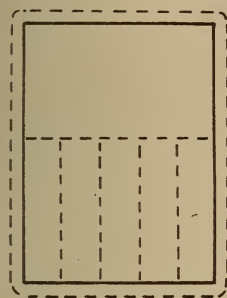


FIG. 11.

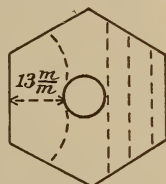


FIG. 12.

of combustion would be proportional to the thicknesses (formula 34, p. 113) and the ratios for dissimilar grains would be still greater (p. 115).

TABLE XXIV.—*Durations of combustion Π_a and maximum rates of increase of pressure $\left(\frac{d P_t}{dt}\right)_{\max.}$ of smoking powders.*

Kind of powder.	Den- sity of load- ing.	Charge of—						Remarks con- cerning grains (sam- ples).	
		One grain.			Several grains.				
		Thick- ness, a.	Dura- tion of com- bus- tion, Π_a .	$\left(\frac{d P_t}{dt}\right)$ max.	a	Π_a	$\left(\frac{d P_t}{dt}\right)$ max.		
		Mm.	Thou- sandths of a second.	Tons per square cm. per second.	Mm.	Thou- sandths of a second.	Tons per square cm. per second.		
Powder, mark 30.40..	{	0.4	13.52	7.64	498.00	6.73	7.95	459.8	Similar.
		.6	19.40	6.31	11.25	6.24	Dissimilar.
		.6	15.30	10.56	212.15	4.08	8.86	206.8	Similar.
Powder, mark 26.34..	{	.6	18.00	8.88	1,148.00	5.00	7.53	*1,036.0	Dissimilar.
		.3	19.00	8.33	321.50	4.00	8.09	*253.1	Spherical.
			13.00	23.75	4.00	16.55	Similar.
Chocolate PRH.....	{	.6	13.00	25.19	6.00	23.23	Dissimilar.
			13.00	12.67	248.50	4.10	11.69	250.1	
			13.00	20.36	309.50	5.00	18.39	303.4	
Powder PRW	{	.6	13.00	13.85	243.60	2.00	13.30	241.2	Do.
		.3	13.00	21.15	5.00	22.27	(*)	
				11.05	6.40	10.42	
Black prismatic from Dünaberg.	{	.6	13.00	9.96	5.10	9.30	Do.
				8.49	5.00	8.21	
					

* Duration of combustion before these experiments is reckoned from the beginning of inflammation, and in the latter from beginning of compression.

Such a result (i. e., almost complete independence of duration of combustion of smoking powders upon their thicknesses) was observed both for high densities of loading (i. e., for great pressures) as well as for low densities. This deduction does not change, whether the duration of combustion be reckoned from the beginning of inflammation or from the beginning of compression of the copper disk.

The comparisons presented in Table XXIV of the quantities $\left(\frac{dP_t}{dt}\right)$ max. or what is the same thing, of the maximum rates of increase of pressures, show that they are almost independent of thickness—i. e., that the law of development of pressures (and consequently the law of evolution of gases) obtained upon burning samples of the same substance but of different thicknesses, is almost the same, and also that the curves expressing these laws almost coincide throughout through their whole extent for each pair of samples, as is evident from fig. 13. In other words, the law of evolution of gases for the same material and the same final pressure is almost independent of the thickness and the form of the grain.

43. *The influence of cohesion of the particles upon the character of combustion of powders.*

As the combustion of smoking powders, as has been shown, does not depend to any sensible degree on the thickness of the grain, experiments were instituted to determine the circumstances which control this phenomenon.

In the case, e. g., of powders built up by pressure from very small grains—that is, in cases where feeble pressure was employed in manufacture (when the density of the ultimate powder grain was less than that of the original grains)—the interstices existing in the final grains permitted flame to penetrate freely through their substance, and the whole grains disintegrated almost at once into the elements composing the grains themselves, which determined in the given case the rate of combustion. The same may be stated in relation to powders prepared from grains and mill cake.* In consequence of this investigations were made into the influence of the degree of cohesion both as existing between elementary particles and between separate grains of a powder on the duration of its combustion. The cohesion between the particles (grains) depends, as we shall see, not only on the magnitude of the applied pressure and the density of the ultimate grain, but also upon the degree of moisture in the original substance.

*See E. Sarrau's *Recherches théoriques sur le chargement des bouches à feu*, p. 12, ed. 1882.

A.—*Combustion of powders prepared by pressure from mill-cake.*—As the mixture (incorporation) of the component parts doubtlessly exercises an influence on the combustion of powder, the cakes used in these experiments were prepared from powders experimented with by pulverizing them, sifting them through silk, and removing the remnants of the grains. Subsequently the cakes were again pressed, while in a dry state, into special cylindrical shapes by the use of a hydraulic press, employing different pressures. In this way powder cylinders were prepared of a diameter of 20 mm. and a height of 10.5 mm.; also cylindrical pellets of the same diameter and 3.5 mm. in height. Two

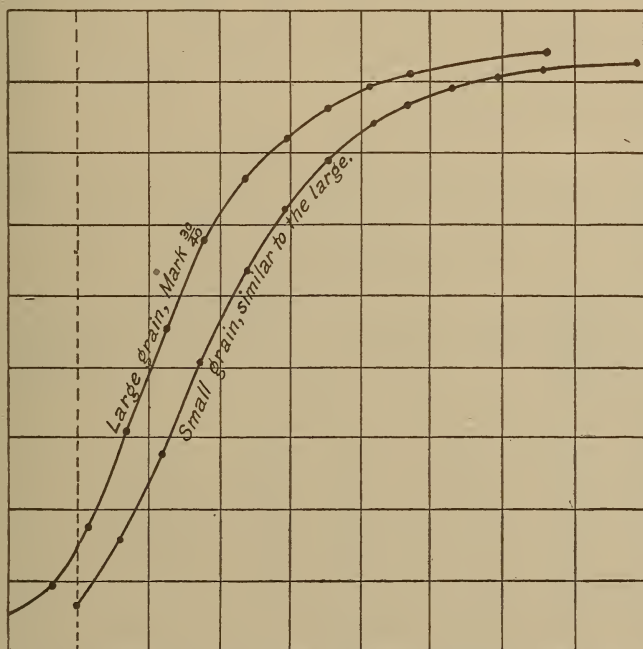


FIG. 13.—Scale: Abcissæ, 10 mm. = $\frac{1}{1000}$ sec.; Ordinates 10 mm. = 200 kgr.

cylinders or six pellets composed a charge for a density of loading of 0.6, and one-half of this quantity for a density of loading of 0.3. The cross sections of the pressure pistons were so chosen that the magnitudes of the compressions were the same for both densities of loading (± 1). Two series of experiments were performed; the first (Table XXV, and fig. 14) with a powder substance PRW and with reduced copper crusher disks, which increased the compression to 4 mm. in lieu of the 2 mm. usually obtained; the second (Table XXVI and fig. 14) was determined with the same substance and also with the powder 26 34, for which experiments were made only with pellets weighing 2.12 grams and whose heights varied inversely with their densities.

TABLE XXV.—*Duration of combustion of grains of powder cake of material P R W, pressed dry.*

Pressure used in pressing (kilograms per square centimeter).	Density of grains.	Large grains, diameter 20 millimeters.			Small grains, diameter 20 millimeters.		
		Height (millimeters).	Duration of combustion for densities of loading (thousandths of 1 second).		Height (millimeters).	Duration of combustion for densities of loading (thousandths of 1 second).	
			0.6	0.3		0.6	0.3
3,600	1.922	10.44	98.07	125.00	3.52	37.58	46.17
*1,000	1.785	11.19	† 87.13	† 108.80	3.70	35.68	43.50
960	1.760	11.47	48.33	74.57	3.82	32.12	42.13
720	1.724	11.65	17.36	28.78	3.88	14.80	23.46
*660	1.635	12.09	5.22	12.01	4.06	5.43	8.74

* About.

† Mean.

Sifted dust; density, 0.645— $\left\{ \begin{smallmatrix} 5.97 \\ 7.25 \end{smallmatrix} \right\}$ 6.61.TABLE XXVI.—*Duration of combustion of pellets weighing 2.12 grams and of the same diameter, 20 millimeters, prepared by pressure from dry cake of the substance P R W and 26/34.*

FROM CAKE OF SUBSTANCE P. R. W.

Pressure applied on pressing (kilograms per square centimeter).	Density of pellets.	Height of pellets.	Duration of combustion in thousandths of a second for—		Ratio of duration of combustion for densities 0.3 and 0.6.
			$\Delta=0.6$	$\Delta=0.3$	
3,600	1.922	3.48	34.58	43.61	1.26
1,800	1.889	3.54	35.55	41.71	1.18
1,100	1.868	3.58	33.93	43.80	1.29
1,000	1.781	3.75	29.55	39.28	1.32
960	1.760	3.80	28.75	34.32	1.19
725	1.737	3.85	25.80	30.52	1.18
720	1.724	3.88	14.80	23.35	1.57
715	1.719	3.90	6.29	15.74	2.50
700	1.685	3.97	4.63	12.03	2.58
680	1.651	4.05	4.67	8.30	1.77
480	1.548	4.32	2.36	5.74	2.43
360	1.441	4.64	2.22	5.23	2.36
60	1.029	6.50	3.14	6.45	2.06
(*)	(*)	3.01	5.85	1.94

FROM CAKE OF SUBSTANCE 26/34.

3,600	1.944	3.44	18.19	23.11	1.26
2,400	1.938	3.45	17.35	22.75	1.28
1,200	1.881	3.56	18.18	23.60	1.27
800 to 700	1.853	3.61	16.86	19.49	1.16
	1.847	3.62	15.61	21.44	1.37
	1.817	3.68	14.95	18.43	1.22
600	1.788	3.74	10.56	11.56	1.10
500	1.735	3.86	3.38	5.16	1.53
.....	1.635	4.09	.87	2.30	2.50
180	1.530	4.43	1.00	2.00	2.00

* Powder cake, density, 0.645.

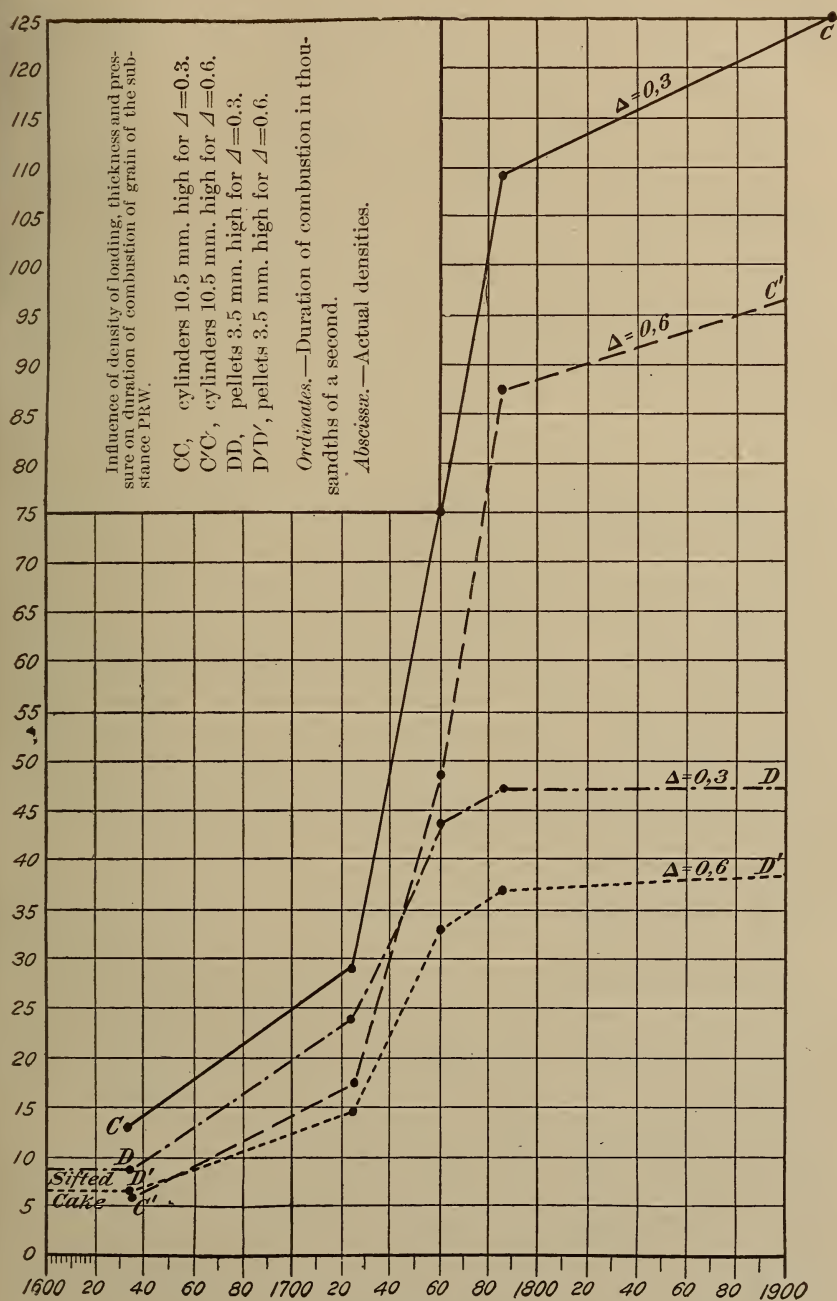


FIG. 14.

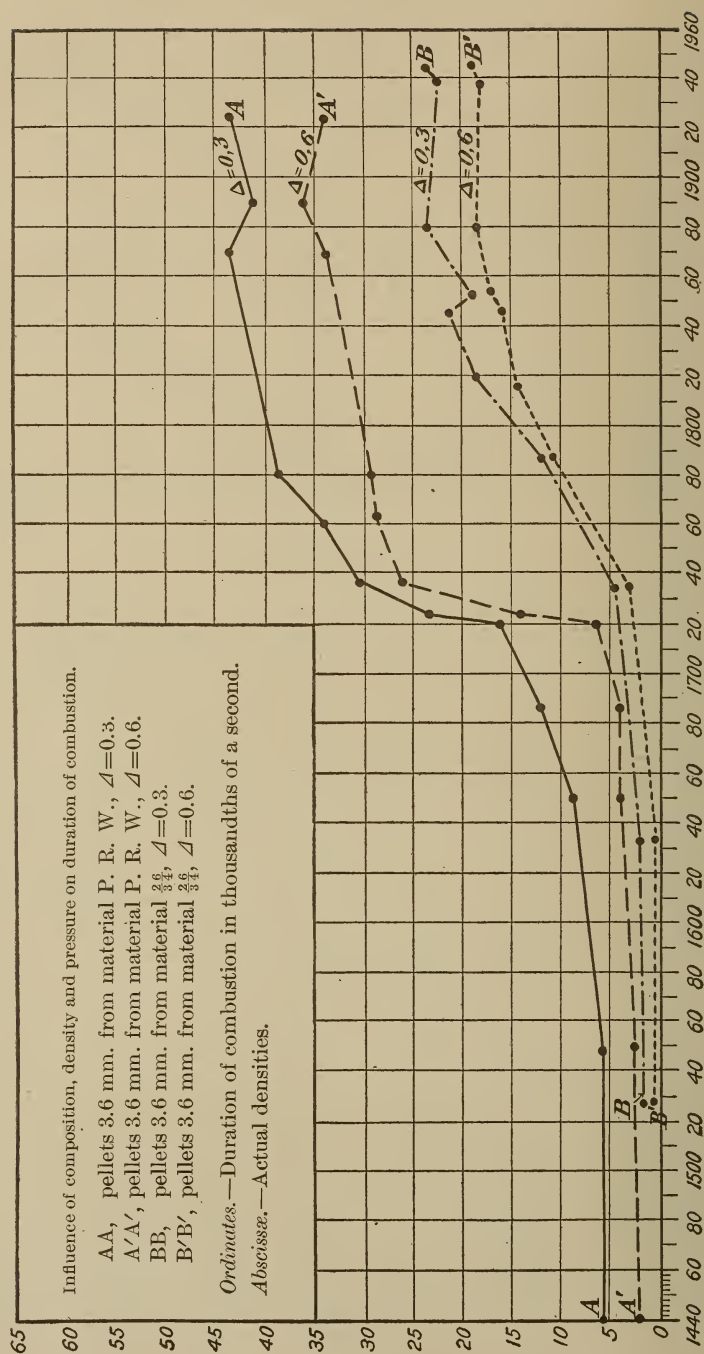


Fig. 15.

In the second series of experiments the influence of weak cohesion between particles was investigated more circumstantially. The crusher disks employed were of the usual types. The developed curves were shorter, which rendered it more difficult to determine the exact duration of combustion, (§41) but this did not influence the general deductions afforded by one and the other series of experiments.

The figures of Tables XXV and XXVI, and also of the diagrams constructed from them (figs. 14 and 15), show at a glance:

(1) That for grains of low densities, up to 1.640 approximately, duration of combustion of grains not only does not depend, at high and low densities of loading, on the dimensions of the grain, but is almost the same in both cases as the duration of combustion of powder cake.

(2) Beginning with densities of from 1.640 to 1.720, duration of combustion, which remains independent of the dimensions of the grain, steadily increases together with the density of the grain and rises to a value almost four times greater than the duration of combustion of powder cake.

(3) The density of 1.720 would seem to constitute a critical point, beginning with which, on further increasing the density of the grain to 1.80, the duration of combustion increases very rapidly with the density, and the influence of thickness becomes more noticable, but the proportionality between thickness and duration of combustion is not established. (4) On increasing the density of the grain above 1.80 the duration of combustion continues to grow very slowly. It seems to strive toward a certain limit, which is reached approximately for a point 1.85, beyond which it hardly changes further with density, but rather seems to diminish somewhat. At this point there is noticed an almost complete relationship between the prolongation of combustion and the thickness of the grain. (5) The substance of which the grain is composed does not change the general character of the previous conclusions, but influences only the absolute value of duration of combustion, which, in all cases, is greater for chocolate than for black powders.

B. Combustion of powders prepared by pressure from small grains.—Experiments to determine in this case the influence of the degree of compression on the value of the duration of combustion were conducted for a density of loading 0.6 upon pressed cylindrical grains of a diameter of 20 mm. and height from 20 to 10 mm. The compression of each kind of grain was produced sometimes at different pressures, chosen as far as possible so that densities of great and small grains would be alike. Experiments were made with three forms of initial grains. (See Table XXVII.) The samples of powder prepared from the grains employed in the manufacture of chocolate powder, specimen No. 9bis, were pressed with different contents of moisture and subsequently the final grains were dried down to a moisture content of 0.8 per cent and were burned in this condition. For purposes of comparison the durations of combustion of the original grains are also given in this table.

TABLE XXVII.—*Combustion of smoking powder, prepared by pressure from small grains.*

(1) GRAINS OF SMALL-ARM POWDER, MARK F.

Designation of samples and pressure on pressing grains.	Dimensions of grains in millimeters.	Density.	Duration of combustion for $\Delta=0.6$ in thousandths of a second.	Remarks.
Free grains F.....	Heights from 1.2 to 0.6 diam.	1.740	2.26	Composition 75-8-15 (black charcoal). Pressed dry.
600 kilograms per square centimeter.	20 { 20.10 11.80	1.670 1.700	2.74 2.78	
900 kilograms per square centimeter.	20 { 20.10 10.90	1.830 1.840	3.92 4.05	
1,200 kilograms per square centimeter.	20 { 20.10 10.40	1.900 1.910	10.73 12.54	
1,500 kilograms per square centimeter.	20 { 20.10 10.35	1.920 1.930	38.02 24.15	
1,800 kilograms per square centimeter.	20 { 20.10 10.25	1.946 1.955	52.69 34.61	

(2) FROM GRAINS EMPLOYED IN PREPARING CHOCOLATE POWDER, SAMPLE NO. 10.

Free grains.....	From 1.4 to 3.00.	1.808	6.91	Pressed dry.
600 kilograms per square centimeter.	20 { 20.10 10.90	1.831 1.840	12.04 11.43	
1,200 kilograms per square centimeter.	20 { 20.10 10.60	1.891 1.891	14.71 25.08	

(3) FROM GRAINS EMPLOYED IN PREPARING CHOCOLATE POWDER, SAMPLE NO. 9BIS.

Free grains.....	From 2.5 to 4.00.	1.820	9.80	Dry, i. e., containing 0.8 per cent of moisture.
			11.48	Containing 0.91 per cent of moisture.
1,320 kilograms per square centimeter.	20 { 20.10 11.65	1.728	12.53	Pressed with a content of 0.8 per cent of moisture.
1,140 kilograms per square centimeter.		1.720	11.91	
3,600 kilograms per square centimeter.	20 { 20.10 10.30	1.937 1.946	19.30 18.64	Before pressing grains contained 2 per cent moisture, after pressing dried to 0.8 per cent moisture.
900 kilograms per square centimeter.	20 { 19.80 11.35	1.836	14.44	
720 kilograms per square centimeter.		1.820	13.79	
1,200 kilograms per square centimeter.	20 { 19.85 10.80	1.891	18.75	
720 kilograms per square centimeter.		1.891	18.08	
1,800 kilograms per square centimeter.	20 { 19.85 10.70	1.958	51.24	
1,440 kilograms per square centimeter.		1.940	34.15	
3,600 kilograms per square centimeter.	20 19.90	1.950	73.53	

TABLE XXVII.—*Combustion of smoking powder, etc.*—Continued.

(3) FROM GRAINS EMPLOYED IN PREPARING CHOCOLATE POWDER, SAMPLE NO. 9BIS—Continued.

Designation of samples and pressure on pressing grains.	Dimensions of grains in millimeters.	Density.	Duration of combustion for $\Delta=0.6$ in thousandths of a second.	Remarks.
120 kilograms per square centimeter.	20 { 19.70 11.25	1.830 1.850	17.93 17.75	Before pressing grains contained 3.5 per cent of moisture and after pressing 0.8 per cent moisture.
180 kilograms per square centimeter.	20 { 19.70 10.75	1.923 1.945	28.71 26.58	
240 kilograms per square centimeter.	20 { 19.70 10.70	1.933 1.945	24.81 45.34	
3,600 kilograms per square centimeter.	20 { 19.70 10.65	1.950 1.980	78.42 30.87	

It is evident from the figures given in Table XXVII, that:

(1) In relation to specimens prepared from the grains of powder F_1 :

(a) The duration of combustion of samples increases progressively in accordance with the force of compression, but does not depend at all upon the dimensions of the final grains, even when their duration of combustion exceeds by as much as five times that of the original grains.

(b) Upon further increase of pressure (above 1,500 kilograms), the influence of the final dimensions of the grains becomes more and more perceptible.

(2) In relation to the specimens prepared from grains for powders No. 10 and 9bis:

(a) Results with these samples indicate a considerable resistance to compression on the part of the dry material (we shall consider as dry material a substance containing 0.8 per cent moisture) when in the form of small but dense grains. Thus for the highest pressure obtainable in practice (3,600 kilograms per square centimeter) the duration of combustion of samples prepared from dry grains was doubled in comparison with duration of combustion of the free initial grains, but did not depend upon thickness, although the density of the final grains considerably exceeded that of the original grains.

(b) On increasing the content of moisture in the pressed grains the duration of combustion of the subsequently dried final grains increased, notwithstanding the fact that their density was less as compared with samples prepared from drier grains. For an original content of moisture of 2 per cent the duration of combustion was doubled for compression under a pressure of $\frac{1,200 \text{ kgr.}}{1.020 \text{ cm}^2}$, and for a content of moisture of 3.5 per cent for pressure something greater than $120 \frac{\text{kgr.}}{\text{cm}^2}$, but these values did not depend upon the dimensions of final grain.

This independence of duration of combustion upon thickness occurs for an initial content of moisture of 3.5 per cent, and in the case (pres. $180 \frac{\text{kgr.}}{\text{cm}^2}$), when the duration of combustion is trebled.

(c) The influence of thickness becomes more evident the higher the pressure rises and the drier the original grains; when the moisture is 2 per cent it becomes observable on pressing under a pressure of $\frac{1,800 \text{ kgr.}}{1.400 \text{ cm}^2}$, and when the moisture is 3.5 per cent for a pressure of $240 \frac{\text{kgr.}}{\text{cm}^2}$, but in the latter case, and in general, for greater pressures anomalies are observed which evidently relate to inequalities of compression of the final grains and to the difficulty of preparing homogeneous samples.

(d) The combination of experimental results with samples prepared from grains of powder 9 bis shows that the duration of combustion of the final grains depends not so much upon the mean densities of the grains as it does, generally speaking, upon the *plasticity* of the original mass. Thus, durations of combustion of original grains are doubled for samples prepared from the initial grains with a content of moisture of 0.8, 2, and 3.5 per cent, corresponding to densities of dry grains of 1.940, 1.890, and 1.840, respectively.

All the above experiments upon the combustion of samples prepared by pressure from small grains permit us to arrive at the same general conclusions such as are arrived at above in relation to the combustion of samples prepared from mill-cake; the whole difference relates to the values of the densities (of the final grains) to which the peculiarities of combustion above indicated are related. These values of the densities evidently depend on the densities of the original grains and their degree of moisture. We observe that in the above experiments upon samples prepared from small grains, only the three first characteristic phases of combustion of powder may be observed.

44. *Conditions of the combustion of smoking powders in concentric layers.*

It was stated above (paragraph 43) that the duration of combustion of grains prepared from mill-cake out of the substance of smoking powders for densities above 1.85 remained almost constant and was almost proportional to the thickness of the grain. This latter, as was stated in paragraphs 39-40, constitutes one of the characteristic indications of combustion of grains in concentric layers. In addition to the experiment there deduced, Table XXVIII gives the durations of combustion of grains of densities above 1.9 prepared from mill-cake of the substance of powders PRW and ³⁰/₄₀. We observe that in the case of relationship between thickness of grain and duration of combustion, the mean speeds of combustion given in the next before the last column of Table XXVIII should be one and the same for similar grains, and in the case of dissimilar grains the speeds should increase with diminution of *extent* of grain—i. e., in the given case, with increase in the height of grain (paragraph 40). Further, when similar powder grains are burned in this manner the maximum increase of pressures $\left(\frac{d P_t}{d t}\right)_{\max.}$ is inversely proportional (39) to thickness of grains, and the graphic conceptions of the development of pressures as a function of time for such grains (see the same) differ from one another only in the scale of times, which should diminish in ratio to the thickness.

The figures for mean speeds of combustion and for the value of $\left(\frac{d P_t}{d t}\right)_{\max.}$ presented in Table XXVIII, and also the curves of development of pressure as represented in fig. 16, for the burning of charges of similar grains, serve to confirm at a glance what has been said, and consequently, the sum and substance of all this goes to show that the materials of which smoking powders are composed, whether chocolate or black, burn both at low and high pressures in concentric layers when a density of 1.90 and over is realized—i. e., their mixtures develop the characteristics of wholly compact substances, a condition which is easily developed when the original material is mill-cake, but which is developed only with great difficulty when the original substance is built up by compression from small dry grains. Such a character of combustion evidently constitutes a limiting and completely exclusive phenomenon which has nothing in common with the combustion of the usually employed forms of smoking powders prepared from brown and black charcoals.

TABLE XXVIII.—*Combustion of grains of density not less than 1.9, prepared by pressure from cake of the powder substances, PRW and ³⁰/₄₀.*

Designation of sub- stances from which the grains are pre- pared.	Dimensions of cylindrical grains (millimeters).			Density of load- ing.	Duration of combus- tion (thou- sandths of a second).	Mean rate of combustion (centimeters per second).	$\left(\frac{dP_t}{dt}\right)$ maxi- mum (tons per square centime- ter per second).	
	<i>Dia- meter.</i>	<i>Height.</i>	<i>Extent.</i>					
Substance ³⁰ / ₄₀	20	1.70	11.800	0.6	11.00	7.72	
		3.40	5.900		20.00	8.50	
		9.60	2.100		56.00	8.57	
		19.30	1.000		79.00	12.20	
	20	* 10.66	1.900	.3	87.40	6.10	20.3	
	10	* 5.35			40.00	6.67	42.7	
	20	† 10.84			70.50	7.69	70.0	
	10	† 5.41			32.40	8.34	136.1	
	Substance P. R. W.	20	3.40	5.900	.6	30.00	5.70
			9.60	2.100		76.00	6.30
19.10			1.000	107.00		8.90	
3.40			5.900			38.25	4.41	4.36
						39.58	4.29	
						92.09	5.65	15.54
						95.52	5.44	
10.00			2.000			128.86	7.79	7.67
						133.12	7.55	

* Dissimilar.

† Similar.

‡ In this series of experiments durations of combustion are reckoned from the beginning of inflammation of the grain.

We must also observe the fact that the combustion of compact masses prepared from the material of which smoking powders are made is very slow and depends directly upon the composition of the substance.

45. *Explanation of the influence of amount of pressure employed in agglomeration of a substance on the character of its combustion.*

In paragraphs 43-44 experiments on the influence of the degree of compression upon combustion of the grains and the powder mass in general were discussed, and it was found possible to establish general deductions as to the mode of change in character of combustion in relation to the force of compression exerted upon the final grains and with respect to the amount of moisture in the original mass; and to show that when the pressure was increased, a certain limit was reached which corresponded to the combustion of compact masses in concentric layers.

Vieille explained this progressive change in character of combustion very simply, assuming that the propagation of combustion in the powder mass proceeds from two causes—the conductivity of the mass itself and the speed with which the highly heated gases penetrate

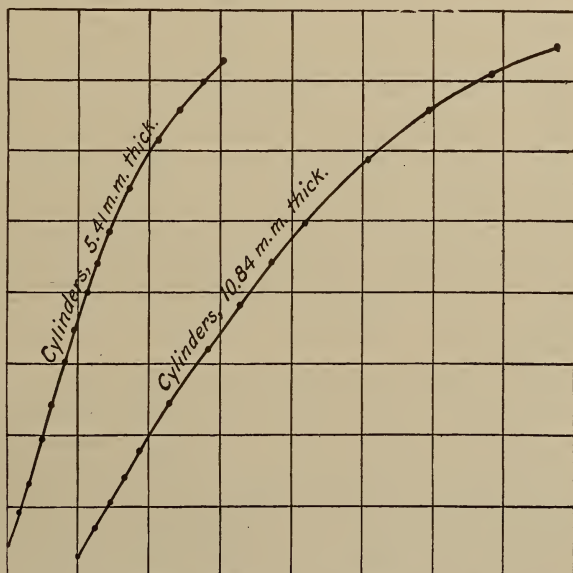
within the grains through channels formed by the pores existing between the particles.* The greater the network of these channels existing between the independent particles of a given grain the more quickly the flame envelops all the separate particles, and in this case complete combustion depends but little upon the conductivity of the substance. On the other hand, when there are no pores, i. e., when the substance is compact, the combustion is propagated exclusively by conductivity of heat. The streaming of the heated gases through the channels between the particles may be compared to the flow of a liquid through capillary tubes. According to Poiseuille the quantity Q of a liquid that flows through capillary tubes may be expressed as

$$Q = \frac{KPD^4}{L},$$

where K is a coefficient characteristic of the liquid, P the pressure, D the diameter of the tube, and L its length. The rate of flow is consequently proportional to $\frac{Q}{D^4}$, and increases with great rapidity with the increase of dimensions of the opening.

When heated gases penetrate through a nonhomogeneous mass, the channels (interstices between the particles), which are of a certain size—we shall designate them as channels of the first order—exert a preponderating influence on the propagation of inflammation. Smaller channels, say ten times smaller, have no significance, as the penetration of gases through them, and consequently the rapidity of inflammation, proceeds at a rate one hundred times less.

Having agreed upon this, let us consider those varieties of powders prepared from fine grains, or, generally speaking, from particles, the



Scale: Abscissae 1 c. m. = $\frac{1}{1000}$ sec.; ordinates 1 c. m. = 400 k. gm.

FIG. 16.

* Compare the hypotheses of Sarrau and also of Sebert and Hugoniot on the speeds of combustion of powder (paragraph 34).

cohesion between which is very weak. In this case there exists a stream of channels of the first class throughout the powder, which correspond to the inflammation of all separate grains or particles without sensible retardation, or in so short an interval of time that it may be neglected in comparison with the duration of combustion of the elementary grains (particles). Such a character of combustion corresponds to the first phase of the influence of the degree of compression upon combustion, and represents evidently many of the examples given above, where duration of combustion of the final grains and of the original substance (fine grains or cake) of which the final grains are prepared are the same.

Upon stronger compression at the time of pressing, a portion of the channels of the first class disappear, as the particles and grains of the original substance grow together, so to speak, and their adjacent surfaces coincide more and more completely. In consequence of this, although there exists in the final grain a number of channels of the first order, yet they no longer separate the original grains (particles) but delimit separate groups of the latter, which constitute separate nuclei of a polyhedral form. The size of these nuclei or nodules increases with increase of pressure, but the dimensions remain considerably less than those of the final grain, and at the limit a certain part of the channels of the first order still remains. On account of this the mean duration of combustion of these nodules may be even as high as three times that of the original grains (duration of combustion), but does not depend upon the outer dimensions of the original grain. This constitutes the second phase of the influence of compression on the combustion of powder. The grains of smoking powders employed at the present time for ordnance purposes belong only to this type, and, therefore, their combustion proceeds in an irregular manner.

Upon further increasing the compression, the system of channels of the first order becomes so reduced that the dimensions of the nuclei bounded by them become of the same order as the dimensions of the final grains. This gives rise to sharply-marked irregularities in combustion, which are especially noticeable on burning samples prepared from fine grains; but, nevertheless, the influence of the thickness of the final grains upon duration of combustion remains noticeable. This constitutes the third phase of the influence of compression on the character of combustion of the powder.

Such materials are not employed in practice on account of the extreme irregularity of their combustion. Irregularity of combustion and together therewith the third phase of the phenomenon under consideration continue, even for increased pressures, as long as all

channels of the first order are not closed, and the influence of the thickness of the grain does not become complete.

From this moment on, the fourth phase of the influence of compression on the character of combustion begins; the structure of the final grain approaches the limiting condition and corresponds to complete homogeneity of the substance, and the duration of combustion changes but little henceforward with further increase of pressure. It is easily understood, in fact, that the speed of combustion in the case in point depends chiefly upon the evolution of heat, but only in a slight degree upon the speed of penetration of the heated gases through channels of the second order whose magnitude continues to diminish gradually, so that finally, when the practical limit of possible pressure is reached, the speed of combustion even diminishes somewhat. Compact masses of smoking powders, considered in paragraph 44, represent this fourth phase and especially pertain to the limiting condition of it through the character of their combustion.

46. *Influence of composition, methods of mixture, duration of incorporation under edged runners, composition of the original material, variety of charcoal, and moisture upon duration of combustion.*

Density exerts, as we have seen, along with certain other factors, a practical influence upon duration of combustion of ordinary powder grains, while, in the case of "compact" substances developed through application of very high pressures, combustion does not depend upon it. On this account compact substances are most suitable for investigating the influences upon duration of combustion, of composition and physical condition of the elements, methods of incorporation and prolongation thereof, and percentages of moisture. Experiments to this end were carried out for a density of loading of 0.6—i. e., for developed pressures of about $3,000 \frac{\text{kgr.}}{\text{cm.}^2}$. The samples were prepared from mill cake like that referred to in paragraph 43, which was compressed under a pressure of $3,600 \frac{\text{kgr.}}{\text{cm.}^2}$.

Table XXIX shows the results of certain experiments performed by Vieille, and gives the mean velocities of combustion of various materials prepared from cylinders of 20 mm. in height.

TABLE XXIX.—*Showing influence of composition, method of mixing, duration of incorporation under edge runners, kinds of charcoal, and moisture on duration of combustion, i. e., on the mean rate of combustion.*

Kinds of powder substance and methods of incorporation.	Mean rate of combustion of cylinders 20 mm. high.
	<i>Cm. per sec.</i>
1. Powder $\frac{2}{11}$, of composition 75-10-10 and 5 parts paraffin	8.46
2. Powder PRW	8.90
3. Powder PRH	10.20
4. French powder of composition (* 78-3-19) with chocolate charcoal from thorn wood, charred—	
77 per cent	10.56
90 per cent	10.17
5. The same (*), but with charcoal from alder, charred—	
78 per cent	10.93
89 per cent	10.09
6. Sevran powder with brown charcoal	11.50
7. Powder $\frac{26}{34}$	12.20
8. Powder $\frac{30}{40}$	13.85
9. Powder Okhta	14.97
10. Very fine Okhta	18.00
11. The same as 4, but mixed under stamps	3.56
12. Powder of composition (78-4-18, incorporated under edge runners 1 hour)	8.16
13. The same, 2 hours 30 minutes	9.93
14. Powder PRH—	
Wholly dry	7.19
Containing 1.4 per cent moisture	7.04
Containing 2.4 per cent moisture	6.64
15. Powder $\frac{26}{34}$ —	
Wholly dry	10.73
Containing 0.94 per cent moisture	9.96
Containing 1.55 per cent moisture	9.85

*Incorporation continued 2 hours 30 minutes under edge runners.

On comparing these figures together we observe the following:

(1) Samples from 1 to 10 show that the speed of combustion may be varied through very wide limits for the methods of manufacture employed, through the introduction into the composition of the powder of paraffin and of charcoals of various degrees of carbonization, and that the introduction of paraffin and partly charred coals exerts the greatest effect on diminution of speed of combustion, although charcoal prepared from the alder possesses in this relation a certain superiority over charcoal prepared from thorn wood.

(2) Samples 11 and 4 show that duration of combustion is trebled by incorporating under edge runners for two and one-half hours, in comparison to the same when the substance is mixed under stamps. It should be noted, however, that powder when not worked under runners is *weaker* than powders thoroughly worked. This may be due to the cooling effect produced by very prolonged combustion and to the imperfect reaction between the elements when mixed together coarsely. Thus, in the examples under consideration, the pressures (for $\Delta=0.6$) were 2,416 kilograms and 2,948 kilograms per square centimeter, respectively.

(3) Samples 12 and 13 show that continuance of incorporation under edged runners increases the speed of combustion.

(4) Samples 14 and 15 show that moisture reduces the speed of combustion. This influence amounts to 3 or 4 per cent for 1 per cent of moisture.

47. *Differences of combustion of different powder grains of the same manufactured lots.*

Experiments to this end were performed upon the chocolate powders PRW and PRH, two samples of the latter being employed, one of which was powder manufactured for the 24 cm. gun and the other powder for guns of from 27 cm. to 42 cm. caliber. The experiments were performed in bombs employing a density of loading of 0.6. Samples were taken from one-half of the prisms, as explained in paragraph 42, in the form of large sections, while the second halves of the prisms were reserved to verify original experiments and to determine the physical characteristics of the material. Results of these experiments are given in Table XXX. They consist in determinations of the duration of combustion of grains and of the maximum rates of development of pressures.

TABLE XXX.—Duration of combustion Π_a and the quantity $\left(\frac{dP_t}{dt}\right)_{\max.}$ for different grains of powders of the same lots.

Powders.					
P. R. W.		P. R. H.			
		For the 24 cm. gun.		For the 27-42 cm. gun.	
Π_a	$\left(\frac{dP_t}{dt}\right)_{\max.}$	Π_a	$\left(\frac{dP_t}{dt}\right)_{\max.}$	Π_a	$\left(\frac{dP_t}{dt}\right)_{\max.}$
<i>Thousands of a second.</i>	<i>Tons per sq. cm. per sec.</i>	<i>Thousands of a second.</i>	<i>Tons per sq. cm. per sec.</i>	<i>Thousands of a second.</i>	<i>Tons per sq. cm. per sec.</i>
10.57	407.3	11.50	422.4	11.60	453.8
				11.75	
11.27	365.2	11.91	11.62	
				11.62
12.03	12.03	12.34
				13.36
12.08	12.53	12.43
				12.86
12.32	12.66	12.88
				12.58
12.47	13.10	12.96
				12.53
12.52	13.12	363.7	14.25	323.7
				15.02
13.04	13.17	15.40
13.28	467.2	13.47	16.55	255.8
13.85	13.60	18.01
14.08	13.80	18.64
				20.58
15.30	15.07	17.33
				21.80
16.59	15.38	22.44
*13.03	15.92	296.5	22.46
				22.58
†1.10	17.72	29.31
				27.25	211.3
		19.61	28.24	
				27.44	
		19.98	20.54
				31.02
		20.30	23.30
				45.08
		25.34	33.42
				45.63	126.0
		27.21	189.1	37.11	
		*15.86	*20.74
		†2.99	†6.71

* Arithmetic mean.

† Probable variation.

The figures as found are arranged in the table in the order of increasing values of Π_a . The values of $\left(\frac{dP_t}{dt}\right)$ show that the latter change almost in the inverse ratio to Π_a . The values of Π_a given in the denominators of the quantities for the powder PRH for the 27-42 cm. are results obtained by duplicating experiments with the same grain.

As is evident from the table, the grains experimented with arrange themselves in order of duration of combustion as follows:

	Powders.		
	PRW.	PRH.	
		24 cm.	27-42 cm.
	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>
Number of grains burnt in from $\frac{10.5}{1,000}$ to $\frac{15.5}{1,000}$ second.....	92.3	65.0	40.9
Number of grains burnt in from $\frac{15.5}{1,000}$ to $\frac{19.5}{1,000}$ second.....	7.7	15.0	13.7
Number of grains burnt in from $\frac{19.5}{1,000}$ to $\frac{23.5}{1,000}$ second.....		10.0	22.7
Number of grains burnt in a time longer than $\frac{23.5}{1,000}$ second		10.0	22.7

These results lead to the following deductions:

(a) The greatest difference for the powders experimented with is found for the powder PRH for the 27 to 42 cm. gun, which contains a considerable admixture of slow-burning grains, as is indicated by the magnitude of the probable errors, which is about two and one-fourth times as great as that found for the powder PRH for the 24-cm. gun, and almost six times as great as that for the PRW powder.

(b) The densities of grains are evidently related to the rapidity of their combustion; thus, in the case of powder PRH two grains, whose duration of combustion was 0.035 second, possessed a density of 1.882, and six which burned in 0.012 possessed a density of 1.853. The external differences of the grains, i. e., their color and structure, bore no relation to their rapidity of combustion.

(c) A repeated observation of the duration of combustion of the same grains (the denominators of the quantities in the next to last column, Table XXX) for the powder PRH shows that in the case of grains whose duration of combustion was less than 0.0155 second the two observations did not differ from one another by more than 0.001 second; while, when the slowness of combustion of the grain was increased, the difference became greater, rising in extreme cases to as high as from 0.01 to 0.001 second.

The cause of this difference must be sought in the conditions of manufacture of the grains in question. In fact, as long as the compression of the powder does not exceed the second phase (paragraph 45), which implies the disintegration of the grain into more or less

regular nodules when it is burned, the duration of whose combustion may be as high as four times that of the original substance, then up to this limit the combustion of the final grain occurs in a regular manner. When, however, in order to secure slowness of combustion, the compression of the original material is increased to a certain limit, the third phase appears; which corresponds to the combustion of nodules of large dimensions, and which are of the same order as the dimensions of the final grain, whereby the marked irregularities observed in the combustion of very dense prisms result.

(d) This irregularity, which is met with in the manufacture of powder, explains the phenomenon sometimes observed and which is adduced as proof of the combustion of smoking powders in concentric layers. We referred to the fact stated in paragraph 32, that sometimes there are found in front of the gun not only unburned portions of grains, but even whole grains (prisms), which preserve the form of the original grain, and this is especially true in the case of chocolate powder. It should be noted, however, that the amount of powder expelled from the gun was a very small quantity in the majority of cases, and therefore the character of combustion of this small portion may not serve as an index of the mode of combustion of the whole charge.

On account of the rarity of the occurrence of these pieces and what is said in relation thereto in paragraph 45, their appearance seems ascribable to the fact that during compression some of the grains are exposed to the extreme pressure, and at the same time, in most cases, to an unequally distributed pressure, so that parts and even whole grains may be built up as compact masses (paragraph 44) which burn very slowly in comparison with other grains, and which, moreover, burn in concentric layers.

(e) It should be remarked, in general, that the powder most uniform in its combustion, namely, the powder PRW, affords the best ballistic results, both in regard to uniformity as well as to magnitudes of velocities developed for a given pressure. A mixture of slow and quick burning powders intended to develop the same velocities as are developed by a homogeneous mass leads to bad results; the velocity required is developed only for a very great pressure. Homogeneity of powder is of first order of importance in respect to ballistic relations.

From what has been said above it is evident that in practice the greater the density of the final grains for a given density of loading of the initial substance, the greater the irregularity that may be expected in the action of the smoking powder, even though the difference in the densities of independent grains be inconsiderable.

48. Particulars of the combustion of polished powders.

Powders of the Pebble type and powders mark 30/40 and 26/34, C₁ and C₂, considered above, are prepared in the form of pellets and are pressed from a mixture of grains and cake. The final grain is subse-

quently polished, which results in its surface being covered with a dense coating. The object of this is to insure the progressive combustion of the grain, under the assumption that combustion proceeds in concentric layers and that at first the outside burns more slowly while subsequently the combustion of the interior part of the grain proceeds more rapidly. In order to illustrate the signification of the outer coating in relation to the combustion of the whole grain, observations were made among others upon combustion of grains whose surfaces were preserved in a perfectly polished state, of their interior substance, and of the outer coating of the powder 26/34. Similarly experiments were made upon combustion of whole grains of powders C_1^* and C_2^* and of the same grains split in halves in a direction perpendicular to their thickness. Such a comparison was instituted for the reason that the dimensions of the grains were small and the separation of the polished coating was very difficult to effect. Samples from the interior and from the outside (of thicknesses of from 4 mm. to 5 mm.) of the powder 26/34 were taken, as described in paragraph 42. All these experiments were conducted with a density of loading of 0.6.

Table XXXI presents the results of observations of the combustion of powder 26/34 and fig. 17 illustrates diagrammatically the laws of development of pressures as functions of time as deduced from the observed curves:

TABLE XXXI.—Combustion of whole grains and portions of grains of powder 26/34.

No. of experiment.	Whole grains.			Centers of grains.			Outer surfaces.		
	Duration of combustion.	$(\frac{dP}{dt})$ max.	Final pressure.	Duration of combustion.	$(\frac{dP}{dt})$ max.	Final pressure.	Duration of combustion.	$(\frac{dP}{dt})$ max.	Final pressure.
	Thousandths of a sec.	Tons per sq. cm. per sec.	Kgr. $\frac{cm.^2}{cm.^2}$	Thousandths of a sec.	Tons per sq. cm. per sec.	Kgr. $\frac{cm.^2}{cm.^2}$	Thousandths of a sec.	Tons per sq. cm. per sec.	Kgr. $\frac{cm.^2}{cm.^2}$
1	10.3	1,182	2,941	9.36	1,124	2,892	{ 11.92	550.2	2,758
							{ 12.81	385.3	2,955
2	(†)	966	2,950	10.50	1,369	2,777	{ 13.80	425.4	2,849
							{ 13.10	431.1	2,687
3	9.4	1,162	2,873	6.00	1,281	2,878	{ 11.40	453.0	2,839

† Incomplete curves.

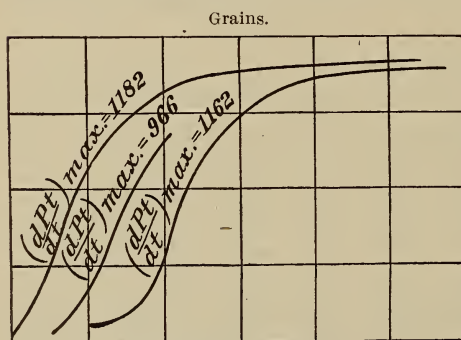
The following may be stated for powders of this type on the basis of these data:

(1) The duration of combustion of whole grains and their mean parts and also the law of development of the pressure of gases is almost one and the same thing.

*The thickness of grain of these powders was 6.5 mm. and 8 mm. for density of loading from 1.75 to 1.775; the number of grains per kilogram, 1,750 to 640.

(2) The duration of combustion of the exterior of the grain and especially the quantity $\left(\frac{dP_t}{dt}\right)_{\max.}$ corresponding thereto, differ considerably from the same quantities as obtained for the combustion of the whole grain and for its mean parts. Keeping in view that the outer polished surface may be removed at will for a depth of from 4 mm. to 5 mm., and that the value of $\left(\frac{dP_t}{dt}\right)_{\max.}$ for it is at least twice what it is for the whole grain, it may be stated that the outer layer, which is exposed to the action of polishing, burns in a completely different manner than the interior mass of the powder grain, approximating in its character of combustion to that of a compact mass.

(3) Notwithstanding this, if attention be not given to the period of combustion of the powder up to the development of a pressure of 300 $\frac{\text{kg.}}{\text{cm.}^2}$, the influence of the outside of the grain upon the combustion of the whole appears almost unappreciable, since for the case under consideration a pressure of 300 $\frac{\text{kg.}}{\text{cm.}^2}$ corresponds to a combustion of scarcely one-tenth of the whole of the charge.



Combustion of powder.
Scale: Ordinates 10 m. m. = 800 kg.; abscissae 10
m. m. = $\frac{1}{1000}$ sec.

FIG. 17. (A)

(4) The general form of the diagram of pressures developed as a function of the time shows that, contrary to the purposes of polishing (p. 139), there occurs at the beginning an excessive evolution of gases and at the end a very slow evolution, so that it is extremely difficult to determine the point of coincidence of the final element of the curve with the tangent, and, therefore, to determine the true value of the duration of combustion, which changes with the fineness of the curve (41).

(5) It follows, therefore, that in the given case duration of combustion may not serve as a characteristic of the powder in the relation considered, and that comparisons of such powders may be made only

for the value of $\left(\frac{dP_t}{dt}\right)_{\max.}$

(6) The peculiar characteristic of the combustion of the powders under consideration fully indicates that their initial material consists of a mixture of grains and cake; this lack of homogeneity results in the

vigorous development of gases at the beginning of combustion, which falls, however, after the lapse of a short time, in consequence of the more rapidly burning elements being completely consumed (particles of cake), while at the final stage of combustion the evolution of gases occurs as it does for more dense materials.

In order to illustrate more completely the character of combustion of polished powders, the elements of combustion of whole grains of powder 26/34, and of

their mean parts, was determined by curves obtained from pressures developed in manometers (crusher gauges) through the employment of copper disks of small resistance, which were 4.9 mm. in height and 3 mm. in diameter. Although these disks did not afford the possibility of determining the law of development of pressures for values greater than $800 \frac{\text{kgr.}}{\text{cm}^2}$, because they were reduced at this limit into pellets; yet

the initial portions of the curves of compression obtained from them

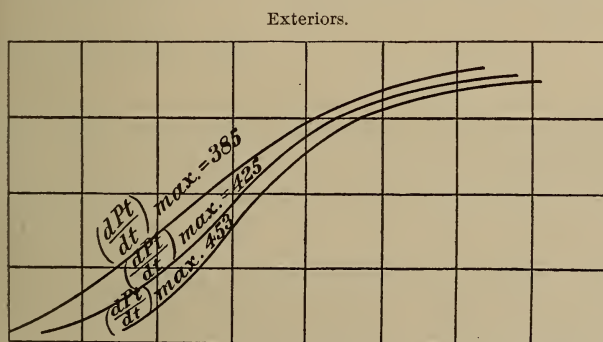


FIG. 17. (C).*

only for pressures less than $300 \frac{\text{kgr.}}{\text{cm}^2}$. Combustion at great pressures is almost identical in both cases. We may observe that if the combustion of the whole grain proceeded in concentric layers, the influence of polishing would be indicated as a diminution of evolution of gases for combustions of at least from one-fourth to one-third of the whole mass of the grain—i. e., for a density of loading $\Delta = 0.6$,

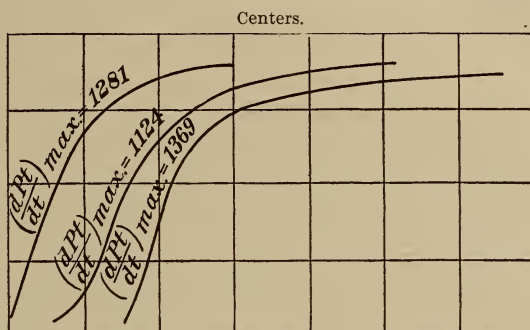


FIG. 17. (B).*

determined with great accuracy the law of development of very small pressures. These latter are shown in fig. 18, which shows at a glance that sensible differences of combustion of whole grains and their mean portions occur practically

*Scale of diagrams; ordinates, 10 mm. = $800 \frac{\text{kgr.}}{\text{cm}^2}$; abscissal, 10 mm. = $\frac{1}{1000}$ sec.

up to the moment when the pressures rise as high as from 700 to 900 $\frac{\text{kg.}}{\text{cm}^2}$; while in actual practice this influence extends through the period of combustion covering pressures less than 300 $\frac{\text{kg.}}{\text{cm}^2}$. It should be borne in mind that in the practical employment of these powders the influence of the polished surfaces is found still less than in the case of the grains used in these experiments, which were chosen from among those whose polished surface had remained intact; while, on account of conditions under which the powder is kept in practice, many grains have their surfaces broken and cracked.

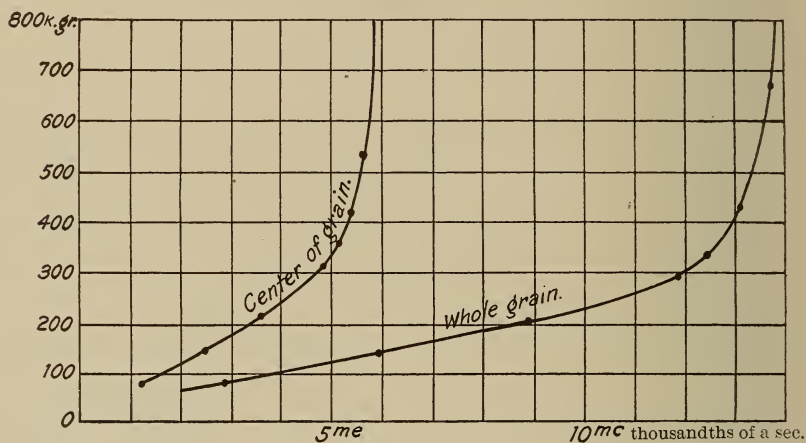


FIG. 18.

Results of observations upon the combustion of powders C_1 and C_2 are given in Table XXXII, which show that no actual differences in the combustion of whole and split grains were observed. The investigation of the initial phase of combustion of whole grains and half

TABLE XXXII.—Elements of combustion of powders C_1 and C_2 .

Kind of powder.	Whole grains.			Half grains.		
	Πa	$\left(\frac{dp}{dt}\right)_{\text{max.}}$	Maximum pressure.	Πa	$\left(\frac{dp}{dt}\right)_{\text{max.}}$	Maximum pressure.
	Thousandths of a sec.	Tons per sq. cm. per sec.	$\frac{\text{Kgr.}}{\text{cm}^2}$	Thousandths of a sec.	Tons per sq. cm. per sec.	$\frac{\text{Kgr.}}{\text{cm}^2}$
C_1	6.13	1,498	2,839	7.32	1,252	2,725
	7.17	1,442	2,849	6.00	1,400	2,744
C_2	6.54	1,963	3,015	5.40	1,787	3,067
	6.03	1,782	3,219	9.13	1,576	3,115

grains of the C_1 powder, as expressed by the curves of compression of the copper disks of small resistance, did not, according to Vieille,

admit of the detection of any sensible differences in their character of combustion, even at low pressures. In general, it may be said in relation to the curves developed for pressures obtained from these powders, that they exhibit the characteristics inherent in powders prepared from the mixture of mill cake and small grains.

These experiments afforded reason for assuming that on firing charges of whole and half grains from cannon similar results would be obtained. To verify this, firing was conducted with a 90-mm. field-gun battery, the charges being taken from powder C_1 ; some charges consisted of normal grains and others of half grains, in relation to which it may be stated that the number of half grains in 1 kilogram of powder taken was actually twice that of the number of whole grains. Results of this firing are presented in Table XXXIII:

TABLE XXXIII.—*Results of firing charges of whole and split grains of powder C_1 , from the 90-mm. gun.*

Weight of grains.	Number of shots.	Whole grains.		Half grains.		Difference.	
		Velocity at 35 meters from the muzzle.	Pressure at base of bore.	Velocity at 35 meters from the muzzle.	Pressure at base of bore.	In velocities.	In pressures.
<i>Kgr.</i>		<i>M. per sec.</i>	<i>Kgr. cm²</i>	<i>M. per sec.</i>	<i>Kgr. cm²</i>	<i>M. per sec.</i>	<i>Kgr. cm²</i>
1.50	1	376.1	1,098	380.3	1,156	+4.2	+58
1.75	1	413.3	1,397	416.0	1,752	+2.7	+355
1.90	2	434.0	1,696	440.4	2,234	+6.4	+538

The grains that were split in half by planes perpendicular to their thickness should have developed, under the assumption of combustion in concentric layers, almost double the speed of combustion of the material, and, in accordance with Sarrau's formulæ (see formulæ 87-89, farther on), increased the initial velocity by 40 meters per second and doubled the pressure. We actually find, however, that the diminution of grains has almost no effect on velocities, and although pressures are increased, especially when large charges are fired, yet they are far from being increased as much as they would be according to the formula.

If we compare the maximum increase in pressure, $538 \frac{\text{kgm}}{\text{cm}^2}$, with the least gain in corresponding velocity, 6.4 meters, we are forced to the conclusion that in this case we are dealing with an abnormal phenomenon, as for one meter gained in velocity the pressure increased 85 kilograms, while under normal conditions it should not have increased more than 43 kilograms—twice as small. This shows that the marked increase in pressure indicates the existence of local abnormalities of pressure, which are due to insufficiently rapid inflammation in the case of quick-burning powder, and which would not occur with a polished or special form of grain (e. g., a prismatic grain pierced by an axial channel).

49. *Comparison of prismatic and coarse-grain (pebble type) powders.*

The experiments cited in paragraphs 47 and 48 permit us to note the inherent difference in the character of the combustion of prismatic powders obtained by pressing individual prisms out of small grains and powders of the coarse-grain type (Pebble), obtained in the form of pellets pressed from a mixture of grain and powder cake. Although neither type of these powders burns in concentric layers, nevertheless the general characteristics of their combustion are quite different. The methods employed in each system of manufacture to obtain powders that burn slowly, are also different.

When prismatic powders are burned, the evolution of gases, i. e., the growth of pressure, reckoning from the moment when the pressure attains the value of 300 atmospheres, varies between narrow limits during the progress of this experiment, while when powders of the pebble type are employed the rate of evolution of gases varies between wide limits. We may judge of this from the value of the ratio

$\frac{\left(\frac{dPt}{dt}\right)_{\max.}}{P}$, where $\frac{P}{\Pi_a}$ represent the mean rate of evolution of

pressure, and $\left(\frac{dPt}{dt}\right)_{\max.}$, the maximum velocity of the same phenomenon.

In the case of chocolate prismatic powders the value of this ratio is 1.7; with Pebble powders, from 2.5 to 3. In consequence of this the character of combustion of the former of these powders is determined uniformly, both by the duration of combustion Π_a as well as by the quantity

$\left(\frac{dPt}{dt}\right)_{\max.}$; for the latter it is determined only by the quantity

$\left(\frac{dPt}{dt}\right)_{\max.}$ For all other conditions equal, the quantity $\left(\frac{dPt}{dt}\right)_{\max.}$ may change within wide limits for chocolate prismatic powders (e. g.,

from 240 to 630 atmospheres per square centimeter per second), on varying dimensions and density of the original grains, and also the density of the final grain, which peculiarity is attributable to the properties of the brown charcoal, which contracts on drying. In the case of powders of the pebble type this quantity remains almost constant (from 1,400 to 1,100 atmospheres per square centimeter per second) for all varieties, both those that burn most rapidly as well as those that burn most slowly, while the influence of the outer dimensions of the finished grain and its polished surface only slightly reduces the rapidity of combustion.

While the influence of these circumstances (dimensions of finished grains and exteriors of grains) affects the original slowness of combustion but little, they are very important from the ballistic standpoint, for they permit, on the one hand, the flame to penetrate through the whole length of the charge, so that great alterations in local pressures are avoided (see what is stated concerning results in Table XXXIII), and on the other, they allow the copper rotating band on the projectile to enter the rifling at a low velocity—i. e., under conditions for which resistance to cutting is found to be least. The slow burning in question evidently occurs for cases where the polished surfaces of the grains are not injured, which is a condition very difficult to attain under present-day conditions of keeping powder.

To the intactness of the outer surface of the grain certain anomalous phenomena noted in service firing of powders may correspond. From what has already been stated it is evident that the method employed for the fabrication of these powders does not permit the preparation of such slow-burning varieties as the prismatic.

50. *Mode of combustion of smokeless powders.*

The results of determination of elements of combustion of pyroxyline powder (mark 148) and nitroglycerin powders (Noble's ballistite) and cordite (mark 5) are given below in Table XXXIV. The "extent" of the varieties of powders experimented with was so great that, notwithstanding the absence of similarity of grains, the durations of combustion for the same densities of loading were considered as proportional to the thicknesses (par. 40), if the powders burn in concentric layers, and consequently, the mean speeds of combustion must be constant.

TABLE XXXIV.—Elements of combustion of smokeless powders.

Kind of powder and dimensions of grain.	Densities of load- ing.	Π_a	$\left(\frac{dP_t}{dt}\right)_{\max.}$	P.	Mean rates of combus- tion.
<i>Pyroxylene, lot 148.</i>					
Plates 20 by 20 millimeters for a thickness of grain of—		<i>Thou- sandths of a sec.</i>	<i>Tons per sq. cm. per sec.</i>	<i>Kgr. cm.²</i>	<i>Cm. per sec.</i>
0.53 millimeter	0.1	7.93	206.70	1,021	3.34
1.01 millimeters		17.06	85.20	974	2.96
1.48 millimeters		22.70	56.60	970	3.26
1.98 millimeters		30.78	42.04	869	3.22
0.54 millimeter2	3.48	886.30	2,282	6.65
1.00 millimeters		9.18	364.30	2,094	5.41
1.47 millimeters		17.13	230.40	2,220	4.29
1.99 millimeters		22.95	160.50	2,104	4.34
<i>Noble's ballistite.</i>					
Square prisms 40 millimeters long, and of thickness:					
4.75 millimeters2	11.60	305.50	2,588	17.56
.....		12.37	312.00	2,772	19.40
3.85 millimeters		10.06	359.60	2,485	19.08
2.95 millimeters		7.96	478.00	2,504	18.87
.....		8.10	478.20	2,578	18.02
<i>Cordite, mark V.</i>					
Plates 25 by 21, m. m. and of thickness:					
0.75 millimeter1	2.72	544.40	1,265	13.79
	.2	1.83	2,386.00	2,635	20.49
	.3	1.31	6,092.00	4,832	28.62
2.36 millimeters1	7.42	161.70	1,160	16.03
	.2	5.50	658.70	2,542	21.45
	.3	4.08	1,889.00	4,554	28.92

An examination of the figures in Table XXXIV leads us to the conclusion that, overlooking the considerable variation in thickness of the powder, the mean velocity of combustion remains about constant for each variety of powder for given densities of loading, and therefore all kinds of pyroxyline powders employed burn almost completely in concentric layers, especially the nitroglycerin varieties. The mean speeds of combustion vary from 5 to 6 cm. per second for pyroxyline powders to 30 cm. for cordite.

As the duration of combustion of the smoking powders employed in large guns does not exceed 0.03 second, the least dimensions (in the given case, the thickness) of smokeless powders employed for these guns should be comparatively small, as we see from the examples in Tables IV and V. In order to secure regularity in inflammation and in diminution of the original surface of combustion, the other dimensions of the powder employed should be made as large as possible, and they are, therefore, generally employed in forms in which one dimension is greatly extended; the possibility of employing such a form of grain depends in practice upon the powder substance being very tough, so that it will not break easily.

51. *Law of change in speeds of combustion in relation to pressures, for compact masses, for smoking and smokeless powders.*

As compact masses of both smoking and smokeless powders burn in concentric layers, therefore, on the basis of what is said in paragraph 38, page 111, the results of determining pressures with the explosion bomb allow us to calculate readily the velocities of combustion corresponding to different pressures. Without citing here examples of such calculation, we shall only indicate the general method employed in both cases. Knowing the form of the grain, we may, by a simple geometrical process, calculate the quantity Ψ of powder burned as a function of thickness of the burned grain, and construct the corresponding curve (A). By formula 33 we may calculate the quantity Ψ of powder burned which develops in the bomb the pressure P_t , assuming an initial density of loading Δ and a final pressure P . Denote the curve of Ψ as a function of P_t by (B). Direct measurement of the curve of compression of the disks on burning powder in bombs gives us the quantity P_t as a function of time. Denote this curve by (C). Comparing curves (B) and (C) together, we may construct a new curve (D), expressing the relation between Ψ and time. Comparing subsequently the curves (D) and (A), we obtain the relationship (E) between the burned thickness and the time. From the curves (E) and (C) we may construct a table of pressures and burned thicknesses corresponding to various intervals of time. Dividing these thicknesses by the intervals of time doubled, we obtain the speed of combustion corresponding to the mean pressure for the beginning and end of a given interval of time. If the experiments be conducted for several densities of loading, the mean speeds may be determined for pressures near to one another. From these latter data may be determined the constants in the expression for the speeds of combustion (paragraph 31, p. 101).

The rates of combustion of compact masses of black and chocolate powders, and also of different forms of smokeless powder, as found experimentally, are given in Table XXXV, along with the corresponding pressures, the formulæ expressing the relationship between the velocities and pressures and the velocities calculated thereby. The exponents (ν) of the degrees of pressure in the adduced formulæ of form (31) are given by Vieille, and the constant coefficients are so chosen that the calculated rates of combustion approximate as near as possible to those found by experiment. We may assume the values given in Table XXXV of the rates of combustion for a pressure of one atmosphere as a measure of the rate of combustion of a substance, assuming that the general character of the phenomenon remains the same, and that the rates of combustion between these limits change in accordance with the formula given. From the data given in the table it is evident that rates of combustion and the effect of pressure on changing these rates depend materially on the nature of the substance.

In what relates to the influence of pressure on change of rates of combustion, the substances are divided in the following order: Brown powders ($\nu = \frac{1}{4}$); black-powder substance ($\nu = \frac{1}{3}$); cordite ($\nu = \frac{5}{9}$); ballistite ($\nu = \frac{2}{3}$); and pyroxyline powders ($\nu = \frac{2}{3}$).

These substances assume an entirely different order if arranged in relation to *rapidity of combustion*, for which they occur as follows: Pyroxyline powders ($u_1 = 0.08$); ballistite ($u_1 = 0.27$); cordite ($u_1 = 0.50$); black-powder substance ($u_1 = 0.80$); and chocolate powder ($u_1 = 0.90$).

Pressure affects the rate of combustion of compact masses of smoking powders very little, whence arises the extreme slowness of their combustion, and they are, therefore, from this point of view, unsuited to firing from cannon.

TABLE XXXV.—*Rates of combustion of compact masses of smoking powders and of various varieties of smokeless powders.*

Name of kind of powder.	Observed by experiment.		Formulae of relationship between velocities and pressures.	Velocities calculated by the formulae.
	Pressures.	Corresponding rates of combustion.		
(1) <i>Compact masses.</i>	$\frac{\text{Kgr.}}{\text{Cm}^2}$	Cm. per sec.		Cm. per sec.
Black powders.....	*500.0000 *1,000.0000 *2,000.0000 *2,800.0000 1.0333	6.00 8.00 10.00 11.19	$P^{1/3}$ $u = 10 \text{ I.89966}$	6.30 7.94 10.00 11.19 8.80
Chocolate powders.....	†416.0000 †833.0000 †2,128.0000 †2,672.0000 1.0333	4.16 5.12 5.72 6.44		
(2) <i>Smokeless powders.</i>				
Pyroxyline powders, sample No. 148.....	†366.0000 †511.0000 †860.0000 †1,410.0000 †1,886.0000 †2,414.0000 †3,052.0000 †3,556.0000 1.0333	3.98 4.56 6.55 9.74 11.19 13.64 15.20 16.65	$P^{2/3}$ $u = 10 \text{ I.80862}$	3.78 4.72 6.68 9.29 11.28 13.30 15.55 17.22 .08
Nitroglycerin.				
Cordite.....	†208.0000 †521.0000 †1,036.0000 †1,390.0000 †2,209.0000 †2,652.0000 †4,060.0000 1.0333	9.34 15.94 21.92 26.32 37.08 41.42 48.24	$P^{5/9}$ $u = 10 \text{ I.68861}$	9.47 15.77 23.11 27.21 35.13 39.06 47.72 .50

* From 2 experiments.

† From 4 experiments.

‡ From 5 experiments.

TABLE XXXV.—*Rates of combustion of compact masses of smoking powders and of various varieties of smokeless powders—Continued.*

Name of kind of powder.	Observed by experiment.		Formulae of relationship between velocities and pressures.	Velocities calculated by the formulæ.
	Pressures.	Corresponding rates of combustion.		
<i>Nitroglycerin—Continued.</i>	<i>Kgr.</i> <i>Cm².</i>	<i>Cm. per sec.</i>		<i>Cm. per sec.</i>
Noble's ballistite.....	*311.0000	7.85	10 1.4353 P ^{3/5} u	8.23
	*449.0000	8.40		10.24
	*617.0000	2.30		12.41
	*823.0000	14.23		14.75
	*1,051.0000	17.06		17.08
	*1,299.0000	20.02		19.40
	*1,553.0000	22.24		21.59
	1,798.0000	25.33		22.51
	2,022.0000	26.09		25.29
	1.033327

* From 1 experiment.

On comparing pyroxyline powders and nitroglycerin powders together, we see that although the latter burn considerably more quickly than the former, yet the influence of pressure on change of rate of combustion is less for them; and this latter circumstance is, in Vieille's opinion, the chief cause of the superiority from a ballistic standpoint of nitroglycerin over pyroxyline powders.

52. *Laws of variation of rates of combustion of pyroxyline powders on change of their density and density of loading.*

The Experimental Commission of the Okhta Powder Works performed some very interesting experiments, employing Vieille's method for illustrating the influence of density (δ) (Table XXXVI), of powder and density of loading (Δ) (Table XXXVII), on the value of the speed of combustion of powder in a closed chamber for various pressures. The influence of density of powder was determined from 200 observations upon different specimens for a density of loading of 0.2; the density of the powder was varied from 1.466 to 1.607.

The influence of density of loading was determined for two kinds of powder of density 1.600, for which about 100 observations were taken; the density of loading varied from 0.050 to 0.250, by steps of 0.025.

On the basis of these experiments Colonel Kisnemsky established the following relation:

$$u = A \delta^{12} \Delta^{1/4} P^{2/3},$$

where u is the speed of combustion of powder in $\frac{\text{cm}}{\text{sec.}}$; P , the pressure in atmospheres; A , a constant coefficient, equal in the case of pyroxyline powders to 0.000555.

53. Law of change of rates of combustion and pressure for usual smoking powders—The modulus of progressiveness of powder and its signification.

From what has been stated in paragraphs 42–49 we know that ordinary powder does not burn in concentric layers, and that from the very beginning it disintegrates into nodules (elements) which determine the rate of combustion in a manner that is independent of the dimensions of the original grain. Generally speaking, these nodules are polyhedra of irregular form, and they may be compared to cubes, spheres, and even to dense cylinders whose height equals the diameter of base. The dimensions of these elements are evidently unknown to us, and, what is very important, they (the nodules) are irregular both as to their composition and size. On investigating the composition of even very regular powders—e. g., chocolate prismatic powders prepared from grains of the most homogeneous kind—we find, assuming that the nodules into which the grains disintegrate on combustion are of a spherical shape, that the rates of combustion at first grow regularly and subsequently fall. Such a result is easily reconciled on the assumption that the smaller and less dense particles burn first.

It is not possible in this manner to determine the relationship between rate of combustion and pressure for ordinary smoking powders, as was done for compact masses (paragraph 51), but the desired relationship may be determined approximately by comparing the *moduli of progressiveness* of these powders to the same quantities for compact masses of smoking and smokeless powders, for which the desired relationships are given above (paragraph 51).

TABLE XXXVI.—Rates of combustion for pyroxyline powders of various densities.

Density of powder.	Thickness of ribbon.	Maximum pressure for $\Delta=0.2$.	Obtained by experiment.		Calculated.
			Pressure.	Corresponding rates of combustion.	
	<i>Mm.</i>	<i>Atm.</i>	<i>Atm.</i>	<i>Cm. per sec.</i>	<i>Cm. per sec.</i>
1.466	3.5	1,680	581	2.5	2.4
			986	4.1	3.6
			1,164	4.7	4.1
			551	4.3	4.4
1.529	3.54	1,785	1,075	6.4	6.4
			1,274	8.2	7.1
			561	4.7	4.6
1.545	.77	1,890	1,095	6.9	7.3
			1,596	9.5	9.4
			573	6.3	6.2
1.579	.77	1,910	1,177	8.9	9.9
			1,552	11.0	11.9
			571	7.5	7.5
1.604	.75	2,090	1,060	10.3	11.2
			1,561	15.2	14.6

TABLE XXXVII.—*Rates of combustion of pyroxyline powder for various densities of loading.*

Densities of loading and corresponding pressures.	0.05	0.10	0.15	0.20	0.25
	450	940	1,540	2,080	2,810

OBTAINED BY EXPERIMENT.

Pressures, about—	Corresponding rates of combustion in centime ers, about.				
265	2.4	3.5	4.8
380	3.1	4.0	5.0	4.0
500	4.4	5.3	6.5	5.9
800	6.5	6.8	7.4	7.4
1,200	11.0	10.6	10.7
1,600	13.2	13.0
2,200	17.0

By the Modulus of Progressiveness is denoted the exponent μ of the degree of pressure in the expression $\left(\frac{dP_t}{dt}\right)_{\max} = DP^\mu$ which deter-

mines the influence of pressure on changing the quantity $\left(\frac{dP_t}{dt}\right)_{\max}$.

From equation 34, page 113, we have the following expression for the complete duration of combustion (Π_a) of geometrically similar substances:

$$\Pi_a = q \Pi_M = q \int_0^a \frac{da}{u}$$

or, substituting for u an expression of form 31, page 103,

$$\Pi_a = \frac{q}{A} \int_0^a \frac{da}{P_t^\nu}$$

In consequence of this Π_a , for a certain constant although unknown value a , varies inversely to the quantity P_t , and $\left(\frac{dP_t}{dt}\right)_{\max}$ which is inversely

proportional to Π_a , evidently changes in the same direction as P_t^ν .

In Table XXXVIII are given the moduli of progressiveness for ordinary smoking powders, for compact masses prepared from them and for certain other substances. Experiments to obtain the same were performed in a closed chamber for various densities of loading; the grains experimented with were similar in form, so that the values of $\left(\frac{dP_t}{dt}\right)_{\max}$ obtained, and, consequently, the moduli of progressiveness were

fully comparable with one another.

From the figures presented in this last table we see that the moduli of progressiveness for ordinary black (excluding powder of mark $^{26/34}$)

and chocolate powders lie between the corresponding moduli for smokeless powders and for compact masses of smoking powders, for which we know (Table XXXV) the exponents ν , which determine the influence of pressure on change of velocity.

TABLE XXXVIII.—Moduli of progressiveness of different substances.

Name of substance.	Modulus of progressiveness.	Name of substance.	Modulus of progressiveness.
1. Black powder mark ²⁶ / ₃₄	2.29	8. Trinitronaphthaline, C ₁₀ H ₃ (NO ₂) ³ , about.....	2.50
2. Other black powders, about.....	1.70	9. Pieric acid (trinitrophenol), C ₆ H ₃ O (NO ₂) ³ , pressed.....	2.69
3. Chocolate powders.....	1.50	10. Pieric acid (trinitrophenol), C ₆ H ₃ O (NO ₂) ³ , cast.....	2.82
4. Compact mass of black powder.....	1.33	11. Favier compound †.....	3.25
5. Compact mass of chocolate powder..	1.25		
6. Pyroxyline* powder.....	1.87		
7. Ballistite.....	1.86		

* According to the determination of the Experimental Committee of the Okhta factory, the moduli of progressiveness of some forms of pyroxyline powder equal 2, and the constant D equals 0.0001935.

† A mixture of binitronaphthalene C₁₀H₆(NO₂)² (12 parts) with nitrate of ammonia NH₄NO₃ (88 parts).

As the values $\left(\frac{dP_t}{dt}\right)_{\max.}$ as stated above, change in the same direc-

tion as P_t^ν , the moduli of progressiveness change in the same manner as the exponents ν , and, therefore, by constructing a curve of the known values of ν as functions of the corresponding moduli of progressiveness, we find that values of ν equal, respectively, to $0.52 \frac{\infty^1}{2}$, § and $0.45 \frac{\infty^5}{11}$ should correspond approximately to the moduli 1.70 and 1.50.

The value of ν obtained by this method for black powders corresponds closely with that assumed by Sarrau on the basis of his hypothesis concerning combustion of powders (paragraph 34).

As the dimensions of the elements which determine the duration of combustion of the final grain of smoking powders are unknown to us, we can not determine in relation to them the constant quantities of formula 31, page 103, and therefore we can not ascertain the true values of the velocities of combustion of these powders.

In the case of powder ²⁶/₃₄ and for other substances presented in the table, the moduli of progressiveness obtained were as great as 2.29 and even 3.25. It is difficult to state what values of the exponent ν should correspond to these moduli, for excessive polishing in this case could not lead to any probable values.

It is to be observed that the powder ²⁶/₃₄, whose modulus of progressiveness is abnormally great when compared with other homogeneous powders, proved most irregular in its action when fired from guns, since various abnormalities were developed on employing it in practice.

In relation to the other substances cited we may remark that they may be reduced by proper incorporation to such a degree of slowness that their use for ordnance purposes does not afford any difficulty;

§ Vieille states the value of this exponent as 0.55.

but the moduli of progressiveness of these substances is very great, and, reasoning by analogy, a very rapid change of speed of combustion in relation to pressure should correspond to them, which may lead, as shown by experiments with some of them, to abnormal pressures.

Generally speaking, the influence of the values of the moduli of progressiveness on the ballistic performance of a powder is not wholly clear. On comparing results obtained from brown and black powders and also from pyroxyline powders and cordite, we come to the general conclusion that the lower the modulus of a powder of a given category is, the more suitable it is for practical purposes.

The wave-like movement of gases which is exhibited by the recording pressure gauges (see second part of this course) occurs with special frequency in the case of slow-burning powders, the speed of whose inflammation is bad. In most cases this phenomenon is very marked at the beginning of combustion, but diminishes subsequently, and is hardly to be observed at the time of formation of maximum pressure. It is evident that high moduli of progressiveness increase wave movements and may lead to dangerous abnormalities in pressure.

Although these phenomena may be considerably diminished and even wholly removed by proper inflammation of the charge, yet they will, nevertheless, occur frequently with powders whose moduli of progressiveness are high.

From the above we may conclude that the successful employment of high-explosive substances in ordnance is connected with the lowering of their moduli of progressiveness, which, in the smokeless powders of present-day types, exceed in all cases those which are found for smoking varieties, especially for chocolate powders.

54. Determination of the relative quantity Ψ of burned powder as a function of time on variation of pressure for grains burned in concentric layers.

Let us assume that, during the interval of time t , the speed of combustion will be u . During the subsequent elementary interval of time dt there burns, in a direction normal to the burning surface, an infinitely thin layer of thickness $u dt$, and throughout the whole time t there burns in the same direction a layer of thickness $\int_0^t u dt$. At constant pressure the speed u would be constant and the thickness of burned grain would be ut .

We shall now determine the form of the function sought in the case of the spherical grain. On the basis of what has been stated the radius of the unburned portion of the sphere to the instant t , is

$$r - \int_0^t u dt,$$

and, therefore, proceeding as indicated in paragraph 29, page 94, we obtain

$$\Psi = 1 - \left[1 - \frac{1}{r} \int_0^t u dt \right]^3$$

As the expression employed for variable velocities takes the form 31, page 103, therefore,

$$\begin{aligned}\Psi &= 1 - \left[1 - \frac{A}{r} \int_0^t P_t^r dt \right]^3 = \\ &= 3 \frac{A}{r} \int_0^t P_t^r dt \left[1 - \frac{A}{r} \int_0^t P_t^r dt + \frac{1}{3} \left(\frac{A}{r} \int_0^t P_t^r dt \right)^2 \right]\end{aligned}$$

Comparing this expression with that found in paragraph 29, we note that all the difference between them consists in the substitution of the

ratio $\frac{t}{H_a}$ in the expression $\frac{A}{r} \int_0^t P_t^r dt$. Bearing in mind subsequently

that $2r$ is the least dimension of the grain, we may, denoting it by a , write the following general expression for all forms of grain for the function Ψ under variable pressures:

$$36 \quad \Psi = \kappa \frac{2A}{a} \int_0^t P_t^r dt \left[1 - \lambda \frac{2A}{a} \int_0^t P_t^r dt + \mu \left(\frac{2A}{a} \int_0^t P_t^r dt \right)^2 \right]$$

in which κ , λ , and μ are constant coefficients, depending on the form of the grain; they are presented in Table XXI, page 98; A and r —a constant coefficient and constant exponent for the given powder entering into the expression 31, page 103, for velocities under variable pressure P_t . They depend upon the nature of the substance of the powder, and are given in some cases in Table XXXV, page 148; and a is the least dimension of the mean powder grain. (Paragraph 11, p. 28.)

In paragraph 30, page 97, we saw that under conditions of constant velocities of combustion and pressure a powder burns more *uniformly* the greater the *extent* of surface the grain possesses in two directions. If the pressures, and consequently the speeds of combustion increase in subsequent intervals of time, the quantity of gases evolved will also increase—i. e., the powder will burn *progressively*, and the quantity of evolved gases will increase with increase of time. It therefore follows that the form of smokeless powders used leads to progressive combustion in the bore of the gun, which is very advantageous from a ballistic standpoint.

We may satisfy ourselves as to the progressiveness of combustion of such grains on increases of pressure in the following manner:

The quantity of powder burned is proportional at each moment to the product of the surface of combustion into rate of combustion. Let the initial surface be s_1 and the speed u_1 . At the end of the interval of time t these quantities will be s_t and u_t . As under our assumption, combustion proceeds in concentric layers, the corresponding

quantities of powder burned at the moment in question will be Bs_1u_1 and Bs_tu_t ; the ratio of these quantities may be greater, equal, or less than unity—i. e.,

$$\frac{s_t u_t}{s_1 u_1} \begin{matrix} > \\ = \\ < \end{matrix} 1$$

whence

$$u_t \begin{matrix} > \\ = \\ < \end{matrix} u_1 \frac{s_1}{s_t}$$

Let us apply this to certain forms of grains. Let us take one of the least suitable (paragraph 30)—the spherical, with radius r , and let us denote by $a_t = A \int_0^t P_t^r dt$ the diminution of radius in the interval of time t . We have

$$s_1 = 4\pi r^2 \text{ and } s_t = 4\pi(r - a_t)^2$$

$$\frac{s_1}{s_t} = \left(1 - \frac{a_t}{r}\right)^{-2}$$

The value of this ratio, when the radius diminished nine-tenths of its original magnitude—i. e., when nearly all of the whole grain is burned, will be

$$\frac{s_1}{s_t} = 100$$

i. e., the speed of combustion is $u_t \begin{matrix} > \\ = \\ < \end{matrix} 100 u_1$.

It was shown previously (paragraphs 32 and 51) that the rates of combustion of powders changed with pressure to such a considerable extent only in exceptional cases, and therefore the quantity of powder burned, in the case of spherical grains, in each successive increment of time with increasing pressures is less in most cases than in the preceding, especially when the first and last moments of combustion of the grain are compared, but the difference will not be so great as for a constant rate of combustion. (Paragraphs 29 and 30.) The same may be said in relation to cubical and cylindrical grains whose height equals their diameter.

More favorable conditions occur for other forms of grains, even in the case where the velocity of combustion increases with the pressure in a considerably less degree.

Thus, for example, let us consider powder in the form of strips of the dimensions 2 by 20 by 200 mm. The initial and final surfaces of combustion will be

$$s_1 = 8\,880 \text{ sq. mm. and } s_t = 7\,128 \text{ sq. mm.}$$

In order that, under these conditions, the quantity of powder burned in the last moment should not be less than that burned in the first, the speed of combustion should be

$$u_t \geq 1.25 u_1.$$

For the cylindrical grains with a channel, i. e., for the tubular grain whose dimensions satisfy the necessary conditions, the least dimension, the extent, and the initial surface of combustion are the same as for the previous grain, i. e.,

$$2r \cong 9 \text{ mm.}, 2r^1 \cong 5 \text{ mm. and } h = 200 \text{ mm.}$$

whence we obtain

$$s_1 = 8\,880 \text{ sq. mm. and } s_t = 8704 \text{ sq. mm.}$$

or, the speed of combustion should be

$$u_t \geq 1.02 u_1.$$

In actual practice, however, the difference between initial and final rates of combustion is considerably greater.

In this manner we see that when the combustion proceeds in concentric layers and is accomplished under a gradual increase of pressure for all forms of grains, with the exception of the spherical, cubical, and the cylindrical, whose height equals its diameter, there occurs in greater or less degree a progressive increase in the evolution of gases, and this will be the greater the greater the *extent* of the grain.

Tubular grains, for the same *extent*, satisfy the condition of progressive combustion to a most favorable degree; at the same time these forms of grains provide the most regular and rapid inflammation, which is very important (53), but under the condition that their combustion progresses actually in concentric layers.

55. *Determination of the quantity Ψ as a function of the time on change of pressure for ordinary smoking powders.*

We saw from 53 that it was impossible to deduce an exact expression for smoking powders showing the relationship of the rate of combustion to pressure. All that could be determined was one exponent ν in expression 31, page 103. The combustion of these powders proceeds in an extremely irregular manner. If we assume, however, that the final grains disintegrate on combustion into their ultimate elements (nuclei) such as cubes, we may assume approximately that the combustion of the latter is accomplished in concentric layers, and consequently that the corresponding quantity of powder burned may be determined by formula 36, in which the meaning of the letters occurring as factors remain the same as before and for which ν is

given, in paragraph 51, while the quantities $\kappa = 3$, $\lambda = 1$, and $\mu = \frac{1}{3}$ (Table XXI); the values A and a , and Π_0 , remain unknown, however.

Until very recently the values of Π_0 were determined in accordance with Sarrau's * assumption, either on the basis of the results of firing from cannon (paragraph 78) or by the empirical formula proposed by him

$$\Pi_0 = K \left(\frac{a}{1.875 - \delta} \right)^{\frac{1}{2}}$$

in which K is a coefficient depending on the composition and the circumstances of manufacture of the powder; a is the least dimension of the grain and δ its density.

Vieille† proposes the following method for determining the quantities in question: On the basis of formula 31, page 103.

$$A = \frac{u_0}{P_0^\nu}$$

where u_0 is the speed of combustion corresponding to a certain constant pressure P_0 , equal, e. g., to that of the atmosphere, on the assumption that the combustion of the grain proceeds at this pressure in the same manner as when at great pressures, i. e., that the duration of combustion of the final grain depends in this case on the time of combustion of the nodules into which it (the grain) disintegrates when burned at great pressures.

In consequence of what has been stated, the expression $\frac{2A}{a}$ entering into formula 36, page 154, may be written as

$$\frac{2A}{a} = \frac{2u_0}{P_0^\nu a} = \frac{1}{P_0^\nu \Pi_0}$$

or

$$\Pi_0 = \frac{a}{2A \cdot P_0^\nu} = \frac{qa_1}{2A \cdot P_0^\nu}$$

where $\Pi_0 = \frac{a}{2u_0}$ is the duration of combustion under the pressure P_0 of the elements (nodules) into which the final grain disintegrates; a_1 is the thickness of similar elements assumed for comparison, and therefore an arbitrary quantity; and q , is the relation of similarity, an unknown quantity.

* Sarrau's "Recherches théoriques sur le chargement des bouches à feu," 1882, p. 13.

† See his work referred to in paragraph 35, pp. 377 and beyond.

Bearing in mind the expression for Π_a (paragraph 53, p. 151), for the complete duration of combustion of the grain for variable pressures between 0 and P, we may write

$$\frac{\Pi_a}{\Pi_0} = \frac{2 P_0^\nu}{a_1} \int_0^a \frac{da}{P_t^\nu} = B'$$

where B' is a constant quantity, since the quantities P_0^ν , a_1 and $\int_0^a \frac{da}{P_t^\nu}$ which enters into the second member of the written expression are constants;
whence

$$\Pi_0 = \frac{\Pi_a}{B'}$$

As complete duration of combustion Π_a for these grains in a closed chamber varies regularly in inverse ratio to $\left(\frac{dP_t}{dt}\right)_{\max.}$, therefore,

$$\Pi_0 = \frac{1}{B \left(\frac{dP_t}{dt}\right)_{\max.}}$$

and consequently,

$$37 \quad \frac{2 A}{a} = \frac{B \left(\frac{dP_t}{dt}\right)_{\max.}}{P_0^\nu}$$

where B , is a constant quantity for an arbitrary final pressure P developed on burning smoking powder in a closed chamber.

In this manner the determination of the quantity $\frac{2 A}{a}$ entering into formula 36 and also of the quantity Π_0 , reduces to the determination of $\left(\frac{dP_t}{dt}\right)_{\max.}$ on burning powder in a closed chamber. To this end Vieille conducted special experiments with a final pressure of 2,500 $\frac{\text{kg.}}{\text{cm.}^2}$, determined by the static table of estimated compressions of copper disks.* In Table XXXIX are given the results of the values of Π_0 determined by Vieille, which are compared with the same as found by Sarrau as a result of firings from cannon, and the values $\frac{2 A}{a}$ (37).

It is evident from the figures obtained by Vieille that the constant coefficient B corresponding to a final pressure P (2,500 kilograms) equals 1.396.

The data in Table XXXIX show that although the figures found for Π_0 and $\frac{2 A}{a}$ from experiments in a closed chamber do not represent absolute values of the quantities sought, nevertheless, Vieille's method

* See Part II of this course.

enables us to classify very different powders in a simple manner in accordance with their degree of brusqueness; and the order obtained in so doing is the same as that determined by Sarrau's method, based upon results of firing from guns; powder $\frac{2}{3}\frac{6}{4}$, which differs generally in its properties (53) from other powders, constitutes an exception to this rule.

TABLE XXXIX.—Values of Π_0 and $\frac{2}{a} \frac{A}{a}$ for smokeless powders.

Name of powder.	Guns for which the powder is designed.	Value of $\left(\frac{dP_i}{dt}\right)_{\max.}$ for a final pressure of 2,500 $\frac{\text{kgr.}}{\text{cm.}^2}$	Values found for Π_0 .		Values of $\frac{2}{a} \frac{A}{a}$, found from the values of $\left(\frac{dP_i}{dt}\right)_{\max.}$
			Sarrau.	Vieille, from the values of $\left(\frac{dP_i}{dt}\right)_{\max.}$	
		<i>Tons per cm.² per sec.</i>			
C ₁	90 millimeters	* 1, 100	* 0.360	* 0.039	2.733
Wetteren, $\frac{13}{16}$	* 725	* .763	* .546	1.801
SP ₂	} 24 centimeters	* 638	* .763	† .621	1.585
26		{ * 776	* 1.000	* .510	1.928
34					
PB ₃ } Chocolate	[14 centimeters-16 centimeters	* 427	* .880	* .927	1.063
PB ₁ }	[27 centimeters-42 centimeters	* 169	* 2.070	† 2.343	.421

* See Table LVI of the work of Vieille above referred to.

† Vieille gave for these powders the corresponding values 0.639 and 2.323.

Chapter VI.

THE THEORETICAL DETERMINATION OF THE PRODUCTS OF DECOMPOSITION OF A POWDER; THEIR COMPOSITION; VOLUMES OF GASES AND OF SOLID PRODUCTS; HEAT EVOLVED ON COMBUSTION; TEMPERATURE OF DECOMPOSITION AND FORCE OF A POWDER.

56. *Volume of gaseous molecules; exothermic and endothermic reactions; the fundamental principles of thermochemistry.*

We know from chemistry that the vapors of the gram-molecules of every body at a temperature 0° and a pressure of $1.0333 \frac{\text{kgr.}}{\text{cm}^2}$, occupy the same volume, equal, as many assume, to 22.32 liters. In consequence of this, we may include in the volume of the gaseous products of decomposition of powders and explosive substances such bodies as H_2O , K_2S , etc., which are converted into vapor only at very high temperatures. It follows, therefore, that if we know the chemical equation of decomposition of a substance, and desire to determine the volume v_0 of gases, including water vapor, formed from one-gram of it, we must consider together the number of gram-molecules of all bodies formed, both those that are gaseous and those capable of being converted into vapors at high temperatures; then multiply this number by 22.320 and divide the product by the weight of the gram-molecule of the decomposed substances or mixture of substances.

For the various chemical and physical transformations of bodies, i. e., for each reaction, a definite amount of heat is either given off or absorbed.

Reactions and transformations in which heat is evolved are styled *exothermic*; such, for example, are most chemical formations of bodies from their constituent elements, and, also, solution, condensation of gases and of vapors, change from the liquid to the solid state, etc.

Those reactions and transformations in which absorption of heat is observed are styled *endothermic*; such, for example, are most chemical decomposition of bodies into their component elements, and also the vaporization of liquids, the melting of solid bodies, etc. It remains to be added, however, that some compounds which are endothermic at ordinary temperature become exothermic at higher or lower temperatures. This is because the elements or their compounds undergo isomeric changes at these temperatures. Such is acetone; and C_2H_2 is

an endothermic compound when considered as formed from the diamond. If we consider it as formed at $4,000^{\circ}$, then each uniting atom of hydrogen or of carbon evolves from 34 to 42 calories of heat. The cause of this is the transformation of carbon into vapor at $4,000^{\circ}$, by which its energy is increased so that in its union with hydrogen heat is evolved, i. e., the compound becomes exothermic.*

The investigation of these phenomena is the object of thermochemistry. The following fundamental principles of thermochemistry and their results are of direct interest to us in the present relation:†

(1) The *principle of molecular work* may be stated as follows: The quantity of heat evolved or absorbed in any chemical reaction is measured by the sum of all chemical and physical works that occur at the time of this reaction. This quantity of heat must evidently correspond exactly to the work which would have to be done to restore the body (or system of bodies) to its original condition.

It follows from this that when exothermic compounds are decomposed into their component elements an absorption of heat should occur, while, on the contrary, when endothermic compounds are decomposed into their elements an evolution of heat should result.

As not only a union of elements but also a change in physical state of a body frequently occurs in chemical reaction, it is necessary, in order to measure the chemical force (chemical affinity) correctly, to allow for that part of the heat observed at the time of the reaction which corresponds to the physical change in condition of the bodies. We must also bear in mind the special condition of the elements and their compounds, the former of which may exist in various allotropic states, and the latter in various isomeric states; whence comparisons must be made upon similar states, both of the reacting as well as of the formed bodies. In view of this, in order to compare chemical phenomena in as clear a manner as is possible, it is best to refer all reacting bodies to the solid state and to absolute 0, or at least to ordinary temperatures. If all reactions are referred to the gaseous state they should be referred to constant volume, under the condition that all uniting and formed bodies possess the same physical properties and do not depend upon external work. We may not refer bodies to the liquid state or consider reaction in solution because of the great changes in the specific heats of liquids on account of their loss of heat by evaporation, on account of the unknown action of the solvent (water) on the body, etc. In view of this, the thermochemical tables indicate the quantity of heat evolved on chemical reactions as either referred to gases reacting at constant volume and forming gaseous bodies, or else to solid bodies, taken, in all cases, at a temperature of about 15° C.

* See Berthelot, "Thermochimie," ed. 1897, vol. 1, p. 14.

† See Berthelot, "Sur la force," etc., 3d ed., vol. 1, p. 174 et seq.

(2) *The principle of initial and final state* is included in the following: If a system of simple or compound bodies taken under definite conditions changes into another state, having undergone certain physical and chemical changes in which the system performs no external work, then the amount of heat given off or absorbed in such reactions depends exclusively on the initial and final state of the system, and does not depend in any way upon the nature and order of the changes of the various successive products, so that the whole of the heat given off at the time of the series of successive changes (physical or chemical) is equal to the algebraic sum of the heats evolved by each partial transformation. Therefore, knowing the heat of formation (positive or negative) from the elements of an initial system of bodies which is decomposed by a certain reaction into definite permanent products, the heats of formation of which from the elements are also known, it is easy to determine the quantity of heat evolved by the reaction under consideration, i. e., on the change of the system considered from its initial into its final state. Let the heats of formation from elements of the bodies of the final state be Q_1, Q_2, \dots, Q_n , the heats of formation from the elements of the bodies of the initial state be q_1, q_2, \dots, q_n ; and the heat evolved on the change from the initial to the final state be Q . Assuming, temporarily, as the initial state a system formed from simple elements, and as the final state the given ultimate constitutions, i. e., the system of bodies evolving at the time of their formation from the elements the heat $Q_1 + Q_2 + \dots + Q_n$; and observing that on account of the second principle of thermochemistry this quantity of heat must equal the sum of the heats evolved upon successive changes, i. e., must equal the sum of the heats evolved on changing from the elements into a system of bodies corresponding to the present initial state, plus also the heat evolved on changing it to a given final state, we have,

$$Q_1 + Q_2 + \dots + Q_n = q_1 + q_2 + \dots + q_n + Q,$$

whence we obtain

$$Q = \Sigma Q_i - \Sigma q_i$$

Dividing Q by the weight of the initial system of bodies, we obtain Q_v or Q_v' (paragraph 18)—the quantity of heat evolved by one gram of the given substance or mixture of given substances on decomposition, in accordance with the given law.

(3) *The principle of maximum work* may be stated as follows: Every chemical transformation which proceeds without assistance of external work endeavors to create that body or system of bodies for which is evolved the maximum quantity of heat, as to this corresponds most stable system of compounds. Although this principle itself may not be applied to all chemical reactions, yet it possesses significance in relation to explosive compounds.

In Table XL are presented, for various considerations, the quantities of heat (columns 4 and 5) evolved on formation from the simple elements of the various bodies which enter into the composition of powders and their products of decomposition. The quantities of heat of formation are given for one gram-molecule of the bodies (column 4) in large calories* and for one gram of the substance (column 5) in small calories.* In the same table are given the chemical formulæ of the bodies (column 2); the weights of their gram-molecules* (column 3);‡, the volume of vapor (for $t=0^\circ$ and $P_0=1 \text{ atm}$) of one gram of the substance (column 6),† if it is transformed into vapor at any elevated temperature; the weight of one liter of the vapor (column 7)† of the same substances (for $t=0^\circ$ and $P=1 \text{ atm.}$); the densities of the bodies ‡ that are solid or liquid at ordinary temperatures and pressures (column 8); their specific volumes (column 9); the melting points (column 10) and the boiling points (column 11) of the various bodies at ordinary pressure.

*The figures are taken from Berthelot's work "Sur la force," etc., and also from the *Annuaire* for 1895, published by Le Bureau des Longitudes.

† In calculating the numbers given in these columns there have been assumed as fundamental data, the weights of the gram-molecules of the elements given in column 3; also, that the volume occupied by one gram of hydrogen (for $t=0^\circ$ and $P_0=1 \text{ atm}$) is 11.16 liters.

‡ Unless otherwise specially stated, these figures are taken from the works above referred to, and also from Londolt and Börnstein's "Physikalische-Chemische Tabellen," 2d ed., 1894; also from Sarrau's work "Théorie des Explosifs" (Vol. VII "Mémorial des Poudres et Salpêtres").

Potash	K ₂ O	94.2	98.2	959.27	2.656	.377
Carbonate of ammonia	(NH ₄) ² CO ₃	96.0	1.600	.625
Nitrate of ammonia	(NH ₄) NO ₃	80.0	88.6	1, 107.50	4.840	.207
Ferrous sulphide	FeS	88.0	23.8	270.45	1.170	.855
Ethyl alcohol	C ₂ H ₅ O	46.0	<i>f</i> 59.8	1, 300.00	485.22	2.06100	.789	1.97	78.0
Ethyl ether	C ₄ H ₁₀ O	74.0	<i>f</i> 65.3	882.43	301.62	3.31540	.786	1.36	34.9
Nitroglycerin	C ₃ H ₅ N ₃ O ₉	227.0	<i>f</i> 94.5	416.30	1.600	.625
Nitrocellulose ^k	C ₂₄ H ₄₀ - <i>n</i> O ₃₀ - <i>n</i> (NO ₃) _{<i>n</i>}	648+45 <i>n</i>	1921.6-26.5 <i>n</i>	1.806	.554	112-113.0
Nitromannite	C ₆ H ₈ N ₄ O ₁₈	482.0	179.1	414.60
Bi-nitronaphthalin	C ₁₀ H ₆ N ₂ O ₄	218.0	-5.7	-26.10
Tri-nitronaphthalin	C ₁₀ H ₄ N ₃ O ₆	263.0	3.3	12.50
Picric acid	C ₆ H ₃ N ₃ O ₇	229.0	46.8	204.40

^a By Devar's investigation for a pressure of 55 mm.

^b By Oshearski's investigation.

^c By Viol's investigation.

^d Diamond.

^e Graphite.

^f At temperature above 800° C.

^g Amorphous.

^h About —

ⁱ At red heat.

^j Gaseous.

^k Here *n* indicates the number of atoms of nitrogen entering into the composition of the molecule of Vieille's nitrocellulose. According to the latest investigations of A. Nastukov, the molecular weight of cellulose is expressed by the formula 40 (C₆H₁₀O₅) and consequently that of the nitric ethers of cellulose must be ten times greater than by Vieille's formula. (See what is stated on page 11.) See paper of A. Nastukov "On certain oxy-celluloses and the molecular weight of cellulose in Journal Russian Phys.-Chem. Soc., Vol. XXXII.

^l The quantity of heat evolved on the formation of nitro-cellulose is given on the basis of E. Tcheltzov's determination.

57. Possibility of forming equations of decomposition of explosive substances and powders in various special cases—The signification of certain changes in composition of the products of decomposition of powder.

From what was stated in Chapter III it is evident that the properties of powders and their action in a closed chamber depend essentially on their products of decomposition, upon the heat evolved at the time of explosion, upon the temperature of decomposition, and upon the force of the powder. We also saw that the determination of these quantities is based upon the results of experiment.

On the other hand, if we knew exactly the initial composition of a powder and if we could foretell the composition of its products of decomposition, then, as is evident from paragraph 56, we would be able to calculate the elements ($v'_0, Q_v', t^\circ, f', \alpha$) which characterize the powder in relation to its action in a closed chamber without having to resort to costly experiments. In fact, we could calculate from the equation of decomposition of powder the volume and covolume of the gases and the quantity of heat evolved, and then, by employing the data in Table XL and the formulæ 25 given on page 81 we could calculate the temperature and force of the powder.

It is not difficult to see the great use and importance, both in a scientific and a practical way, of the theoretical solution of these questions, and therefore many scientists have given attention to them; but unfortunately, up to the present time, the laws which control the progress of chemical reactions of decomposition of powder and all explosive materials in general at different pressures are far from being explained. These pressures, as shown in paragraphs 15 and 17, exercise a direct influence on the qualitative and also on quantitative composition of products of decomposition of the various kinds of powders experimented with.

With the present state of the science, the composition of the decomposition products of substances exploded in a closed chamber may only be foretold for those cases where there exists more oxygen than is necessary to completely consume C and H into CO_2 and H_2O , as such fully meet the requirements of the third principle of thermochemistry. Such materials are—e. g., nitroglycerin, nitromannite, $\text{C}_6\text{H}_8\text{N}_6\text{O}_{18}$, some mixtures of pyroxylylene with nitrates (e. g., $\text{C}_{24}\text{H}_{29}\text{N}_{11}\text{O}_{42} + 20.5 (\text{NH}_4)\text{NO}_3$), smoking powder of the composition $10 \text{KNO}_3 + 1.5\text{S}_2 + 8\text{C}$, etc.; and also those (endothermic compounds) which resolve on decomposition into their original constituent elements, such as nitric oxide, NO, etc. All these bodies decompose in accordance with the equations given below to which the values of V'_0 in c.m.^3 ; Q_v' , in small calories; α in cubic centimeters; t , $^\circ\text{C}$, and f in $\frac{\text{kgr.}}{\text{c.m.}^2}$

for one gram of the substance correspond, for 1 gram of the body.

Nitroglycerin: $2\text{C}_3\text{H}_5\text{N}_3\text{O}_9 = 6\text{CO}_2 + 5\text{H}_2\text{O} + 3\text{N}_2 + 0.50\text{O}_2$; $V'_0 = 712.9$; $Q_v' = 1\ 679$; $t^\circ = 3\ 352^\circ$; $f' = 9.781$; $\alpha = 0.713$.

Nitromannite: $C_6H_8N_6O_{18}=6CO_2+4H_2O+3N_2+O_2$; $V_0'=723.3$; $Q_v'=1434$; $t_1^\circ=3051$; $f=9100$; $\alpha=0.723$.

A mixture of pyroxylyene with ammonium nitrate: $C_{24}H_{29}N_{11}O_{42}+20.5(NH_4)NO_3=24CO_2+55.5H_2O+26N_2$; $V_0'=846.1$; $Q_v'=1095$; $t_1^\circ=2342^\circ$; $f_1=8375$; $\alpha=0.846$.

A mixture of saltpeter, sulphur, and charcoal: $10KNO_3+1.5S_2+8C=6CO_2+5N_2+2K_2CO_3+3K_2SO_4$;

$V_0'=V_0=204.1$; $Q_v'=Q_v=517$; $t_1^\circ=1,936^\circ$; $f=1,706$; $\alpha=0.533$.

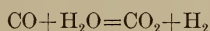
Nitric oxide: $2NO=N_2+O_2$;

$V_0'=V_0=744$; $Q_v'=Q_v=720$; $t_1^\circ=3,309^\circ$; $f=10,086$; $\alpha=0.744$.

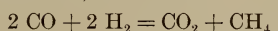
All the powders now in use lack sufficient oxygen, and, therefore, can not fulfill completely the third principle of thermochemistry, as a result of which the composition of their products of decomposition changes with the pressure (i. e., with the density of loading), the reaction occurring in such a manner that, on increasing the final pressure, the ultimate constitution of the products of decomposition becomes more and more stable.

It was stated in paragraphs 15 and 17 that, among others, the amounts of the substances CO_2 , CH_4 , CO , H_2 , and H_2O evolved on the decomposition of smokeless powders changed in relation to the pressure in such a way that on increasing the pressure the quantity of the first two increased, but that of the latter three diminished; sometimes, however, the quantity of hydrogen increases—e. g., on decomposing Vieille's endekanitrocellulose. Such change of composition of products of decomposition occurs chiefly in the way of diminishing the CO and increasing the CO_2 , to which result* two chemical reactions contribute.

The first of these is the transformation of the carbonic oxide into carbonic acid at the expense of water, whereby free H_2 is evolved.†



The second reaction, it is assumed, begins either when the first has sufficiently reduced the percentage of water vapor in the products of decomposition and enriched the mixture with free hydrogen, H_2 , or else when the explosive substance itself contains so little oxygen that it is impossible to form a sufficient amount of water vapor. As a result of this reaction CH_4 is formed, as follows:



*See Sarrau et Vieille's "Note relative à l'influence du rapprochement moléculaire sur l'équilibre chimique des systèmes gazeux homogènes," 1887, inserted in *Mémorial des Poudres et Salpêtres*, Vol. II, p. 337.

†See page 37 on the causes of formation of H_2 and CH_4 as products of decomposition of smoking powders.

Both of these reactions are exothermic—i. e., they proceed with the evolution of heat (the first with 10.1 calories and the second with 32.8 calories per gram-molecule of the reacting carbonic oxide), and, therefore, they fulfill the conditions of the formation of a system of bodies through the greatest possible evolution of heat (see the third principle of thermo-chemistry, stated above). We may further remark that the first of these reactions does not change the volume of gases, reckoning water as existing in the form of steam; the second reaction produces bodies whose general volume is 50 per cent less than the volume of the bodies entering into the reaction.

The reactions just considered may explain the transformation of known products of decomposition for one density of loading (for one pressure) into those obtained at another; but they in no way explain the course of the reaction itself and the composition of the products at the moment of decomposition at different pressures.

It might be possible, knowing the products of decomposition (e. g., of smoking powders, Table IX) at various densities of loading, to unite to them an expression for corresponding pressures and establish empirical formulæ expressing the desired relation, but we are forced to abandon this on account of the complexity of the matter.

Although the exact solution of the question interesting us is not possible, nevertheless it frequently becomes useful to possess approximate conceptions relating to possible decomposition on explosion of a given substance whose content of oxygen is not sufficient to convert its whole content of C and H_2 into CO_2 and H_2O . In this case if the oxygen in the powder is more than is required for changing all the C into CO, we may assume, in accordance with the remark of Mallard and Le Chatelier* that on explosion there are only formed oxygen compounds and free H_2 and N_2 ; that all the carbon is converted into CO, and that the excess of O_2 is distributed in a certain manner between the CO and H_2 forming CO_2 and H_2O . Such compounds as CH_4 may be neglected on account of their relative insignificance in amount.

58. *Decomposition of smokeless powders.*

For smokeless powders in actual use, whose composition may be expressed by the empirical formula $C_p H_{2q} N_{2r} O_s$, the assumptions made above in paragraph 57 are sufficient for the establishment of certain useful conclusions.

On the basis of what was said, the general equation of decomposition of these powders will be



whence

$$p = \eta_1 + \alpha_1$$

$$q = \varepsilon_1 + \gamma_1$$

$$r = \beta_1$$

$$s = 2\eta_1 + \alpha_1 + \varepsilon_1$$

* See *Mémorial des Poudres et Salpêtres*, Vol. II, p. 445.

or

$$\begin{aligned}\eta_1 &= s - p - q + \gamma_1 \\ \alpha_1 &= 2p + q - s - \gamma_1 \\ \varepsilon_1 &= q - \gamma_1\end{aligned}$$

from which, comparing these designations with those previously assumed in paragraph 61, we find that

$$\alpha_1 = \frac{\alpha}{28}, \quad \beta_1 = \frac{\beta}{28}, \quad \gamma_1 = \frac{\gamma}{2}, \quad \varepsilon_1 = \frac{\varepsilon}{18}, \quad \eta_1 = \frac{\eta}{44}$$

From the above equations it follows that:

(1) Under the conditions considered, a given substance may decompose very differently.

(2) The volume of gases V'_0 (in liters) at the temperature $t=0^\circ$ and for $P_0=1.0333 \frac{\text{mgr.}}{\text{cm}^2}$, reckoning water as gaseous, will be

$$V'_0 = 22,32 (\eta_1 + \alpha_1 + \beta_1 + \gamma_1 + \varepsilon_1) = 22,32 (p + q + r)$$

i. e., V'_0 does not depend upon the method of decomposition of the given substance.

(3) Denoting the quantity of heats evolved on formation of $\text{C}_p\text{H}_{2q}\text{N}_{2r}\text{O}_s$, CO_2 , CO , and H_2O (vapor) from the elements by Q , Q_η , Q_α , Q_ε respectively, we find that the quantity of heat evolved by the explosion of the substance $\text{C}_p\text{H}_{2q}\text{N}_{2r}\text{O}_s$, will be

$$Q'_v = \eta_1 Q_\eta + \alpha_1 Q_\alpha + \varepsilon_1 Q_\varepsilon - Q = s (Q_\eta - Q_\alpha) + p (2 Q_\alpha - Q_\eta) + q (Q_\alpha + Q_\varepsilon - Q_\eta) + \gamma_1 (Q_\eta - Q_\alpha - Q_\varepsilon) - Q$$

Bearing in mind subsequently the partial values of Q_η , Q_α and Q_ε (see Table XL) we find that for water in the form of steam

$$Q'_v = 68.3 s - 42.3 p - 10.1 (q - \gamma_1) - Q$$

and for liquid water

$$Q_v = 68.3 s - 42.3 p + 0.7 (q - \gamma_1) - Q$$

This quantity of heat relates to the gram-molecule of $\text{C}_p\text{H}_{2q}\text{N}_{2r}\text{O}_s$.

The expressions found for Q'_v and Q_v show that the quantity of heat evolved on decomposing a substance of the composition above indicated changes with the content of H_2 in the products of combustion.

(4) The combustible gases of the products of decomposition, under the assumptions made, will only be CO and H_2 .* Their general sum,

$$\alpha_1 + \gamma_1 = 2p + q - s$$

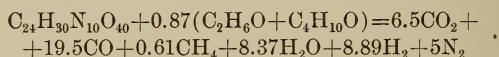
remains constant for a given powder. The more of these gases will evidently be given off the richer the given substance in carbon (p) and in hydrogen (q) and the poorer it is in oxygen (s). It therefore follows that the admixture of such solvents as alcohol and ether that exist in a powder facilitate the evolution of a greater quantity of those gases which inflame on admixture with the atmosphere, if the temperature favors such a result.

* In actual practice marsh gas is to be added to these (see par. 17, p. 47).

For a given content of solvent, powders prepared from some nitro-celluloses will evolve the more combustible products the lower they are in nitrogen, for increase of the latter leads in itself to a diminution in the content (q) of hydrogen and to an increase in the content (s) of oxygen. (See Table IV.)

These deductions, relating to explosive materials in general, were established by Mallard's Subcommission* (for investigating questions relating to the employment of explosive substances in the presence of fire damp), on studying the character of explosion of various substances in the open air. The experiments showed, among other things, that the products of decomposition of nitrocelluloses for a content of nitrogen higher than 12.8 per cent do not inflame in the air; whereas, the products of octo and of enneanitrocelluloses, which contain a great quantity of combustible gases, burn in the air after explosion. As applied to the gun, this property has a deep significance, for it shows that the gases from smokeless powders, under certain conditions, inflame when the breech is opened. Their greater or less inflammability depends, however, not only on the quantity of burning gases, but also on the degree of expansion of the products of decomposition of the powder in the gun, as this directly influences the temperature established in the bore of the gun. This will be the higher, for a given bore pressure, the less the useful work that is performed by the unit weight of the powder—i. e., the greater the weight of the charge, and this latter increases in a given gun, other conditions being equal, with increase in thickness of the grain.

We shall now consider pyrocollodion powder, the actual composition of the products of decomposition of which (Table XL) may, neglecting ash, be expressed, in accordance with the assumption of E. Tchelsov, by the following nearly correct equation:

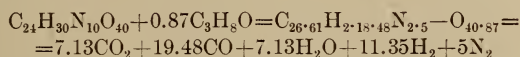


We shall henceforward denote this equation as the complete equation of combustion.

The approximate equations of decomposition of powder do not include marsh gas and depend, as already stated, on the quantity γ of hydrogen included in the products of decomposition. The latter is to a certain degree arbitrary and is limited only by the following conditions: In the first place, the quantities η_1 , α_1 , and ϵ_1 of the substances CO_2 , CO , and H_2O , should not be negative; and in the second place the greatest value of γ can not exceed q , for which should be fulfilled the conditions: (1) $\gamma_1 \geq p + q - s$; (2) $\gamma_1 \leq 2p + q - s$, or $\gamma_1 \leq q$, having regard for which of the latter limits will be the least. In the given case γ_1

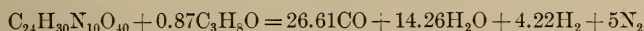
* See extract from "Mémorial des Poudres et Salpêtres," Vol. II, 1884-89, pp. 397-403.

may be included between the limits 4.22 and 18.48. Assuming for γ_1 , as Mallard and Le Chatelier propose, a mean value between these limits—i.e., $\gamma_1 = 11.35$, we find $\eta_1 = 7.13$, $\alpha_1 = 19.48$, and $\varepsilon_1 = 7.13$; besides this, $\beta_1 = \gamma$; therefore the approximate expression, so to speak, for the “mean” decomposition of pyrocollodion powder will be



Under these assumptions, the most probable case of decomposition of this powder will be:

(1) When the whole excess of oxygen goes to the formation of water, for which $\gamma_1 = 4.22$.



and (2) when the whole excess of oxygen goes to the formation of carbonic acid, for which $\gamma = 18.48$.



Proceeding with these equations (complete and approximate) of decomposition of powder, as indicated in paragraph 57, we may calculate the volume of gases, the quantities of heat evolved, reckoning water as gaseous, the temperature of decomposition, the covolume, and the force of the powder. The values obtained are given in Table XLI along with certain experimental numbers taken from Table XIX.

TABLE XLI.—*Volumes of gases and quantities of heat, temperatures, and forces calculated by the equations of decomposition of pyrocollodion powder.*

Corresponding to 1 gram of pyrocollodion powder.	By equations of decomposition.				From Table XIX.
	Complete.	Approximate.			
		Without water.	Mean.	Without carbonic acid.	
Volume of gases, V'_0 , in cm^3	948.300000	972.000000	972.000000	972.000000	945.300000
Quantity of heat, Q'_0 in sm. cal	836.400000	877.500000	823.600000	752.200000	883.900000
Coeff. of spec. heat $\left\{ \begin{array}{l} a \\ b \end{array} \right.$251736 .000097	.261252 .000099	.256617 .000096	.251893 .000094	*, 220116 *, 000091
Temperature of decomposition in degrees C	2,107°	2,337°	2,074°	1,979°	2,177°
Force of powder, f in $\frac{\text{kgr.}}{\text{cm}^2}$	8,543	9,602	8,635	8,285	8,800
Covolume, α in cm^3948300	.972000	.972000	.972000	.945300

* From Table XV, p. 66, they equal $a + a_1$, and $b + b_1$, respectively.

The figures in the second and fourth columns of Table XLI show that the quantities characterizing the properties of a powder (which have been obtained in various ways) are quite close to one another and that their mean values as given in the third and fifth columns are close to experimental values as shown in the last column. In any case, the data collected in this table show that, as far as concerns the chief properties, viz, covolume, energy, and temperature of decomposition, and, consequently, the force of a powder of a given composition

$C_p H_{2q} N_{2r} O_s$, we may arrive at sufficiently accurate conclusions on the basis of the theoretical conceptions given above without resorting to the experiments cited in Chapter III.

It is useful to consider the following simple formulæ for approximately determining the covolume α and, consequently, the volume V'_0 , the temperature of decomposition t_1^0 and the force f of a pyroxyline powder when its content of nitrogen is given as a percentage, N per cent—i. e.,

$$\alpha = 5.7 (N)^{-0.7}$$

$$t_1^0 = 31.7 (N)^{5/3}$$

$$f = 1,180 (N)^{4/5}$$

These formulæ were established by the Experimental Committee of the Okhta powder factory on the basis of the approximate formulæ of Mallard and Le Chatelier from the reactions of decomposition in the case of various nitrocelluloses, and from the results of determination of the quantities α , t_1^0 and f for samples of pyroxyline powders.

For the sample of pyrocollodion powder, above cited, which possesses 12.18 per cent of nitrogen,* we obtain $\alpha = 0.991$; $V'_0 = 991$;

$t_1^0 = 2,044^\circ \text{C.}$; $f = 8,718 \frac{\text{kgr.}}{\text{cm.}^2}$, which, in the absence of experimental

data, affords indications relating to the properties of this powder sufficiently close to what these actually are.

59. Decomposition of smoking powders.

The theoretical solution of the question of the decomposition of the ordinary forms of smoking powders at different pressures is more difficult than the solution of the same question for smokeless powders. On account of the greater variety of the products of decomposition of the former we shall confine ourselves to the consideration of a salt-peter-sulphur-charcoal powder, completely dried, prepared from black charcoal, assuming that it forms upon decomposition only the products CO_2 , CO , N_2 , K_2CO_3 , K_2SO_4 and K_2S .

The general equation of decomposition of this powder when its decomposition corresponds to the formation of products of complete combustion is given above on page 166. The general equation of decomposition of powders of this class of any desired composition is, under the above assumptions,

$k \text{KNO}_3 + y \text{C} + c \text{S}_2 = \eta_1 \text{CO}_2 + \alpha_1 \text{CO} + \beta_1 \text{N}_2 + \gamma_1 \text{K}_2\text{CO}_3 + \delta_1 \text{K}_2\text{SO}_4 + \varepsilon_1 \text{K}_2\text{S}$,
where

$$\eta_1 + \alpha_1 + \gamma_1 = y$$

$$2\eta_1 + \alpha_1 + 3\gamma_1 + 4\delta_1 = 3k$$

$$\beta_1 = \frac{k}{2}$$

$$\gamma_1 + \delta_1 + \varepsilon_1 = \frac{k}{2}$$

$$\delta_1 + \varepsilon_1 = 2c.$$

* See Table XI, p. 47

These equations give

$$\begin{aligned}\eta_1 &= 2k - y - 4c + 4\varepsilon_1 \\ \alpha_1 &= 2y - 2\frac{1}{2}k + 6c - 4\varepsilon_1 \\ \beta_1 &= \frac{k}{2} \\ \gamma_1 &= \frac{k}{2} - 2c \\ \delta_1 &= 2c - \varepsilon_1\end{aligned}$$

where ε_1 —i. e., the content of sulphide of potassium remains an indeterminate quantity.

In fact, smokeless powders may decompose in an infinite number of ways, even in the case when their particles are especially well incorporated together. Nevertheless, in this case, as also for smokeless powders, we may make the following deductions:

(1) The volume of gases does not depend upon the equation of decomposition of the powder, since, if we consider K_2S as a non-volatile body at high temperatures,* we have

$$V_0' = 22.32 (\eta_1 + \alpha_1 + \beta_1) = 22.32 (y + 2c)$$

i. e., V_0' depends only on the original composition of the powder.

(2) The quantity of heat evolved Q_v' will be

$$\begin{aligned}Q_v' &= \eta_1 Q_{\eta_1} + \alpha_1 Q_{\alpha_1} + \gamma_1 Q_{\gamma_1} + \delta_1 Q_{\delta_1} + \varepsilon_1 Q_{\varepsilon_1} - k Q_k = \\ &= k (2Q_{\eta_1} - 2\frac{1}{2}Q_{\alpha_1} + \frac{1}{2}Q_{\gamma_1} - Q_k) + 2c (3Q_{\alpha_1} - 2Q_{\eta_1} - Q_{\gamma_1} + Q_{\delta_1}) + \\ &\quad + y (2Q_{\alpha_1} - Q_{\eta_1}) + \varepsilon_1 (4Q_{\eta_1} - 4Q_{\alpha_1} - Q_{\delta_1} + Q_{\varepsilon_1})\end{aligned}$$

where we denote by Q_{η_1} , Q_{α_1} , Q_{γ_1} , Q_{δ_1} , Q_{ε_1} and Q_k , the respective heats of formation of CO_2 , CO , K_2CO_3 , K_2SO_4 , K_2S , and KNO_3 from the elements. Substituting here their values from Table XLI we obtain

$$Q_v' = 144 k - 42.3 y - 90.4 c + 32.4 \varepsilon_1$$

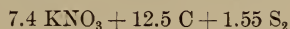
i. e., the quantity of heat evolved on the decomposition of smoking sulphur powder changes with the content of sulphide of potassium in the products, which, as we have seen, remains arbitrary.

(3) The quantity of carbonate of potash in the products of decomposition should be constant, since

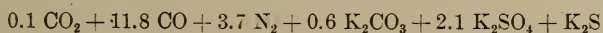
$$\gamma = \frac{k}{2} - 2c$$

which is contrary to observed facts.

We shall apply this to the powders P, R. L. G., and F. G., whose compositions and products of decomposition are given in Tables VI, VIII, IX, and X. Dry powders of these types may be expressed by the formula



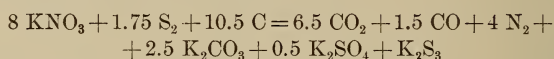
and the principal products of decomposition, on the basis of adduced theoretical considerations, assuming $\varepsilon_1 = 1$, are



* See footnote on p. 44.

It is easy to verify the fact that on these assumptions, the volume of the gases becomes exceedingly great (349 cubic centimeters for a very small content of carbonic acid) * and that the quantity of heat given off proves very small (433 small calories), in comparison with the results actually observed in the experiments of Noble and Abel, apart from the fact that the products of decomposition themselves are markedly different from those obtained experimentally.

From studying the experiments of Noble and Abel, Doctor Debus † found it possible to express the results of the decomposition of these powders by the following equation:



The quantities of products of decomposition here obtained actually agree quite closely with those obtained by the experimenters, but the composition of the powder differs considerably from the actual, as is shown in Table XLII, where are given, in the second column, the composition of the powder and of the products of decomposition according to Debus's equation; in the third column, the values actually found by Noble and Abel for fully dried powders; and in the last column, the same quantities obtained according to the above-deduced theoretical equation.

The chief cause of the difficulty in establishing the decomposition, even of a known powder by a chemical equation, lies in the fact that the charcoal employed in its manufacture is not pure carbon, but a certain product consisting of more or less charred celluloses. In consequence of this, if we assume all the charcoal as carbon, we find an excess of C in comparison with the quantity of oxygen O₂ contained in the saltpeter, which leads to a diminution in the quantity of carbonic acid in comparison with that actually observed. Having established his empirical equation, Debus avoided the difficulty under consideration by purposely increasing the quantity of saltpeter in his powder and diminishing the quantity of carbon.

It follows from what has been stated that at the present state of development, theoretical conceptions do not permit us to foretell, even in an approximate manner, the composition of the products of decomposition of smoking powders and therefore the properties of these powders.

* See last column of Table XL.

† See Berthelot's *Sur la Force*, etc., p. 299 and beyond.)

TABLE XLII.

Name of component parts of powder and of products of decomposition.	Content in percentages.		
	By Debus's equation.	Actually observed values.*	By theoretical equation.
Saltpeter	77.25	75.48	75.02
Charcoal	12.04	14.36	15.04
Sulphur	10.71	10.16	9.94
CO ₂	27.34	27.11	.44
CO	4.02	1.82	33.13
N	10.71	11.34	10.38
K ₂ CO ₃	32.98	34.48	8.32
K ₂ SO ₄	8.32	2.52	36.68
K ₂ S ₃ , K ₂ S+S, and K ₂ S	16.63	†13.09	11.05

* Computed from Tables VIII and VI for entirely dry powder.

† Assumed as a compound of potassium sulphide with free sulphur.

Chapter VII.

LAWS OF MOTION OF THE PROJECTILE AND DETERMINATION OF PRESSURE IN THE BORE OF THE GUN.

60. *General conceptions and deductions of the fundamental expression for the work of gases.*

First of all we shall establish fundamentally the following definitions: We shall designate the *weight of the charge* by ω ; G represents the *weight of the projectile*; L_0 is the *length of the whole bore of the gun* from the muzzle section to the base; l_0 is the *actual length of the powder chamber* (i. e. the length from the base of the bore to the base of the projectile when the latter is in the position it occupies on firing); $L = L_0 - l_0$ the *length of the path traveled by the projectile in the bore*; c is the *caliber of the gun*; c_n is the *diameter of the bore* measuring over the rifling; s the *area of the cross section of the bore of the gun* (it equals the area of the bore over the rifling, increased by the area of the riflings themselves). If we denote by p the *width of the land* and by q the *width of the rifling*, then

$$s = \frac{p \frac{c^2}{4} + q \frac{c_n^2}{4}}{p + q}$$

We shall denote as the *reduced caliber*, c' , of the gun, the diameter of the circle whose area equals s , i. e.,

$$c' = \sqrt{\frac{4s}{\pi}}$$

The powder chamber may possess any desired section; its full volume is V_1 ; this is calculated in accordance with the cross section of the chamber. We shall denote as the *reduced diameter of the chamber* d' , the diameter of the cylinder whose volume and length equal, respectively, the volume and length of the chamber, i. e.,

$$d' = \sqrt{\frac{4V_1}{\pi l_0}}$$

The *reduced length of the chamber*, l_1 , is the length of that cylinder whose volume equals the volume of the chamber and whose diameter equals the reduced caliber—i. e., $l_1 = \frac{V_1}{s}$

The density of loading Δ , in accordance with the above definitions, will be $\Delta = \frac{\omega}{V_1} = \frac{\omega}{\frac{\pi d^2}{4} l_0}$, and $V_1 = \frac{\omega}{\Delta}$.

As the density of the powder equals δ , the *volume of the powder grains* constituting the charge ω will be $\frac{\omega}{\delta}$.

We shall denote as the *unoccupied original volume of the powder chamber* V_Δ the empty space in the chamber after loading—i. e.,

$$V_\Delta = \frac{\omega}{\Delta} - \frac{\omega}{\delta} = \frac{\omega}{\Delta} \left(1 - \frac{\Delta}{\delta}\right) = s l_1 \left(1 - \frac{\Delta}{\delta}\right).$$

The *reduced length of the unoccupied volume of the powder chamber* l_Δ is the length of the cylinder whose volume equals V_Δ and whose area equals s , i. e.,

$$l_\Delta = \frac{\omega}{s\Delta} \left(1 - \frac{\Delta}{\delta}\right) = l_1 \left(1 - \frac{\Delta}{\delta}\right)$$

The volume of the bore itself corresponding to the path l traveled by the projectile is $V_l = sl$.

The volume of the chamber corresponding to the unit weight of charge is $v_1 = \frac{V_1}{\omega}$, and the same fraction of the volume of the bore

and chamber taken together is $v = \frac{V_l + V_1}{\omega}$.

The pressure in the bore of the gun when the whole charge is just consumed at the instant of movement of the projectile will be, on the basis of formula 20, page 78, $P_1 = \frac{f\omega}{V_1 - \alpha\omega} = \frac{f\omega}{s l_1 - \alpha\omega} = \frac{f\omega}{s l_1'}$, in which is introduced the notation

$$l_1' = l_1 - \frac{\alpha\omega}{s}.$$

This quantity, l_1' represents the reduced length of the powder chamber, diminished by the reduced length of the cylinder, whose volume equals the covolume of the gases evolved by the charge.

Speaking in general terms, when fire is communicated to the charge, a portion of the powder burns before the remaining grains of the charge are ignited and before the projectile moves from its initial position. The gradually increasing pressure of gases finally overcomes the resistance of the projectile to motion, and the latter begins to move with a certain acceleration. Simultaneously with this, if not sooner, the unconsumed portion of the charge begins to move also. The density of the products of decomposition continually changes, both on account of the prolonged combustion of the powder and in consequence of the increased volume occupied by it and by the products of decomposition, which is due to the movement of the projectile. Evidently

then, at any given moment, the density of the products of decomposition will not be a constant quantity throughout the whole space occupied by them, and the pressure of the powder gases will not be the same at different sections of the bore. The change of pressure on passing from one section of the bore to another arises also from the following causes: If we divide the whole extent occupied by the powder gases between the base of the projectile and the base of the bore into thin layers, the first layer of the gases, counting from the base of the projectile, overcomes, on expanding, the resistance of the projectile to movement; the second layer must overcome, besides this, the resistance of the first layer; consequently the pressure on this layer from the rear must be greater than the pressure on it from the front, and the latter equal to the pressure on the rear of the first layer, etc.; the same may be said in relation to the distribution of pressures in the rear layers close to the base of the bore if the gun recoils; the gases of the layer farthest to the rear overcome the resistance of the gun and carriage to recoil; the next to rear layer must also overcome, in addition to resistance of gun and carriage, the resistance of the rear layer, and therefore the pressure on its front face must be greater than the pressure on its rear face, etc. In view of this there exists a section in the bore of the gun in which the pressure will attain a maximum, while in front and rear of this section pressures will gradually diminish, and as, at the moment of formation of maximum pressure, the gun travels, in comparison with the shell, a very short distance, the section of greatest pressure at each given moment must correspond very closely with the base of the bore and that of lowest pressure—with the base of the projectile.

In this manner the combustion of each portion of the grain will be accomplished at a different pressure. This circumstance leads us, as we saw in Chapter III, to the fact that the products of decomposition, and, consequently, the quantity of heat evolved by the unit weight of newly burning powders, will vary to a certain extent.

The heat of the products of decomposition is expended, not only in communicating vis viva to the whole system (projectile, carriage, and products of combustion), but in overcoming various harmful resistances, and also in heating the walls of the gun. The amount of cooling due to the latter may be, as we shall see, very considerable under certain conditions. The combined influence of these, as well as prolonged combustion of the powder, produces constant changes of temperature of the products of combustion, so that this temperature is different at each given moment, not only at different sections of the bore, but also in different parts of the same section, being least nearest the circumference.

A part of the gases is lost through the vent, if such exists, and by escape around the projectile, on account of the imperfect obturation

existing there. Finally, under certain special conditions, the distribution of the charge in the bore of the gun will lead to the creation of wave-pressures. (See Part II of this course.)

All the above circumstances indicate at a glance the exceeding complexity of the phenomenon occurring in the bore of the gun at the time of combustion of the charge. It is exceedingly difficult to express it by methods of mathematical analysis.

For simplification of solution of the problem, it is necessary to reduce to a mean the actual conditions of the combustion of powder in the gun and the composition of the products of decomposition, and we shall therefore assume (1) that all the grains of the charge are ignited at the same time which is the moment of displacement of the projectile; (2) that the density of the products of decomposition, their pressure and temperature, change only with time, and at each given moment are constant for all sections of the bore in rear of the projectile; (3) that the quantity of heat Q_v evolved by the unit weight of newly-burned powder, the volumes of gases v'_0 , page 52, and the force of the powder, f , page 78, remain constant throughout the whole time of the burning of the charge and are equal to their mean values determined from burning powder in closed chambers; (4) that there is no loss of products of decomposition either through the vent or through the windage, and (5) that there is no wave motion of the products of combustion.

Making these assumptions, and observing what has been stated in Chapters III and V concerning the combustion of powder in closed chambers, we may determine at each moment the quantity of powder burned and the quantity of heat evolved from it.

Subsequently, in forming the equations of motion of the projectile we shall introduce special expressions to determine the influence of various circumstances attending the motion of the projectile and which possess secondary signification.

Among these circumstances we may indicate the following: (1) the recoil of the gun with the carriage; (2) the influence of the rifling; (3) the rotating movement of the projectile; (4) the effect of the resistance of the air to the movement of the projectile, the friction of the latter, and of the products of combustion, etc.; (5) the internal work of the gases; (6) the effect of the cooling of the products of decomposition by the walls of the gun.

However, as the influence of all these circumstances is assumed as proportional to the vis viva of the projectile, we may partly account for their combined influence in forming equations of motion of the projectile by multiplying the mass m of the projectile by a certain coefficient $\varphi > 1$. The value of this coefficient must be determined from the results of experiment.

Assume that in a time t the corresponding quantity of powder

burned by weight, i. e., the quantity of products of decomposition formed will be $\omega\Psi$; their temperature, reckoning from absolute zero, T ; the volume occupied by the products of decomposition of the burned portion of the charge $=v\omega\Psi$. In the elementary increment of time dt subsequent to this; (1) the quantity of burned powder is increased by $\omega d\Psi$; (2) the temperature of this new portion of the products of decomposition is immediately lowered from T_1 , to T ; hence, the products of decomposition acquire, as it were, a certain quantity of heat dQ ; (3) the products of decomposition $\omega(\Psi + d\Psi)$ after expansion perform certain external work dT^* and the volume of the gases increases by dV ; (4) at the same time the temperature of the products of decomposition is lowered by dT , which corresponds to the change of the quantity of heat by dQ_2 .

It follows, therefore, that on performing an amount of external work dT there is expended a portion of internal energy of the gases corresponding to the quantity of heat $dQ_1 - dQ_2$. Denoting the mechanical equivalent of heat by J we obtain

38

$$J(dQ_1 - dQ_2) = dT$$

This is the fundamental expression of thermodynamics for the adiabatic process—i. e., for that process in which the products of decomposition neither impart heat to nor receive heat from their surrounding envelope—i. e., their whole internal energy is devoted to performing certain external work.

The integration of equation 38 depends on the form of the expression for dQ_1 and dQ_2 , i. e., on the variety of the powder and the character of its combustion. Speaking in general terms, powders, e. g., smoking powders (paragraph 15, p. 35), decompose into gaseous and nongaseous products. We may assume that in the unit weight of the latter are included gaseous bodies ζ and nonvolatile bodies γ , so that $\zeta + \gamma = 1$. In the case of smokeless powders, the products of decomposition of which are wholly gaseous (paragraph 17, p. 46), we may, without sensible error, substitute in all formulæ the values $\gamma = 0$ and $\zeta = 1$.

It was stated on page 44, paragraph 16, that the nongaseous and gaseous products of decomposition were completely admingled together. We must therefore assume with Noble and Abel † that there occurs at each moment a constant interchange of heat and temperature between the two kinds of products of decomposition. In consequence of this, denoting by η and C_v the specific heats for constant volume of the nonvolatile and gaseous products of decomposition, we find that to each change of temperature of the products of decomposition by the amount ΔT there corresponds a certain quantity of heat ΔQ :

$$\Delta Q = (\zeta C_v + \gamma \eta) \Delta T$$

* *Translator's note.*—Hereafter distinguish between the heavy T representing external work, and the light T representing temperature.

† See their "Researches on Explosives."

In consequence of this and what has already been stated under headings 2 and 4, page 179, we obtain, neglecting the quantities of the second order,

$$\begin{aligned}dQ_1 &= (\zeta C_v + \gamma \eta) (T_1 - T) \omega d\Psi \\dQ_2 &= (\zeta C_v + \gamma \eta) \omega \Psi dT\end{aligned}$$

and therefore expression 38 assumes the following form:

$$dT = J \omega (\zeta C_v + \gamma \eta) [(T_1 - T) d\Psi - \Psi dT] = J \omega (\zeta C_v + \gamma \eta) d[\Psi (T_1 - T)].$$

Observing that the value T_1 is constant for a given powder and that $RT_1 = f'$ (page 78), where R possesses the signification given in 15 (page 67), and does not refer to the unit weight of gases but to the weight ζ evolved by the unit weight of the charge, we obtain

$$dT = \omega f J \frac{\zeta C_v + \gamma \eta}{R} d \left[\Psi \left(1 - \frac{T}{T_1} \right) \right]$$

moreover, as

$$J = \frac{R}{\zeta (C_p - C_v)}; \quad C_p - C_v = 0.00008848 \frac{v'_0}{\zeta}$$

(page 55).

$$RT = P(v - \alpha) = f \frac{T}{T_1}$$

(page 78); then by substituting (compare with the value for k , page 60).

$$\beta = \frac{\zeta (C_p - C_v)}{\zeta C_v + \gamma \eta} = \frac{0.00008848 \frac{v'_0}{\zeta}}{\zeta C_v + \gamma \eta}$$

the adduced expression for elementary work may be written also as

$$39. \quad dT = \frac{\omega f}{\beta} d \left[\Psi \left(1 - \frac{T}{T_1} \right) \right] = \frac{\omega}{\beta} d[\Psi(f - P(v - \alpha))]$$

The integration of this is possible: (1) when β is constant, i. e., when the specific heats C_v and η are reckoned as independent of the temperature and are equal to the mean values

$$\begin{aligned}C_v &= \Sigma a + \Sigma b_1 t_1 \\ \eta &= \Sigma a_1 + \Sigma b_1 t_1\end{aligned}$$

and (2) when C_v and η are actual specific heats and are lineal functions of the temperature (page 55) of the form

$$\begin{aligned}C'_v &= \Sigma a + 2 \Sigma b_1 t \\ \eta^1 &= \Sigma a_1 + 2 \Sigma b_1 t\end{aligned}$$

substituting in the latter t° for $T^\circ - 273^\circ$ and substituting in the expression for β , we obtain for the latter another signification β' , namely,

$$\frac{1}{\beta'} = A + BT,$$

in which to abbreviate notation there are introduced the values

$$\begin{aligned}B &= 2 \frac{\Sigma \zeta b + \Sigma \gamma b_1}{0.00008848 v_0^1} \\ A &= \frac{\Sigma \zeta a + \Sigma \gamma a_1}{0.00008848 v_0^1} - 273 B\end{aligned}$$

In this case the expression 39 assumes the form

$$40. \quad dT = \omega f (A + BT) d \left[\Psi \left(1 - \frac{T}{T_1} \right) \right]$$

61. *The laws of movement of the projectile and distribution of pressures in the bore of the gun, when the whole powder is burnt at the moment of displacement of the projectile—The potential of a powder—Comparison of different powders in their action in the same gun.*

In the given case $\Psi = 1$ at the beginning of combustion, and therefore expressions 39 and 40 assume the forms

$$dT = -\frac{\omega f}{\beta} \frac{dT}{T_1} = -\frac{\omega R}{\beta} dT$$

and

$$dT = -\omega f (A + BT) \frac{dT}{T_1} = -\omega R (A + BT) dT$$

After integrating between limits 0 and T , and T_1 and T , we obtain

$$T = \frac{\omega f}{\beta} \left[1 - \frac{T}{T_1} \right]$$

$$T = \omega f \left(1 - \frac{T}{T_1} \right) \left[A + \frac{B}{2} (T_1 + T) \right] = \omega f \left[\frac{\Sigma \zeta a + \Sigma \gamma a_1}{0.00008848 v'_0} + \frac{\Sigma \zeta b + \Sigma \gamma b_1}{0.00008848 v'_0} (t_1 + t) \right]$$

As in the case under consideration $v^1 = v$ (page 177), and in general $dT = P \omega d(v - \alpha)$, therefore equating this with the expression written above for elementary work we obtain

$$P d(v - \alpha) = -\frac{R}{\beta} dT$$

$$P d(v - \alpha) = -R (A + BT) dT$$

Besides this, expression 18 (page 77), also obtains in all cases; i. e., we always have $P(v - \alpha) = RT$, and, therefore, dividing the above expression by it we obtain

$$-\beta \frac{d(v - \alpha)}{v - \alpha} = \frac{dT}{T},$$

and

$$-\frac{d(v - \alpha)}{v - \alpha} = A \frac{dT}{T} + B dT.$$

After integrating between the limits T_1 and T , and between v_1 and v we obtain

$$\frac{T}{T_1} = \left(\frac{v_1 - \alpha}{v - \alpha} \right)^\beta$$

or

$$\frac{v_1 - \alpha}{v - \alpha} = \left(\frac{T}{T_1} \right)^{\frac{1}{\beta}}$$

and

$$\frac{v_1 - \alpha}{v - \alpha} = \frac{\left(\frac{T}{T_1} \right)^A}{e^{BT_1} \left(1 - \frac{T}{T_1} \right)}$$

Moreover, on the basis of the same expression 18, and of the definitions assumed on page 176, we have

$$\frac{P}{P_1} = \frac{v_1 - \alpha}{v - \alpha} \frac{T}{T_1} = \frac{l_1'}{l_1' + l} \frac{T}{T_1}$$

and, therefore, we finally obtain, reckoning specific heats as constant,

$$41. \left\{ \begin{aligned} \frac{P}{P_1} &= \left(\frac{v_1 - \alpha}{v - \alpha} \right)^{1+\beta} = \left(\frac{l_1'}{l_1' + l} \right)^{1+\beta} = \left(\frac{T}{T_1} \right)^{\frac{1+\beta}{\beta}}, \\ \left(\frac{v_1 - \alpha}{v - \alpha} \right) &= \frac{l_1'}{l_1' + l} = \left(\frac{T}{T_1} \right)^{\frac{1}{\beta}}, \\ T &= \frac{\omega f}{\beta} \left[1 - \left(\frac{v_1 - \alpha}{v - \alpha} \right)^\beta \right] = \frac{\omega f}{\beta} \left[1 - \left(\frac{l_1'}{l_1' + l} \right)^\beta \right] = \\ &= \frac{\omega f}{\beta} \left[1 - \frac{T}{T_1} \right], \\ \beta &= \frac{\zeta C_p - C_v}{\zeta C_v + \gamma \eta} = \frac{0.00008848 v_0'}{\zeta C_v + \gamma \eta} \end{aligned} \right.$$

and, reckoning specific heats as true,

$$42. \left\{ \begin{aligned} \frac{P}{P_1} (*) &= \frac{\left(\frac{T}{T_1} \right)^{A+1}}{e^{BT_1} \left(1 - \frac{T}{T_1} \right)}, \\ \frac{w_1 - \alpha}{w - \alpha} &= \frac{l_1'}{l_1' + l} = \frac{\left(\frac{T}{T_1} \right)^A}{e^{BT_1} \left(1 - \frac{T}{T_1} \right)}, \\ T &= \omega f \left[1 - \frac{T}{T_1} \right] \left[A + \frac{B}{2} (T_1 + T) \right] = \\ &= \omega f \left[1 - \frac{T}{T_1} \right] \left[\frac{\zeta C_v + \gamma \eta}{0.00008848 v_0'} + \frac{\sum \zeta b + \sum b_1}{0.00008848 v_0'} t \right] = \\ &= T + \omega f \frac{\sum \zeta b + \sum \gamma b_1}{0.00008848 v_0'} \frac{t_1 - t}{T_1} t; \\ B &= 2 \frac{\sum \zeta b + \sum \gamma b_1}{0.00008848 v_0'}, \\ A &= \frac{\sum \zeta a + \sum \gamma a_1}{0.00008848 v_0'} - 273 B. \end{aligned} \right.$$

* The equation for pressures, although of somewhat different form, was first given by Mallard and Le Chatelier. Consult their work, page 58, for particulars.

If we put b and b_1 , equal to zero in the last expressions, and $\Sigma \zeta a + \Sigma \gamma a_1 = \zeta C_v + \gamma \eta$, i. e., if we assume the specific heats as constant and equal to their mean values, all formulæ 42 are converted into 41.

A comparison of the expressions 41 and 42 shows that for the same reduction of temperature of the products of combustion and for $t^0 > 0$, the work of expansion of the gases for true specific heats T (42) is something greater than the value of T obtained from (41) for mean specific heats. The difference between them attains a maximum value for that expansion of gases which reduces their temperature to $t^0 = \frac{t_1^0}{2}$.

For $t^0 = 0$, both expressions give the same values for work performed; for $t^0 < 0$, T (42) $<$ T (41).

If we denote the velocity of the projectile during the time t by v , the vis viva acquired by it will be $\frac{Gv^2}{2g}$. As only a part of the work of the gases is expended in imparting vis viva to the projectile, therefore, dividing it by the coefficient ϕ , greater than unity, we obtain,

$$43 \quad v = \sqrt{2g \frac{T}{G\phi}}$$

where $\frac{1}{\phi}$ denotes the coefficient of the useful effect of the powder in a gun of a given caliber, and is determined by comparing the results of firing actually obtained with those calculated by the adduced formulæ. The other portion of the work of the gases is expended, in imparting vis viva to the other elements of the system, in overcoming various resistances, etc. In order to obtain the initial velocity of the projectile, it is necessary to substitute in place of T in expression 43 its greatest value, corresponding to the complete expansion of the gases in a given gun. The greatest value of T is obtained for constant specific heats from the third equation 41 by substituting in it l for L , page 176; in order to obtain the value for true specific heats, it is necessary to determine, first, from the second equation 42 by experiment, the ratio $\frac{T}{T_1}$ for $l=L$ and subsequently the value of the maximum work T from the third equation 42.

It remains to determine the time of movement of the projectile. In general terms we have

$$v = \frac{dl}{dt}$$

whence

$$t = \int_0^L \frac{dl}{v}$$

On account of the complexity of the expressions for v (43), as obtained after substituting in the third equations of systems (41) and (42), the

integral expressing the value of t can not be found, even approximately. Instead of this, we may, by aid of one of the formulae for quadrature, calculate the area limited by the curve $\frac{l}{v}$ and the axis l , beginning with values of v corresponding to $l=0.01$ dem., $l=0.02$ dem., $l=0.03$ dem., up to $v=V_0$. These areas give the times in which the projectile traverses the distances $L-0.01$; $L-0.02$; $L-0.03$; etc., respectively. Subsequently, by interpolation, we may find the time in which the shell traverses the whole distance $L=L_0-l_0$.

As formulæ 41 to 43 are adduced on the assumption that all the powder is consumed at the moment the projectile begins to move, their application to ordinary cases in practice, where the powder continues to burn throughout the whole time of movement of the projectile, affords the highest values for work, pressures, and velocities possible in a given gun under actual conditions, while the values for t and T are least, under the same conditions.

Introducing, however, corresponding coefficients of correction into the formulæ considered we may employ them for approximate calculations in all practical cases. These coefficients should be determined on the basis of results of firing from cannon similar in structure of bore and conditions of loading, and also near to one another in caliber.*

Formulæ (41 and 42) allow us to determine the maximum work which may be performed by the products of combustion of 1 kilogram of powder expanding to an infinitely great volume, $v=\infty$ i. e., to absolute zero, when $T=0$ or $t=-273^\circ$, as a result of which both expressions give different values for the work performed. However, as we do not know the changes that occur in specific heat at such low temperatures, calculations of this kind should be regarded as only approximate.

It is more accurate to calculate by the same formulæ the work performed by the products of combustion of 1 kilogram of powder in cooling through expansion down only to ordinary temperature, e. g. to $t=17^\circ$ C, for which the quantity of heat given off on combustion of the powder is determinate.

On the other hand, the maximum work which powder is capable of performing is obtained when all heat given off by it is converted into work, i. e.,

$$T_{\max} = JQ_v'$$

This expression represents the store of work contained in the powders or the potential of its energy for ordinary temperature.

* A similar method of calculating the velocities and pressures was proposed: In Noble in Abel's works, already cited (see footnote, p. 31); by Krupp (see his *Calcul des vitesses des projectiles et des pressions des gaz aux differentes longueurs de l'ame*), and also the works of Professor Kaiser, *Construction der gezogenen Geschützrohre*.

The values of T_{\max} calculated by both methods should agree closely which is confirmed by the figures given in Table XLIII, in which are given the potentials of energy of smoking powders (Spanish, pebble, and chocolate), and of smokeless powders (pyrocollodion and ballistite), which are the forms of powder usually employed. The data in Tables XV and XIX were used in calculating these values.

TABLE XLIII.—Constant quantities in formulæ 41 and 42 and the potentials of energy of powders.

Name of powder.	$\frac{f}{\text{cm.}^2}$ kgr.	α cm. ³	T_1	$\frac{\zeta C_p + \gamma \eta}{\zeta C_v + \gamma \eta} = 1 + \beta$	A	B	Potential of the energy in 1 kilogram of powder.*	
							By formula 41 or 42 $t^\circ = 17^\circ$.	From the expression $JQ^{1.4}$.
							Ton-meters.	Ton-meters.
Spanish	2,297	0.679	2,510	1.06283	6.4189	0.006824	$\frac{323}{323}$	324
Pebble	2,651	.594	2,427	1.07729	5.2224	.005716	$\frac{302}{302}$	303
Chocolate	2,793	.602	2,335	1.07584	5.4692	.005918	$\frac{323}{323}$	324
Pyrocollodion.	8,800	.945	2,440	1.20666	2.0781	.002028	$\frac{375}{376}$	377
Ballistite	9,375	.807	3,138	1.15617	2.0799	.002585	$\frac{546}{546}$	563

*The numerators were calculated by formula 41 and the denominators by formula 43.

The values for the potentials of these powders presented in Table XLIII do not afford us, however, the means of comparing the latter together in their application to cannon, as it was assumed that each powder cooled down to ordinary temperature, independent of the pressure that it developed at the moment of decomposition.

In comparing powders in relation to their use in cannon, it is necessary to refer them to the same gun. Let us assume that the strength of the gun permits us to develop in it a pressure P_1 not greater than $2500 \frac{\text{kgr.}}{\text{cm.}^2}$; that the volume of the powder chamber equals $\frac{\pi(1.14c)^2}{4} l_0 \approx 0.325\pi c^2 l_0$; that the volume of the whole bore $\approx 1.325\pi c^2 l_0$; i. e., the length of the powder chamber is one-fifth of the whole length of the bore. If we assume that all the powder is burned at the instant the projectile commences to move, then the weight of the charge introduced into the chamber and developing a pressure P_1 will be

$$\omega \approx \frac{0.325\pi c^2 l_0 P_1}{f + \alpha P_1}$$

If we denote the same quantities for pebble powder taken as a unit of comparison by the index, p , then

$$\omega = \omega_p \frac{f_p + \alpha_p P_1}{f + \alpha P_1}$$

Bearing in mind the first expression for ω , the number of "volumes of expansion" of the gases in the gun will be

$$44 \quad \frac{l'_1 + L}{l'_1} = \frac{V - \alpha \omega}{V_1 - \alpha \omega} \frac{1.325 \pi c^2 l_0 - \alpha \omega}{0.325 \pi c^2 l_0 - \alpha \omega} \frac{2.408 + 3.08 \alpha}{f} \frac{P_1}{f}$$

The density of loading is $\Delta = \frac{P_1}{f + \alpha P_1}$

The weight of charge ω_p taken in relation to pebble powder as a unit is calculated in Table XLIV for the same powders as in Table XLIII.

A comparison of the figures in these tables shows (1) the powders are arranged differently in reference to their action in guns than they are when arranged in accordance with the values of their potentials, which should be, since in the latter case the quantity of heat is alone considered, while in guns both the pressure developed by the powder and the degree of expansion of its gases must be taken into account; (2) that the useful work in the gun performed by 1 kilogram of powder is from one-third to one-eighth of its potential, according to the kind of powder used, and (3) that the difference between results calculated by formulæ 41 and 42 is insignificant.

62. *The differential equation of motion of the projectile when the powder burns progressively.*

The most complete and simple solution of this question in relation to smokeless powders was afforded in the year 1876 by the eminent French scientist, E. Sarrau.* The principle of his method is explained in paragraphs 61–67, 75–76 of the present course, where the same is also applied to smokeless powders.

TABLE XLIV.—*Work performed by different powders in the same gun, for volume of chamber $0.325 \pi c^2 l_0$ and of bore $1.325 \pi c^2 l_0$; initial pressure $P_1 = 2500 \frac{\text{kgr.}}{\text{cm.}^2}$*

Name of powder.	Weight of charge ω .	Density of loading Δ .	Number of volumes of expansion of gases $\frac{l'_1 + L}{l'_1}$.	P. Pressure of gases at the muzzle of the gun.	Absolute temperature of the gases at the muzzle T.	Work of powder gases.	
						For the whole charge.	For unit weight of powder.
	<i>Kilogram.</i>			<i>Kilogram per cm.²</i>		<i>Ton-meters.</i>	<i>Ton-meters.</i>
Spanish	1.07	0.626	6.36	350	2,235	41.7	40.1
				363	2,315	42.1	40.8
				376	2,119	43.6	43.6
Pebble.....	1.00	.604	5.80	392	2,208	44.2	44.2
				381	2,045	43.9	45.7
Chocolate96	.582	5.14	397	2,126	45.2	47.0
				366	1,756	44.1	119.3
Pyrocollodion37	.224	4.91	398	1,908	46.0	124.4
				414	2,461	46.6	129.5
Ballestite.....	.36	.219	4.74	447	2,660	48.5	134.6

NOTE.—The numerators in the last four columns were obtained by formula 41 and the denominators by formula 42.

* See his work "Nouvelles recherches sur les effets de la poudre dans les armes," 1876.

Formulae 39 and 40, page 181, also relate to the case under consideration, according as we assume specific heats as mean or true. As the assumption of true specific heat would greatly complicate the deduction, we shall limit ourselves to the consideration of the former of these expressions, 39. The variables in this expression have all been defined, and, therefore, integrating between limits $T = 0$, $\Psi = 0$, $P = 0$ and T , Ψ and P , we obtain

$$\text{or,} \quad T = \omega \frac{\Psi}{\beta} \left[f - P (v' - \alpha) \right]$$

$$45 \quad \beta T + P (V' - \omega \Psi \alpha) = f \omega \Psi$$

Assuming the instantaneous inflammation of the charge, the volume V' consists (p. 177): (1) of the unoccupied initial volume V_{Δ} , (2) of the volume of the bore V_b , corresponding to the path l traversed by the projectile, and (3) of the volume $V\Psi$ of the burned part, $\omega\Psi$, of the powder, i. e.,

$$46 \quad V' = \omega \left(\frac{1}{\Delta} - \frac{1}{\delta} \right) + sl + \frac{\omega\Psi}{\delta} = s(l_{\Delta} + l) + \frac{\omega\Psi}{\delta}$$

Assuming that the work of the gases expended in overcoming various resistances and imparting vis viva to various parts of the system is proportional to the vis viva of the projectile, we obtain

$$\left\{ \begin{array}{l} T = \varphi \frac{G}{2g} \left(\frac{dl}{dt} \right)^2 = \frac{1}{2} M \left(\frac{dl}{dt} \right)^2 \\ \text{and} \\ ps = \varphi \frac{G}{g} \frac{d^2l}{dt^2} = M \frac{d^2l}{dt^2} \\ \text{where} \\ M = \varphi \frac{G}{g} \end{array} \right.$$

If we substitute these values for T , P and V in 45 we obtain

$$\frac{\beta}{2} M \left(\frac{dl}{dt} \right)^2 + M \left[l_{\Delta} + l + \frac{\omega\Psi}{s} \left(\frac{1}{\delta} - \alpha \right) \right] \frac{d^2l}{dt^2} = f \omega \Psi_0$$

In relation to the expression $\frac{1}{\delta} - \alpha$ here introduced, we may remark the following: In paragraphs 21 to 24, pages 64 to 90, the signification of the covolume was explained and its value was given for various powders. In the case of smoking powders α varied between 0.679 and 0.443; while for smokeless powders it varied between 1 and 0.80. On the other hand, the reciprocal of the density of the powder, $\frac{1}{\delta}$, varied, for the former from 0.625 to 0.541, while for the latter it was close to 0.625. In view of this, the difference $\frac{1}{\delta} - \alpha$, as obtained for all powders, is, generally speaking, a small quantity, positive or negative.

Besides this, at the commencement of motion Ψ is usually very small, while at the end of the movement, l is a considerable quantity. In consequence of this we may neglect the difference $\frac{1}{\delta} - \frac{1}{\alpha}$ in the term in which it enters, in comparison with the value of $(l_{\Delta} + l)$, which considerably simplifies the differential equation.

In this way the expression under consideration assumes the form

$$48 \quad (l_{\Delta} + l) \frac{d^2 l}{dt^2} + \frac{\beta}{2} \left(\frac{dl}{dt} \right)^2 = \frac{f\omega\Psi}{M}$$

This expression represents the desired differential equation of movement of the projectile for the progressive combustion of the whole of the powder in the bore of the gun.

63. *The integration of the differential equation of motion of the projectile, 48.*

In paragraphs 53 and 54, pages 150–156, were indicated the conditions of the employment of the general expression 36 for the value Ψ of the burned portion of the unit weight of powder, assuming that the combustion was accomplished under variable pressure. Evidently this expression, under the assumptions made in paragraph 60, page 179, fully corresponds to the combustion of the powder in the bore of the gun during the movement of the projectile.

As in the given case,

$$Pt = \frac{\varphi}{s} \frac{G}{g} \frac{d^2 l}{dt^2} = \frac{M}{s} \frac{d^2 l}{dt^2}$$

the expression 36 may, therefore, be written as

$$49 \quad \Psi = \kappa \frac{2A}{\alpha} \left(\frac{M}{s} \right)^{\nu} \int_0^t \left(\frac{d^2 l}{dt^2} \right)^{\nu} dt \left\{ 1 - \lambda \frac{2A}{\alpha} \left(\frac{M}{s} \right)^{\nu} \int_0^t \left(\frac{d^2 l}{dt^2} \right) dt + \mu \left(\frac{2A}{\alpha} \right)^2 \left(\frac{M}{s} \right)^{2\nu} \left[\int_0^t \left(\frac{d^2 l}{dt^2} \right)^{\nu} dt \right]^2 \right\}$$

We shall substitute this value of Ψ in equation 48 and introduce instead of l a new variable z , which is a function of t , whence

$$50 \quad \begin{cases} l = z l_{\Delta}, \\ \frac{dl}{dt} = l_{\Delta} \frac{dz}{dt} \\ \frac{d^2 l}{dt^2} = l_{\Delta} \frac{d^2 z}{dt^2} \end{cases}$$

consequently we obtain

$$51 \quad (1+z) \frac{d^2 z}{dt^2} + \frac{\beta}{2} \left(\frac{dz}{dt} \right)^2 = \frac{f\omega}{M l_{\Delta}^2} \kappa \frac{2A}{\alpha} \left(\frac{M l_{\Delta}}{s} \right)^{\nu} \int_0^t \left(\frac{d^2 z}{dt^2} \right)^{\nu} dt \left\{ 1 - \lambda \frac{2A}{\alpha} \left(\frac{M l_{\Delta}}{s} \right)^{\nu} \int_0^t \left(\frac{d^2 z}{dt^2} \right) dt + \left(\frac{2A}{\alpha} \right)^2 \left(\frac{M l_{\Delta}}{s} \right)^{2\nu} \left[\int_0^t \left(\frac{d^2 z}{dt^2} \right)^{\nu} dt \right]^2 \right\}$$

If we introduce the conventions

$$52 \quad \begin{cases} K = \frac{f\omega}{Ml_{\Delta}^2} \kappa \frac{2A}{a} \left(\frac{Ml_{\Delta}}{s} \right)^{\nu} \text{ and} \\ Z = \int_0^t \left(\frac{d^2z}{dt^2} \right)^{\nu} dt \end{cases}$$

then equation 51 assumes the form

$$53 \quad (1+z) \frac{d^2z}{dt^2} + \frac{\beta}{2} \left(\frac{dz}{dt} \right)^2 = \kappa Z \left\{ 1 - \lambda K \frac{Ml_{\Delta}^2}{f\omega\kappa} Z + \right. \\ \left. + \mu \left(K \frac{Ml_{\Delta}^2}{f\omega\kappa} \right)^2 Z^2 \right\}$$

In place of the independent variable t we shall introduce a new variable, x , determined from the expression

$$54 \quad x = K^b t$$

where K has the signification given in 52 and b is a certain exponent independent of t .

Consequently

$$55 \quad \begin{cases} \frac{dx}{dt} = K^b; \quad dt = \frac{1}{K^b} dx, \\ \frac{dz}{dt} = \frac{dz}{dx} \frac{dx}{dt} = K^b \frac{dz}{dx}, \\ \frac{d^2z}{dt^2} = K^b \frac{d^2z}{dx^2} \frac{dx}{dt} = K^{2b} \frac{d^2z}{dx^2}. \end{cases}$$

As the result of this, the expression Z (52) assumes the form

$$56 \quad Z = K^{(2\nu-1)b} X$$

where

$$57 \quad X = \int_0^x \left(\frac{d^2z}{dx^2} \right)^{\nu} dx.$$

Substituting the values 55 and 56 in the above-transformed equation of motion, 53, we obtain

$$58 \quad (1+z) \frac{d^2z}{dx^2} + \frac{\beta}{2} \left(\frac{dz}{dx} \right)^2 = K^{1+(2\nu-3)b} X [1 - \lambda \varepsilon X + \mu \varepsilon^2 X^2]$$

where

$$\varepsilon = K^{1+(2\nu-1)b} \frac{Ml_{\Delta}^2}{f\omega\kappa}.$$

As the value of the exponent b is arbitrary, we shall assume it so that $1+(2\nu-3)b$ equals 0. This gives

$$59 \quad b = \frac{1}{3-2\nu},$$

$$60 \quad \varepsilon = K^{\frac{2}{3-2\nu}} \frac{Ml_{\Delta}^2}{f\omega\kappa},$$

and therefore expression 58 may be written as

$$61 \quad (1+z) \frac{d^2 z}{dx^2} + \frac{\beta}{2} \left(\frac{dz}{dx} \right)^2 = X - \lambda \varepsilon X^2 + \mu \varepsilon^2 X^3.$$

In this manner the integration of the original differential equation of motion of the projectile 48 is reduced to the integration of equation 61.

Let us assume that we have found the integral satisfying this expression. Its general form will evidently be

$$62. \quad z = z_0 + \varepsilon z_1 + \varepsilon^2 z_2,$$

where $z_0, z_1, z_2 \dots$ are certain functions of x and, consequently, are also functions of the time t . This expression gives

$$63 \quad \begin{cases} \frac{dz}{dx} = \frac{dz_0}{dx} + \varepsilon \frac{dz_1}{dx} + \varepsilon^2 \frac{dz_2}{dx} \\ \frac{d^2 z}{dx^2} = \frac{d^2 z_0}{dx^2} + \varepsilon \frac{d^2 z_1}{dx^2} + \varepsilon^2 \frac{d^2 z_2}{dx^2} \\ \left(\frac{dz}{dx} \right)^2 = \left(\frac{dz_0}{dx} \right)^2 + \varepsilon^2 \left(\frac{dz_1}{dx} \right)^2 + 2\varepsilon \frac{dz_0}{dx} \frac{dz_1}{dx} + 2\varepsilon^2 \frac{dz_0}{dx} \frac{dz_2}{dx} + \dots \\ \left(\frac{d^2 z}{dx^2} \right)^\nu = \left(\frac{d^2 z_0}{dx^2} \right)^\nu + \nu \varepsilon \frac{d^2 z_1}{dx^2} \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-1} + \nu \varepsilon^2 \frac{d^2 z_2}{dx^2} \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-1} + \\ + \frac{\nu(\nu-1)}{1.2} \varepsilon^2 \left(\frac{d^2 z_1}{dx^2} \right)^2 \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-2} + \end{cases}$$

Substituting the latter of these equations in the expression for X , 57, we obtain

$$64 \quad X = X_0 + \varepsilon X_1 + \varepsilon^2 X_2 +$$

in which, for abbreviation, are introduced the designations

$$65 \quad \begin{cases} X_0 = \int_0^x \left(\frac{d^2 z_0}{dx^2} \right)^\nu dx, \\ X_1 = \nu \int_0^x \frac{d^2 z_1}{dx^2} \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-1} dx, \\ X_2 = \nu \int_0^x \frac{d^2 z_2}{dx^2} \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-1} dx + \frac{\nu(\nu-1)}{1.2} \int_0^x \frac{d^2 z_1}{dx^2} \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-2} dx, \\ \dots \end{cases}$$

We now substitute the values found for z , $\frac{dz}{dx}$, $\frac{d^2z}{dx^2}$ and X in the equation of motion (61) and we equate with one another, respectively, the coefficients of the same powers of ε on both sides; this affords us the following system of auxiliary equations replacing the equation 61.

$$66 \quad \begin{cases} (1+z_0) \frac{d^2z_0}{dx^2} + \frac{\beta}{2} \left(\frac{dz_0}{dx} \right)^2 = X_0, \\ (1+z_0) \frac{d^2z_1}{dx^2} + z_1 \frac{d^2z_0}{dx^2} + \beta \frac{dz_0}{dx} \frac{dz_1}{dx} = X_1 - \lambda X_0^2, \\ (1+z_0) \frac{d^2z_2}{dx^2} + z_1 \frac{d^2z_1}{dx^2} + z_2 \frac{d^2z_0}{dx^2} + \frac{\beta}{2} \left(\frac{dz_1}{dx} \right)^2 + \\ + \beta \frac{dz_0}{dx} \frac{dz_2}{dx} = X_2 - 2\lambda X_0 X_1 + \mu X_0^3, \\ \dots \dots \dots \end{cases}$$

To solve our problem, it remains to choose such a system of functions z_0, z_1, z_2 , that the latter, while satisfying equation 66, are themselves converted, as well as their first derivatives, into 0 for $x=0$. These two conditions arise from the circumstance that the path traversed by the projectile and its velocity equal 0 when $t=0$.

It must be observed that these equations are calculated for a given form of similar grains—i. e., for the given values of κ, λ, μ , which are independent of the circumstances of firing.

The determination of the exact forms of the functions z_0, z_1, z_2 , entering into the equation 66 is not possible, and therefore Sarrau applied to this equation a method of integration by series with indeterminate coefficients and exponents.

This method permits us to obtain numerical values of the functions $z_0, \frac{dz_0}{dx}, \frac{d^2z_0}{dx^2}, z_1 \dots$ entering into equations 66 and 62.

Although the calculations of all these quantities by this method do not present any special difficulties, yet they are very long and therefore we shall limit ourselves to the calculation of $z_0, \frac{dz_0}{dx}, \frac{d^2z_0}{dx^2}$; i. e., the determination of the first member of equation 62.*

We may satisfy this equation for small values of x by putting

$$\left\{ \begin{aligned} \frac{d^2z_0}{dx^2} &= ax^m + bx^n + cx^p + \dots \\ \frac{dz_0}{dx} &= \frac{a}{m+1} x^{m+1} + \frac{b}{n+1} x^{n+1} + \frac{c}{p+1} x^{p+1} + \dots \\ z_0 &= \frac{a}{(m+1)(m+2)} x^{m+2} + \frac{b}{(n+1)(n+2)} x^{n+2} + \\ &+ \frac{c}{(p+1)(p+2)} x^{p+2} + \dots \end{aligned} \right.$$

*We shall refer those who are desirous of acquainting themselves with the method of calculation of the quantities z_1, z_2 , and their derivatives to the work of Sarrau, referred to on p. 187.

in which it remains to determine the quantities $a, b, c, \dots m, n, p, \dots$

To do this we shall substitute the written values of $\frac{dz_0}{dx^2}, \frac{dz_0}{dx}$ and z_0 in the above-deduced auxiliary equation, integrate the second member, and then obtain:

$$\begin{aligned}
 & ax^m + bx^n + cx^p + \dots + \frac{a^2}{m+1} \left[\frac{1}{m+2} + \frac{\beta}{2(m+1)} \right] x^{2(m+1)} + \\
 & + \frac{b^2}{n+1} \left[\frac{1}{n+2} + \frac{\beta}{2(n+1)} \right] x^{2(n+1)} + \frac{c^2}{p+1} \left[\frac{1}{p+2} + \frac{\beta}{2(p+1)} \right] x^{2(p+1)} + \dots \\
 & + ab \left[\frac{1}{(m+1)(m+2)} + \frac{\beta}{(m+1)(n+1)} + \frac{1}{(n+1)(n+2)} \right] x^{m+n+2} + \\
 & + ac \left[\frac{1}{(p+1)(p+2)} + \frac{\beta}{(m+1)(p+1)} + \frac{1}{(m+1)(m+2)} \right] x^{m+p+2} + \\
 & + bc \left[\frac{1}{(p+1)(p+2)} + \frac{\beta}{(n+1)(p+1)} + \frac{1}{(n+1)(n+2)} \right] x^{n+p+2} + \dots = \\
 & = \frac{av}{m\nu+1} x^{m\nu+1} + \frac{\nu a^{\nu-1}b}{m(\nu-1)+n+1} x^{m(\nu-1)+n+1} + \\
 & + \frac{\nu(\nu-1)a^{\nu-2}b^2}{2[m(\nu-2)+2n+1]} x^{m(\nu-2)+2n+1} + \dots \\
 & + \frac{\nu a^{\nu-1}c}{m(\nu-1)+p+1} x^{m(\nu-1)+p+1} + \\
 & + \frac{\nu(\nu-1)a^{\nu-2}c^2}{2[m(\nu-2)+2p+1]} x^{m(\nu-2)+2p+1} + \dots + \frac{b\nu}{n\nu+1} x^{n\nu+1} + \\
 & + \frac{\nu b^{\nu-1}c}{n(\nu-1)+2p+1} x^{n(\nu-1)+p+1} + \frac{\nu(\nu-1)b^{\nu-2}c^2}{2[n(\nu-1)+2p+1]} x^{n(\nu-2)+2p+1} + \dots \\
 & + \frac{c\nu}{p\nu+1} x^{p\nu+1} + \dots
 \end{aligned}$$

Equating the exponents of the degree of x of the first terms of both sides of this equation we find

$$m = \frac{1}{1-\nu}$$

Besides this, in the work of Sarrau above referred to, it is necessary to assume that the degrees of x in the expression for $\frac{dz_0}{dx^2}$ increase in arithmetical progression for the denominators by the ratio $m+2$, i. e., that

$$n=2m+2, p=n+m+2=3m+4 \dots$$

If we limit ourselves only to terms of the degrees m, n , and p , the equation in question assumes the form

$$\begin{aligned}
 & ax^m + \left[\frac{a^2}{(m+1)(m+2)} + \frac{\beta a^2}{2(m+1)^2} + b \right] x^{2(m+1)} + \left[\frac{ab}{(m+1)(m+2)} + \right. \\
 & + \left. \frac{ab}{(m+1)(n+2)} + \frac{\beta ab}{(m+1)(n+1)} + c \right] x^{3m+4} + \dots = \frac{a^\nu x^m}{m} + \\
 & + \frac{\nu a^{\nu-1}b}{2(m+1)} x^{2(m+1)} + \left[\frac{\nu(\nu-1)a^{\nu-2}b^2}{2(3m+4)} + \frac{\nu a^{\nu-1}c}{3m+4} \right] x^{3m+4} + \dots
 \end{aligned}$$

Equating now the coefficients of the terms the same degree of x of both sides of the equation, we obtain

$$a = (1-\nu)^{\frac{1}{1-\nu}},$$

$$\frac{a^2}{(m+1)(m+2)} + \frac{a^2}{2(m+1)^2} + b = \frac{\nu a^{\nu-1} b}{2(m+1)},$$

$$\frac{ab}{(m+1)(m+2)} + \frac{ab}{(2m+3)(2m+4)} + \frac{\beta ab}{(m+1)(2m+3)} + c =$$

$$= \frac{\nu(\nu-1)a^{\nu-2}}{2(3m+4)} b^2 + \frac{\nu a^{\nu-1} c}{3m+4}$$

.

Bearing in mind the value assumed above for m , we may, knowing ν , calculate the exponents m, n, p and the coefficients a, b, c entering into the expression for $\frac{d^2 z_0}{dx^2}$, $\frac{dz_0}{dx}$ and z_0 .

These expressions, for all powders taken, whose properties determine the values of β and of ν in the given expressions, afford very converging series for $x < 1$, so that in many cases for values limited to five-tenths we may reject the third and sometimes the second terms; but for values of $x > 1$, these series prove very divergent. In consequence of this, the deduced expressions may be employed for calculating z_0 and its derivatives only between the limits $x=0$ and $x=1$. For values of x greater than 1 we must have recourse to Taylor's theorem.

Let us assume that the functions $\frac{d^2 z_0}{dx^2}$, $\frac{dz_0}{dx}$ and z_0 are calculated for any value of the variable x . The change of these functions by $\Delta \frac{d^2 z_0}{dx^2}$, $\Delta \frac{dz_0}{dx}$ and Δz_0 corresponding to the change of the variable from x to $x+h$, will be

$$\Delta \frac{d^2 z_0}{dx^2} = h \frac{d^3 z_0}{dx^3} + \frac{h^2}{1.2} \frac{d^4 z_0}{dx^4} + \frac{h^3}{1.2.3} \frac{d^5 z_0}{dx^5} + \dots$$

$$\Delta \frac{dz_0}{dx} = h \frac{d^2 z_0}{dx^2} + \frac{h^2}{1.2} \frac{d^3 z_0}{dx^3} + \frac{h^3}{1.2.3} \frac{d^4 z_0}{dx^4} + \dots$$

$$\Delta z_0 = h \frac{dz_0}{dx} + \frac{h^2}{1.2} \frac{d^2 z_0}{dx^2} + \frac{h^3}{1.2.3} \frac{d^3 z_0}{dx^3} + \dots$$

In this way, in order to ascertain the desired variations of our functions, it becomes necessary to calculate the values of the derivatives to x of the third and higher degrees. For this, we take a few derivatives of the first of the auxiliary equations (66), and we then obtain a system of equations,

$$(1+z_0) \frac{d^3 z_0}{dx^3} + (1+\beta) \frac{dz_0}{dx} \frac{d^2 z_0}{dx^2} = \left(\frac{d^2 z_0}{dx^2} \right)^\nu,$$

$$(1+z_0) \frac{d^4 z_0}{dx^4} + (2+\beta) \frac{dz_0}{dx} \frac{d^3 z_0}{dx^3} + (1+\beta) \left(\frac{d^2 z_0}{dx^2} \right)^2 =$$

$$= \nu \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-1} \frac{d^3 z_0}{dx^3},$$

$$(1+z_0) \frac{d^5 z_0}{dx^5} + (3+\beta) \frac{dz_0}{dx} \frac{d^4 z_0}{dx^4} + (4+3\beta) \frac{d^2 z_0}{dx^2} \frac{d^3 z_0}{dx^3} =$$

$$= \nu(\nu-1) \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-2} \left(\frac{d^3 z_0}{dx^3} \right)^2 + \nu \left(\frac{d^2 z_0}{dx^2} \right)^{\nu-1} \frac{d^4 z_0}{dx^4},$$

.

In the first of the equations obtained the only unknown quantity is $\frac{d^3 z_0}{dx^3}$, and therefore it may be calculated. We may then calculate $\frac{d^4 z_0}{dx^4}$ from the second equation and so on. In this way we find all the elements necessary for calculating the desired variations of the function z_0 and its derivatives for a change of x from x to $x+h$, if the values of these be known for any one value of x . Having determined in this way the values of the function z_0 and its derivative for $x+h$, we may, proceeding in the same manner, ascertain the value of these functions for $x+2h$, and so on, employing each time as many terms in the expressions for the variations as will afford results of the desired degree of accuracy, to which it may be added that in order to diminish the number of the terms employed we may also diminish the value of h .

The results of these calculations are presented in Table XLV for black powders, in Table XLVI for chocolate powders, in Table XLVII for pyro-collodion powders, and in Table XLVIII for ballistite.

TABLE XLV.—For black powders, $\nu=0.52$; $\beta=0.07731$.

x	z_0	$\frac{dz_0}{dx}$	$\frac{d^2 z_0}{dx^2}$
0.25	0.00006	0.00099	0.01207
.50	.00102	.00829	.05107
.75	.00531	.02887	.11804
1.00	.01713	.06958	.21097
1.25	.04227	.13490	.32740
1.50	.08756	.23265	.45570
1.75	.16125	.36204	.57863
2.00	.27096	.51998	.68140
2.25	.42306	.69975	.75129
2.50	.62193	.89261	.78594
2.75	.86988	1.09034	.79072
3.00	1.16705	1.28638	.77427
3.25	1.51252	1.47658	.74487
3.50	1.90458	1.65840	.70891
3.75	2.34088	1.83092	.67056
4.00	2.77613	1.99384	.63237
4.50	3.69675	2.29171	.56005
5.00	4.90987	2.55364	.49746
5.50	6.24957	2.79004	.44585
6.00	7.69981	3.00283	.40819
7.00	10.90335	3.37171	.33931
8.00	14.43610	3.68621	.29260
9.00	18.26250	3.96050	.25764
10.00	22.34704	4.20368	.23025

TABLE XLVI.—For chocolate powders, $\nu=0.45$; $\beta=0.07554$.

x	z_0	$\frac{dz_0}{dx}$	$\frac{d^2z_0}{dx^2}$
0.25	0.00015	0.00241	0.02711
.50	.00219	.01702	.09498
.75	.01011	.05210	.19412
1.00	.02965	.11211	.28691
1.25	.05255	.20047	.33731
1.50	.13009	.30107	.46614
1.75	.22118	.43257	.58405
2.00	.34861	.59068	.67660
2.25	.51813	.76798	.73638
2.50	.73352	.95618	.76391
2.75	.99646	1.14789	.76549
3.00	1.30726	1.33761	.74950
3.25	1.66713	1.52184	.72230
3.50	2.06981	1.69844	.68954
3.75	2.51559	1.86659	.65473
4.00	3.00241	2.02582	.62001
4.50	4.09058	2.31926	.55445
5.00	5.31704	2.61089	.49826
5.50	6.68267	2.84761	.45006
6.00	8.16094	3.02415	.40975
7.00	11.41669	3.43914	.34842
8.00	15.02173	3.76348	.30317
9.00	18.93084	4.04846	.26867
10.00	23.10844	4.30289	.24160

TABLE XLVII.—For pyro-collodion powders, $\nu = 0.66$; $\beta = 0.20003$.

x	z_0	$\frac{dz_0}{dx}$	$\frac{d^2z_0}{dx^2}$
0.25	0.00000	0.00005	0.00071
.50	.00007	.00069	.00546
.75	.00052	.00342	.01797
1.00	.00215	.01063	.04188
1.25	.00645	.02498	.07317
1.50	.01543	.04882	.11782
1.75	.03193	.08565	.17717
2.00	.05964	.13909	.25050
2.25	.10314	.21249	.33633
2.50	.16753	.30790	.42549
2.75	.25868	.42466	.50610
3.00	.37982	.55972	.57089
3.25	.53811	.70837	.61424
3.50	.73469	.86449	.63530
3.75	.97074	1.02400	.63730
4.00	1.24660	1.18200	.62446
4.50	1.91390	1.48270	.57207
5.00	3.77362	1.75410	.51234
5.50	4.71281	1.99865	.46562
6.00	5.76846	2.22056	.42271
7.00	8.13280	2.86490	.35099

TABLE XLVIII.—For nitroglycerine powders, $\nu = 0.6$; $\beta = 0.15033$.

x	z_0	$\frac{dz_0}{dx}$	$\frac{d^2z_0}{dx^2}$
0.25	0.00001	0.00023	0.00316
.50	.00028	.00256	.01789
.75	.00176	.01056	.04917
1.00	.00641	.02880	.10020
1.25	.01744	.06247	.17266
1.50	.03937	.11678	.26514
1.75	.07950	.19612	.37184
2.00	.12764	.30275	.48162
2.25	.21828	.43723	.59377
2.50	.34720	.59794	.68775
2.75	.51869	.77612	.73227
3.00	.73582	.96154	.74615
3.25	.99947	1.14734	.73638
3.50	1.30912	1.32862	.71112
3.75	1.66318	1.50234	.67718
4.00	2.05949	1.66706	.63971
4.50	2.96367	1.96779	.56267
5.00	4.01504	2.23820	.49550
5.50	5.19362	2.47128	.43893
6.00	6.48202	2.67880	.39229
7.00	9.34415	3.03375	.32301
8.00	12.53026	3.33020	.27346
9.00	15.99042	3.58418	.23669
10.00	19.68793	3.80601	.20853

The deduced tables present the complete solution of the problem of interior ballistics. In fact, on the basis of the assumptions 50, 55, and 59 and bearing in mind that we are limited to the first term of expression 62 for z , we may write

$$l = l_{\Delta} z_0$$

$$v = K \frac{1}{3-2\nu} l_{\Delta} \frac{dz_0}{dx}$$

$$p = \frac{M d^2 l}{s dt^2} = \frac{M l_{\Delta}}{s} K \frac{2}{3-2\nu} \frac{d^2 z_0}{dx^2}$$

$$t = K \frac{1}{3-2\nu} x$$

The first expression gives the path traversed by the projectile as a function of z_0 , the values of which are given in the second column of the tables. The second expression (v) corresponds to the velocity of the projectile as a function of $\frac{dz_0}{dx}$, and is given in the third column of the tables. The third expression (p) is the pressure on the base of the projectile $\frac{d^2 z_0}{dx^2}$ and its value is given in the last column of the tables.

Finally, the last expression affords us the time t occupied by the shell in traversing the distance l . It is given as a function of x , whose value is shown in the first column of the tables.

As the quantities K (52), b (59), and l_{Δ} (45), are known for given conditions of firing, it follows that we may, by the method indicated, construct the curve of velocities of the projectile, of the pressures on its base, and of the times of its movement. In consequence of the assumptions made, however, the results obtained by calculation are sufficiently accurate only for the beginning of motion, up to the development of the maximum pressure, which corresponds to $\left(\frac{d^2 z_0}{dx^2}\right)_{\max}$. It is evident from the tables that the slower the powder the greater the distance ($l_{\Delta z_0}$) traversed by the projectile before the development of this maximum.

64. *General formulæ of Sarrau for initial velocities and pressures—The principle of similarity of guns.*

If we are not limited to the first term of function z (62), the expressions for velocity and pressure on the base of the projectile become, from 50 and 55,

$$v = K^{\frac{1}{3-2\nu}} l_{\Delta} \frac{dz}{dx}$$

$$p = \frac{M l_{\Delta}}{s} K^{\frac{2}{3-2\nu}} \frac{d^2 z}{dx^2}$$

Substituting here, instead of the derivatives, their equivalents 63, we obtain

$$v = l_{\Delta} K^{\frac{1}{3-2\nu}} \left(\frac{dz_0}{dx} + \varepsilon \frac{dz_1}{dx} + \varepsilon^2 \frac{dz_2}{dx} \right)$$

$$p = \frac{M l_{\Delta}}{s} K^{\frac{2}{3-2\nu}} \left[\frac{d^2 z_0}{dx^2} + \varepsilon \frac{d^2 z_1}{dx^2} + \varepsilon^2 \frac{d^2 z_2}{dx^2} \right]$$

The second members of these expressions are functions of x (54), and consequently the velocities and pressures will be functions of time. In order to express them as functions of the distance traversed, it is sufficient to assume that, from expression (62), x is deduced as a function of z of the form

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2,$$

and substitute this in the expressions found for v and for p , which consequently become functions of z , or, what is the same thing, functions of $\frac{l}{l_{\Delta}}$.

Let us consider more closely expression 62, which gives the distance traversed by the projectile as a function of elapsed time. The first term z_0 of its second member is determined by the first of equations (66) page 192, and is independent of λ and of μ . The second term, z_1

of the expression for z is determined by the second equation of system 66, which is lineal (compare the value of X_1 (65)) with respect to the variable z_1 , and as the term λX_0 , which is independent of it, is proportional to λ , the quantity z_1 is also proportional to λ . In consequence of this, the second terms of the factors included in the brackets in the equations for v and p are equal to the products of $\varepsilon \lambda$ into a certain function $\frac{l}{l_\Delta}$.

In this way, finally, the general formulæ of Sarrau for velocities and pressures may be written as

$$67. \quad \begin{cases} v = l_\Delta K^{\frac{1}{3-2\nu}} \left[F_0\left(\frac{l}{l_\Delta}\right) - \varepsilon \lambda F_1\left(\frac{l}{l_\Delta}\right) + \varepsilon^2 F_2\left(\frac{l}{l_\Delta}\right) \right], \\ p = \frac{M l_\Delta}{s} K^{\frac{2}{3-2\nu}} \left[f_0\left(\frac{l}{l_\Delta}\right) - \varepsilon \lambda f_1\left(\frac{l}{l_\Delta}\right) + \varepsilon^2 f_2\left(\frac{l}{l_\Delta}\right) \right]. \end{cases}$$

By employing these formulæ we may deduce the principle of similarity of guns.

We observe that the functions F and f and also λ are numerical quantities, independent of the caliber of the gun; therefore if ε and the factors before the brackets in the second members are also independent of the caliber, then in guns of different calibers the quantities v and p will be equal for all ratios of $\frac{l}{l_\Delta}$, i. e., for paths l traversed by the projectile, and proportional to the calibers, if l_Δ is proportional to the caliber.

The quantities ε (60), $l_\Delta K^{\frac{1}{3-2\nu}}$ and $\frac{M l_\Delta}{s} K^{\frac{2}{3-2\nu}}$ (the quantity K has the value 52) will not depend on the caliber if the bores of the guns are geometrically similar, i. e., if their dimensions are proportional to the calibers and if the guns are *similarly loaded*, i. e., if the qualities f , of the powder being the same, the grains are similar to one another (i. e., x , λ , μ are the same), and the least dimensions (α) of the grains* are proportional to the calibers; in addition to this, the weights of the charges and projectiles should be proportional to the cubes of the calibers. In this case, l_Δ is evidently proportional to the caliber.

This principle of similarity of guns possesses a significance in practical applications, and, therefore, the possibility of theoretically deducing it from the general equations is especially important.

* What has been stated in paragraph 55 in relation to smoking powders must be borne in mind.

65. *General binomial formulæ of Sarrau for velocities and pressures.*

The quantity ε (60) is, generally speaking, quite small, and therefore it may be assumed that the first two members of expression 67 afford sufficiently approximate values of velocities and pressures, and therefore formulæ 67 assume the form

$$v = l_{\Delta} K^{\frac{1}{3-2\nu}} \left[F_0\left(\frac{l}{l_{\Delta}}\right) - \varepsilon \lambda F_1\left(\frac{l}{l_{\Delta}}\right) \right]$$

$$p = \frac{M l_{\Delta}}{s} K^{\frac{2}{3-2\nu}} \left[f_0\left(\frac{l}{l_{\Delta}}\right) - \varepsilon \lambda f_1\left(\frac{l}{l_{\Delta}}\right) \right]$$

Substituting here for K and ε their values from 52 and 60, we obtain

$$68. \quad \begin{cases} v = l_{\Delta} \left(\frac{f \omega \kappa 2 A}{M l_{\Delta}^2 a} \right)^{\frac{1}{3-2\nu}} \left(\frac{M l_{\Delta}}{s} \right)^{\frac{\nu}{3-2\nu}} \left[F_0\left(\frac{l}{l_{\Delta}}\right) - \right. \\ \left. - \frac{\lambda 2 A}{a} \left(\frac{f \omega \kappa 2 A}{M l_{\Delta}^2 a} \right)^{\frac{2\nu-1}{3-2\nu}} \left(\frac{M l_{\Delta}}{s} \right)^{\frac{2\nu}{3-2\nu}} F_1\left(\frac{l}{l_{\Delta}}\right) \right] \\ p = \frac{M l_{\Delta}}{s} \left(\frac{f \omega \kappa 2 A}{M l_{\Delta}^2 a} \right)^{\frac{2}{3-2\nu}} \left(\frac{M l_{\Delta}}{s} \right)^{\frac{2\nu}{3-2\nu}} \left[f_0\left(\frac{l}{l_{\Delta}}\right) - \right. \\ \left. - \frac{\lambda 2 A}{a} \left(\frac{f \omega \kappa 2 A}{M l_{\Delta}^2 a} \right)^{\frac{2\nu-1}{3-2\nu}} \left(\frac{M l_{\Delta}}{s} \right)^{\frac{2\nu}{3-2\nu}} f_1\left(\frac{l}{l_{\Delta}}\right) \right] \end{cases}$$

66. *Formula for maximum pressure upon the base of the projectile.*

The pressure corresponding to the maximum acceleration of the projectile, i. e., the maximum pressure on the base of the projectile, is of greatest practical importance to us. It is developed in the first moments of the combustion of the charge, when the evolution of the gases is almost constant, and the shell has traversed only a small portion of the length of the bore; in addition to this, ε , generally speaking, is small, and therefore, for values of p close to the maximum, we may equate the term containing f , $\left(\frac{l}{l_{\Delta}}\right)$ to 0.

If we subsequently assume that the maximum value for the pressure p as developed for a certain $\frac{l_2}{l_{\Delta}} = z_2$, and if we denote $f_0\left(\frac{l_2}{l_{\Delta}}\right)$ by Ξ then formulæ 68 for p assume the form

$$p = \Xi_1 \frac{M l_{\Delta}}{s} \left(\frac{f \omega \kappa 2 A}{M l_{\Delta}^2 a} \right)^{\frac{2}{3-2\nu}} \left(\frac{M l_{\Delta}}{s} \right)^{\frac{2\nu}{3-2\nu}}$$

or, denoting the maximum pressure on the base of the projectile by P_p we obtain, after reduction,

$$69 \quad P_p = \Xi_1 \left(\frac{f \omega \kappa 2 A}{M l_{\Delta}^2 a} \right)^{\frac{2}{3-2\nu}} \left(\frac{M l_{\Delta}}{s} \right)^{\frac{3}{3-2\nu}}$$

According to the assumption made, the pressure P_p is developed when the projectile has traversed a path

$$l_2 = z_2 l_\Delta$$

whence it follows that this value of the path traversed by the projectile up to the development of the maximum pressure in the bore, is proportional to the reduced length of the unoccupied initial volume of the chamber l_Δ , page 177, and therefore depends, for the given gun, on the reduced length of the chamber, on the density of loading and on the density of the powder. Other elements of firing, such, e. g., as the weight of the charge, would appear to have no direct influence on the quantity l_2 considered, but this is in general untrue and may be assumed only as a first approximation to formula 69, and only as a consequence of it, in those cases which do not relate to extreme conditions of firing.

A more exact value of l_2 , i. e., the length of the path traversed by the projectile up to the time of formation of maximum pressure in the bore, may be obtained from the general binomial expression 68 by equating its first derivative for $\frac{l}{l_\Delta}$ to 0. Let the exact value z_2 corresponding to P_p be $z_2 + h$, where we may regard h as a small quantity of the first order; then

$$f'_0(z_2 + h) - \varepsilon \lambda f'_1(z_2 + h) = 0$$

Neglecting terms of the second order and observing that $f'_0(z_2) \cong 0$, we

$$h = \varepsilon \lambda \frac{f'_1(z_2)}{f''_0(z_2)}$$

obtain where $f''_0(z_2)$ is negative, since $f_0(z_2)$ is a maximum; and where $f'_1(z_2)$ is positive, since the function f_1 is increasing. In this manner h becomes a negative quantity, and, consequently, the approximate value $l_2 = z_2 l_\Delta$ is greater than that resulting from the binomial expression for p . In a similar manner the pressure P_p is somewhat greater, since in deducing it the negative term of the second equation was neglected (68).

67. *The influence of the characteristics of a powder on its action in the bore of the gun.*

Of the elements determining the properties of a powder in formulæ 68 and 69 we shall consider as constant those quantities upon which the velocity of combustion of the substance depends, i. e., the values A and ν . Of the remaining elements, the quantities f , κ and a only enter into the formula for pressures (69) in the form of a factor $\frac{f\kappa}{a}$,

besides which, in the formula for velocities (68) the factor $\frac{\lambda}{a}$ enters.

It is included in the negative term of the expression for velocity and therefore it is useful to employ those varieties of powders for which $\frac{\lambda}{a}$ is as small as possible for the same value of the factor $\frac{f\kappa}{a}$.

Assume, then, other conditions being equal, that we have two powders the variable characteristics of which are the following:

$$\begin{array}{c} f, \kappa, \lambda, a \\ f', \kappa', \lambda', a' \end{array}$$

For equality of pressures it is necessary that

$$\frac{f\kappa}{a} = \frac{f'\kappa'}{a'}$$

Under this condition the first terms of the expressions for velocities remain the same in both cases, and the ratio of the negative terms becomes

$$\frac{\lambda'}{a'} \cdot \frac{a}{\lambda}$$

If this ratio is less than unity, the second powder gives greater velocity for the same pressure, i. e., is more suitable, and conversely.

INFLUENCE OF THE FORCE OF THE POWDER.

Let us assume that the forms of the grains of the varieties of powder compared are similar to one another, i. e., $\kappa = \kappa'$ and $\lambda = \lambda'$, but that the values of f and a are different. In this case the condition of the equality of pressures reduces to the following:

$$\frac{f}{a} = \frac{f'}{a'}$$

or

$$\frac{a'}{a} = \frac{f'}{f}$$

The ratio of the negative terms in the formula for velocities will consequently equal $\frac{f}{f'}$. Hence it follows that the stronger powder gives the greater velocity for the same pressure if the least dimension (a) of the grains of the powder is proportional to the force of the powder.

THE INFLUENCE OF FORM AND DIMENSIONS OF GRAIN FOR THE SAME POWDER.

In this case the condition of equality of pressures reduces to the following:

$$\frac{\kappa}{a} = \frac{\kappa'}{a'}$$

or

$$\frac{a'}{a} = \frac{\kappa'}{\kappa}$$

The ratio of the negative members in the formula for velocities consequently becomes

$$\frac{\lambda' \kappa}{\lambda \kappa'} = \frac{\lambda' \kappa}{\kappa' \lambda}$$

Let us compare grains of different shape in this relation, assuming as the unit of comparison, spherical grains, for which, according to Table XXI, we have

$$\frac{\lambda}{\kappa} = \frac{1}{3}$$

(a) CYLINDRICAL GRAINS WITH ONE CHANNEL.

For such grains of any desired dimensions, the ratio interesting us is, in accordance with the same table,

$$\frac{\kappa'}{\lambda'} = \frac{(\alpha+1)^2}{\alpha}$$

Consequently, the ratio of the negative terms in the formulæ for velocities is

$$\frac{\lambda'}{\kappa'} \frac{\kappa}{\lambda} = 3 \frac{\alpha}{(\alpha+1)^2}$$

which is always less than unity—i. e., for spherical grains the negative term considered is greater than for cylindrical grains with a channel, and therefore the latter grain is more suitable than the former, and becomes the more suitable, the greater its *extent*.

Thus, if we assume that the value of the negative term in the expression for velocities to equal 100 meters per second for the spherical form of grain, the value of the same term for cylindrical grains with a channel, other conditions being equal, will be

<25 m. per sec. for $\alpha=0.1$, max. extent of grain	10
<75 m. per sec. for $\alpha=1.0$, max. extent of grain	1
<67 m. per sec. for $\alpha=2$, max. extent of grain	2
<25 m. per sec. for $\alpha=10$, max. extent of grain	10
<3 m. per sec. for $\alpha=100$, max. extent of grain	100

(b) DENSE CYLINDRICAL GRAIN.

We have for these (from Table XXI)

$$\frac{\kappa'}{\lambda'} = \frac{4(\alpha+1)^2}{\alpha(\alpha+4)}$$

consequently

$$\frac{\lambda}{\kappa} \frac{\kappa'}{\lambda'} = \frac{3}{4} \frac{\alpha(\alpha+4)}{(\alpha+1)^2}$$

This ratio equals unity for $h = 2r$, i. e., for $\alpha = 2$; in this case the cylindrical grain is not even as suitable as the spherical.

For other dimensions dense cylindrical grains are more suitable than spherical, although not to the same degree as the grains with the channel. In fact, if we assume the value of the negative term for spherical grains as 100 m. per second, the value of the same term for dense cylindrical grains will be—

<26 m. per sec. for $\alpha=0.1$, max. extent of grain	20
<94 m. per sec. for $\alpha=1$, max. extent of grain	2
100 m. per sec. for $\alpha=2$, max. extent of grain	1
96 m. per sec. for $\alpha=4$, max. extent of grain	2
<87 m. per sec. for $\alpha=10$, max. extent of grain	5
<77 m. per sec. for $\alpha=100$, max. extent of grain	50

An analogous deduction is obtained from comparing parallelopipedic grains, and therefore we may say, in general, that in the ballistic relation (compare paragraph 30) grains are the more suitable the greater that their extent in two directions is. It is evident that these deductions relate to powders that burn in concentric layers, such as the smokeless powders employed at the present time. In their application to such powders as smoking powders these conceptions possess only a theoretical value.

68. *Determination of the form of the functions F_0 and F_1 ; substitution of l_Δ in expression (68) for velocities.*

In deducing these functions Sarrau observed the following: The reduced length of the unoccupied initial volume of the chamber l_Δ may, generally speaking, be made as small as desired, and may be even reduced to 0, and the magnitude of the velocity for every value of l_Δ should approach a limit. As in expression 68 for v we multiply F_0 by $l_\Delta^{\frac{1-\nu}{3-2\nu}}$ and F_1 by $l_\Delta^{\frac{3(1-\nu)}{3-2\nu}}$, so the limiting value of v for $l_\Delta = 0$ is obtained by putting

$$F_0\left(\frac{l}{l_\Delta}\right) = B_1\left(\frac{l}{l_\Delta}\right)^{\frac{1-\nu}{3-2\nu}}$$

and

$$F_1\left(\frac{l}{l_\Delta}\right) = D_1\left(\frac{l}{l_\Delta}\right)^{\frac{3(1-\nu)}{3-2\nu}}$$

and consequently

$$\frac{F_1\left(\frac{l}{l_\Delta}\right)}{F_0\left(\frac{l}{l_\Delta}\right)} = D_2\left(\frac{l}{l_\Delta}\right)^{\frac{2(1-\nu)}{3-2\nu}}$$

where B_1 , D_1 , and D_2 are constant factors.

If in expression 68 we remove function F_0 from the brackets, substitute in place of F_0 and F_1 their above-deduced values, then l_Δ in the factor in front of the brackets is abbreviated. In view of this, Sarrau assumes, for greater generalization,

$$F_0\left(\frac{l}{l_\Delta}\right) = B_2\left(\frac{l}{l_\Delta}\right)^\gamma$$

where the exponent γ is determined by experiment.

Substituting this in equation 68, we obtain

$$v = B_2 l_\Delta \left(\frac{2fA\kappa\omega}{aMl_\Delta^2}\right)^{\frac{1}{3-2\nu}} \left(\frac{Ml_\Delta}{s}\right)^{\frac{\nu}{3-2\nu}} \left(\frac{l}{l_\Delta}\right)^\gamma \left[1 - D_2 \frac{2A\lambda}{a} \left(\frac{2fA\kappa\omega}{aMl_\Delta^2}\right)^{\frac{2\nu-1}{3-2\nu}} \left(\frac{Ml_\Delta}{s}\right)^{\frac{2\nu}{3-2\nu}} \left(\frac{l}{l_\Delta}\right)^{\frac{2(1-\nu)}{3-2\nu}} \right]$$

Substituting in this formula instead of l the value L , i. e., the length of the whole path traversed by the projectile in the bore of the gun, we obtain an expression for the initial velocity V_0 .

We observe that by multiplying and dividing the expression l_Δ , page 177, by $\frac{\delta}{\Delta}$, we may give it the following form

$$l_\Delta = \frac{\omega \delta}{s \Delta^2} \cdot \frac{\Delta}{\delta} \left[1 - \frac{\Delta}{\delta} \right]$$

Under ordinary conditions of firing $\frac{\Delta}{\delta}$, generally speaking, is close to one-half, and as the product $\frac{\Delta}{\delta} \left[1 - \frac{\Delta}{\delta} \right]$ then attains its greatest value (one-fourth); therefore, in accordance with Sarrau's assumption, we may put $l_\Delta = \frac{\omega \delta}{s \cdot 4 \Delta^2}$. This considerably simplifies the formulæ for velocities and pressures, and at the same time gives values for l_Δ close to the truth, and for small variations in densities of loading from $\Delta = \frac{\delta}{2}$.

Substituting this, and also the value M (47) in formula 69 for pressure, and in the above-deduced formula for velocity, by replacing l for L in it, we obtain

$$70 \left\{ \begin{aligned} V_0 &= B_3 \left(\frac{2f\Delta\kappa}{a} \right)^{\frac{1}{3-2\nu}} \frac{1}{\delta} \frac{1-\nu}{3-2\nu} - \gamma \frac{\omega}{s} \frac{2-\nu}{3-2\nu} - \gamma \frac{2\gamma - \frac{2(1-\nu)}{3-2\nu} L}{\Delta} \frac{\gamma}{\delta} \left[1 - \frac{1}{s^{\frac{1}{3-2\nu}}} - \gamma \frac{1-\nu}{G^{\frac{1}{3-2\nu}}} \right] \\ &\quad - D_3 \frac{2A\lambda}{a} \left(\frac{2f\Delta\kappa}{a} \right)^{\frac{2\nu-1}{3-2\nu}} \frac{2\nu-1}{\omega} \frac{2\nu-1}{3-2\nu} \frac{1}{G} \frac{2(1-\nu)}{3-2\nu} \frac{L}{\delta} \frac{2\nu}{s^{\frac{2\nu}{3-2\nu}}} \left. \right\} \\ P_P &= \Xi_2 \left(\frac{2f\Delta\kappa}{a} \right)^{\frac{2}{3-2\nu}} \frac{1}{\delta} \frac{1}{3-2\nu} (\omega G) \frac{\frac{1}{3-2\nu} \frac{2}{3-2\nu}}{\frac{2}{s^{\frac{2}{3-2\nu}}}} \end{aligned} \right.$$

in which the following are constants:

$$71 \left\{ \begin{aligned} B_3 &= B_2 4^{\gamma - \frac{1-\nu}{3-2\nu}} \left(\frac{g}{\varphi} \right)^{\frac{1-\nu}{3-2\nu}} \\ D_3 &= D_2 \left(\frac{\varphi}{g} \right)^{\frac{1}{3-2\nu}} \\ \Xi_2 &= \Xi_1 \left(4 \frac{\varphi}{g} \right)^{\frac{1}{3-2\nu}} \end{aligned} \right.$$

Finally, in formulæ 70 we substitute for Δ , s , and L their equivalents (pp. 176, 177), after which the equations assume the form

$$\begin{aligned}
 & V_0 = B \left(\frac{2f\Lambda\kappa}{a} \right)^{\frac{1}{3-2\nu}} \delta^{\frac{1-\nu}{3-2\nu} - \gamma} \times \\
 & \times \frac{\omega^{\frac{\nu}{3-2\nu} + \gamma} (L_0 - l_0)^{\gamma}}{\frac{2}{c'^{\frac{3-2\nu}{2} - 2\nu}} \frac{1-\nu}{G^{\frac{3-2\nu}{2}}} \frac{4\gamma - \frac{4(1-\nu)}{3-2\nu}}{d'^{\frac{4\gamma - \frac{4(1-\nu)}{3-2\nu}}{2\gamma - \frac{2(1-\nu)}{3-2\nu}}}} \frac{1}{l_0}} \left[1 - \right. \\
 & 72 \left. - D \left(\frac{2f\Lambda\kappa}{a} \right)^{\frac{2\nu-1}{3-2\nu}} \frac{2\Lambda\lambda}{a} \omega^{\frac{2\nu-1}{3-2\nu}} G^{\frac{1}{3-2\nu}} \frac{(L_0 - l_0)}{\frac{4\nu}{c'^{\frac{4\nu}{3-2\nu}}}} \frac{2(1-\nu)}{3-2\nu} \right] \\
 & P_P = \Xi \left(\frac{2f\Lambda\kappa}{a} \right)^{\frac{2}{3-2\nu}} \delta^{\frac{-1}{3-2\nu}} \frac{\frac{3}{\omega^{\frac{3}{3-2\nu}}} G^{\frac{1}{3-2\nu}}}{\frac{4}{c'^{\frac{4}{3-2\nu}}} \frac{4}{d'^{\frac{4}{3-2\nu}}} \frac{2}{l_0^{\frac{2}{3-2\nu}}}}
 \end{aligned}$$

in which the constant factors are

$$\begin{aligned}
 & B = B_2 \pi^{\frac{3\gamma-1}{4} + \frac{2-\nu}{3-2\nu} - \gamma} \left(\frac{g}{\varphi} \right)^{\frac{1-\nu}{3-2\nu}} \\
 & 73 \left\{ \begin{aligned} D &= D_2 \left(\frac{4}{\pi} \right)^{\frac{2\nu}{3-2\nu}} \left(\frac{\varphi}{g} \right)^{\frac{1}{3-2\nu}} \\ \Xi &= \Xi_2 \left(\frac{4}{\pi} \right)^{\frac{4}{3-2\nu}} \left(\frac{4\varphi}{g} \right)^{\frac{1}{3-2\nu}} \end{aligned} \right.
 \end{aligned}$$

69.—Sarrau's formulæ for maximum pressure P_B on the base of the bore of the gun.

Besides the importance of knowing the maximum pressure on the base of the projectile, it is exceedingly important to possess formulæ as accurate as possible for determining the greatest pressure on the base of the bore P_B , as this determines the resistance of the gun at the part where the powder chamber is located; besides this, it is more convenient in practice to determine the pressure in the bore of the gun than at the base of the projectile, in consequence of which the latter determinations are employed only in exceptional cases.

In order to deduce pressures at the base of the bore, it is necessary to determine the relationship between pressures on the projectile and in the various sections of the bore in rear of the projectile.

Let us assume that the gun does not recoil, and that in the interval of time from the beginning of movement of the projectile the latter is found at a distance L from the base of the bore of the gun. We shall also assume that the cross section of the bore equals s (page 176). Let us consider the portion of the bore-volume in rear of the projectile, between its base and any section of the bore. Let there be

included in this volume a certain quantity ω_n of products of combustion and also unconsumed powder. This mass moves with the projectile and exists under the influence of exterior forces; the pressure of the gases P_s on the section of the bore in question and the reaction of the projectile equal the pressure $P'_p s$ acting upon it. These forces are exerted in opposite directions parallel to the bore of the gun. Let

us now consider an element whose mass is $\frac{d\omega_n}{g}$ and whose acceleration in the direction of the bore is j . The whole force of inertia of the products of combustion and of the unburned portion of the charge included in the given section of the bore will be, $\frac{1}{g} \int_0^{\omega_n} j d\omega_n$. Consequently,

$$(P - P'_p)_s = \frac{1}{g} \int_0^{\omega_n} j d\omega_n$$

The value of the acceleration changes in accordance with a law that is unknown to us in passing from one element ω_n to another, but in any case j is a continuous function of ω and therefore

$$(P - P'_p)_s = \frac{j_1}{g} \int_0^{\omega_n} d\omega_n = j_1 \frac{\omega_n}{g}$$

where j is the mean arithmetical value of all the accelerations j on varying ω from 0 to ω_n . Let us assume besides this, that

$$j_1 = \Theta \frac{P'_p}{m} \times s = \Theta g \frac{P'_p}{G} \times s$$

where Θ represents the ratio of the mean arithmetical acceleration j_1 to the actual acceleration of the projectile $\frac{P'_p}{m} \times s$ at the moment in question.

Substituting this in the equation found above we obtain

$$P = P'_p \left(1 + \Theta \frac{\omega_n}{G} \right)$$

The value Θ is unknown. Generally speaking, it changes constantly throughout the whole time on passing from one element of mass $\frac{\omega_n}{g}$ to another, but under the condition, however, that $\Theta > 0$, i. e.,

that it is a positive quantity, since the velocities of all the elements of the charge increase with increase of time and consequently the mean acceleration, which is proportional to the Θ , is always positive. In

consequence of this, the ratio $\frac{P}{P'_p}$ will be an increasing function $\frac{\omega_n}{g}$.

Considering the moment when the maximum pressure $P_p' = P_p$ is exerted on the projectile, we find the maximum pressure in the bore of the gun; in this case $\omega_n = \omega$ and $P = P_B$. From the above deduced expression, we have

$$74 \quad P_B = P_p \left(1 + \Theta \frac{\omega}{G} \right)$$

Substituting here instead of P_p its value 72 we obtain

$$P_B = \Xi \left(1 + \Theta \frac{\omega}{G} \right) \left(\frac{2 f A \kappa}{a} \right)^{\frac{2}{3-2\nu}} \frac{1}{\delta^{\frac{1}{3-2\nu}}} \frac{\omega}{\frac{4}{\epsilon^{3-2\nu}} \frac{4}{d^{3-2\nu}} \frac{2}{G^{3-2\nu}}}$$

For convenience in calculation we replace this binominal expression by a monominal one, making use of the capability to change any function within given limits, by a certain power of a variable.

In fact, we assume

$$y = f(x)$$

It is always possible to select such another curve whose equation is

$$y = Nx^n$$

so that both, for a certain value of the variable x , possess a common tangent: in such a case the second expression may be substituted for $f(x)$ with sufficient accuracy for values of the variable near to $x = x_1$.

In view of what is said, we have

$$Nx^n = f(x).$$

$$nNx^{n-1} = f'(x)$$

whence

$$75 \quad \begin{cases} N = \frac{f(x)}{x^n} \\ n = \frac{xf'(x)}{f(x)} \end{cases}$$

In the given case, assuming $x = \frac{\omega}{G}$ and $f(x) = 1 + \Theta \frac{\omega}{G}$, we obtain

$$N = \frac{1 + \Theta \frac{\omega}{G}}{\left(\frac{\omega}{G} \right)^n}$$

$$n = \frac{\Theta \frac{\omega}{G}}{1 + \Theta \frac{\omega}{G}}$$

If $f(x)$ is unknown, as is usually the case, the values N and n are determined from experimental data.

Proceeding in this manner, we may consequently write

$$76 \quad \dots \quad P_B = \Xi_u \left(\frac{2f\Lambda\kappa}{a} \right)^{\frac{2}{3-2\nu} - \frac{1}{3-2\nu} \frac{3}{\omega} + n \frac{1}{G} \frac{1}{3-2\nu} - n} \frac{4}{e' \frac{4}{3-2\nu} d' \frac{4}{3-2\nu} l_0 \frac{2}{3-2\nu}}$$

where

$$\Xi_u = \Xi N$$

The value Θ entering into formula 74 may be found only approximately, on the basis of the results of determining pressures in various sections of the bore of the gun. In the case of smoking powders we may, in accordance with Sarrau's assumption, put $\Theta = \frac{2}{3}$, and in the case of smokeless powders we may put $\Theta = \frac{1}{2}$. (See Part II of the present work.)

70. *Maximum of velocity and its reduction to 0 for certain dimensions a_1 and a_0 of thickness of grain and variable pressure, P_B .*

On increasing the rapidity of combustion of a powder, i. e., on diminishing a , the velocity of the projectile should increase, tending (all other conditions of firing being the same) toward a certain limit. The value of this latter is determined by the velocity which the projectile acquires when the charge is completely consumed at the instant that the projectile begins to move. Such cases were separately considered (paragraph 61), and expressions for velocities (43), page 184, were obtained.

The binomial formula 72, page 206, for velocities, was deduced under condition of progressive combustion of the powder; on diminishing a it does not tend toward any special limit, but passes through a certain maximum. In fact, considering all elements of firing, excepting the least dimension a of the grain, as constant, formulæ 72 becomes a function of one variable a and may be written as

$$V_0 = B_1 a^{-\frac{2}{3-2\nu}} \left(1 - D_1 a^{-\frac{2}{3-2\nu}} \right)$$

where the remaining factors of expression 72 are assembled in the coefficients B_1 and D_1 .

Further on we shall frequently be obliged to consider similar equations and we shall therefore investigate their general form

$$V_0 = B_1 x^p (1 - D_1 x^q)$$

The first derivative, $\frac{dV_0}{dx} = p B_1 x^{p-1} \left[1 - \frac{p+q}{p} D_1 x^q \right]$ reduces to 0 for

$$77 \quad x_0 = \left(\frac{1}{D_1} \frac{p}{p+q} \right)^{\frac{1}{q}}$$

For such a value of x

$$\left(\frac{d^2 v_0}{dx^2}\right)_{x=x_0} = -B_1 p q \left(\frac{1}{D_1 p + q}\right)^{\frac{p-2}{q}}$$

In this manner the expression for V_0 attains a maximum for $x=x_0$ (77) independently of the absolute values of p , q , B_1 and D_1 (the last two are always positive quantities), if p and q possess the same sign.

On the basis of this, V_0 attains a maximum for the following value of a_1 of thickness of grain:

$$78. \quad a_1 = (3 D_1)^{\frac{3-2\nu}{2}} = [3 D. 2 A \lambda]^{\frac{3-2\nu}{2}} (2 f A x)^{\frac{2\nu-1}{2}} \frac{2\nu-1}{\omega} \frac{2\nu-1}{2} G^{\frac{1}{2}} \frac{(L_0-l_0)^{(1-\nu)}}{c'^{2\nu}}$$

whence it is evident that $\left(\frac{a_1}{a}\right)^{\frac{2}{3-2\nu}}$ equals to the treble of the negative term in brackets of expression 72 for velocity. Replacing the latter by this ratio and multiplying V_0 by $\left(\frac{a_1}{a}\right)^{\frac{1}{3-2\nu}}$, we obtain still another expression for initial velocity, as follows:

$$79. \quad V_0 = B \left(\frac{2 f A \kappa}{a_1}\right)^{\frac{1}{3-2\nu}} \delta^{\frac{1-\nu}{3-2\nu}-\gamma} \times \\ \times \frac{\omega^{\frac{\nu}{3-2\nu}+\gamma} (L_0-l_0)^{\gamma}}{c'^{\frac{2}{3-2\nu}-2\gamma} G^{\frac{1-\nu}{3-2\nu}} d'^{4\gamma-\frac{4(1-\nu)}{3-2\nu}} l_0^{2\gamma-\frac{2(1-\nu)}{3-2\nu}}} \left(\frac{a_1}{a}\right)^{\frac{1}{3-2\nu}} \left[1 - \frac{1}{3} \left(\frac{a_1}{a}\right)^{\frac{2}{3-2\nu}}\right]$$

The maximum value for initial velocities consequently becomes

$$(V_0)_{\max.} = \frac{2}{3} B \left(\frac{2 f A \kappa}{a_1}\right)^{\frac{1}{3-2\nu}} \delta^{\frac{1-\nu}{3-2\nu}-\gamma} \times \\ \times \frac{\omega^{\frac{\nu}{3-2\nu}+\gamma} (L_0-l_0)^{\gamma}}{c'^{\frac{2}{3-2\nu}-2\gamma} G^{\frac{1-\nu}{3-2\nu}} d'^{4\gamma-\frac{4(1-\nu)}{3-2\nu}} l_0^{2\gamma-\frac{2(1-\nu)}{3-2\nu}}}$$

The pressure P_B (76) increases to an unlimited extent with the diminution of the dimension a of the grain, while in fact it should tend toward a certain limit which will be reached when all the powder in the chamber is burned at the instant when the projectile begins to move.

Expression 79 shows moreover, that the velocity V_0 not only passes through a maximum for $a=a_1$ but also reduces to 0 for

$$a_0 = \frac{a_1}{\frac{3}{2}}$$

Both these properties of the binomial formula arise from the fact that the assumptions upon which it was based are not wholly in accordance with conditions actually obtaining.

We finally observe that the value a_1 (78) depends not only on the properties of the powder, the weight of the projectile, and the caliber of the gun, but also, considering the value v , on the weight of the charge and the length of the path, $L_0 - l_0$.

In any case in similar guns with a given kind of powder and shape of grain, and with weights of charges and projectiles proportional to the cubes of the calibers, the thicknesses a_1 (78) of the grain are proportional to the calibers.

71. The modulus of a powder.

Sarrau called the ratio $\frac{a_1}{a}$ the modulus of the powder. That is, the modulus of a powder denotes the ratio of the *least* thickness of grain (a_1) to the thickness (a) of the grain of a given powder. This ratio serves as a measure of the "quickness" of a powder for a given caliber of gun—i. e., as a measure of the rapidity of its combustion in the bore of the gun under different conditions.

In fact, a powder burns the more slowly in a given gun the greater the duration of its combustion in comparison with the duration of combustion of grains of the dimensions a_1 —i. e., the more $a > a_1$. Whence it follows that we may actually assume the modulus of the powder—i. e., the ratio $\frac{a_1}{a}$ —which we will denote by Ω as a measure of the "quickness" of the combustion of the powder in a given gun. We may establish a classification of powders in accordance with the value of the modulus, as shown in Table XLVIII.

TABLE XLVIII.—Classification of quickness of powders in accordance with the value of the modulus.

Value of the modulus, Ω	Corresponding classification of powder for a given gun.
$\Omega > 1.0$	Extremely quick.
$.1 \geq \Omega > .9$	Very quick.
$.9 \geq \Omega \geq .8$	Quick.
$.8 > \Omega \geq .7$	Mean.
$.7 > \Omega \geq .6$	Slow.
$.6 > \Omega \geq .35$	Very slow.
$\Omega < .35$	Extremely slow.

We may consider two powders as possessing the same degree of "quickness" (burning at the same rate) in two different guns for which they are intended, if their moduli are the same. From what has been said above in relation to the value a_1 , it follows that the same powder proves of different degrees of "quickness" in guns of different calibers, and even in one and the same gun under different conditions of firing.

Dividing both members of expression 78 by a , we obtain

$$80. \quad \Omega = \left(3D \frac{2\Lambda\lambda}{a} \right)^{\frac{3-2\nu}{2}} \left(\frac{2f\Lambda\kappa}{a} \right)^{\frac{2\nu-1}{2}} \frac{\omega^{\frac{2\nu-1}{2}} G^{12} (L_0 - l_0)^{(1-\nu)}}{c'^{2\nu}}$$

The latter shows that the modulus of a powder is a constant quantity in guns of different calibers, if for the given variety of powder and form of grain the guns are similar and are similarly loaded, and if the least dimensions of the grain a are proportional to the calibers.

72. *The most suitable modulus of power.*

As we have neither changed charge nor the dimensions of grain, the whole pressure on the base of the bore of the gun P_B should not exceed, on firing, a certain maximum, permissible limit, determined by the strength of the gun; and at the same time, in order to develop maximum velocities, the pressure should not be less than this quantity. Therefore, assuming P_B as a constant quantity for a given gun, we obtain from 76, page 209, the expression for weight of charge.

$$81. \quad \omega = \Xi_n^{\frac{3\nu-3}{3+n(3-2\nu)}} \left(\frac{a}{2f\Lambda\kappa} \right)^{\frac{2}{3+n(3-2\nu)}} \delta^{\frac{1}{3+n(3-2\nu)}} \times \\ \times P_B^{\frac{3-2\nu}{3+n(3-2\nu)}} \frac{(c'd')^{\frac{4}{3+n(3-2\nu)}} l_0^{\frac{2}{3+n(3-2\nu)}}}{G^{\frac{1-u(3-2\nu)}{3+u(3-2\nu)}}}$$

For any limiting value of the elements entering into this expression, we obtain the limiting value for the weight of the charge.

Substituting this in formula 79, page 210, in place of ω and observing that $\frac{a_1}{a} = \Omega$, we obtain

$$82. \quad V_0 = \frac{B}{\Xi_n^{\frac{\nu+\gamma(3-2\nu)}{3+n(3-2\nu)}}} \left(\frac{2f\Lambda\kappa}{a_1} \right)^{\frac{1+n-2\gamma}{3+n(3-2\nu)}} \times \\ \times \delta^{\frac{1+n(1-\nu)-2\gamma-n\gamma(3-2\nu)}{3+n(3-2\nu)}} P_B^{\frac{\nu+\gamma(3-2\nu)}{3+n(3-2\nu)}} \times \\ \times \frac{(L_0 - l_0)^{\gamma} c'^{\frac{2\gamma[5+n(3-2\nu)]-(1+n)}{3+n(3-2\nu)}}}{(d'^2 l_0)^{\frac{\gamma[2+n(3-2\nu)]+vn-(1+n)}{3+n(3-2\nu)}}} \frac{G^{\frac{\gamma[1-n(3-2\nu)]+(1+n)}{3+n(3-2\nu)}}}{\times} \\ \times \Omega^{\frac{1+n-2\gamma}{3+n(3-2\nu)}} \left[1 - \frac{1}{3} \Omega^{\frac{2}{3-2\nu}} \right]$$

Considering all the elements in this expression except Ω as constant for the given case, we find (see 77, p. 209) that V_0 becomes a maximum for

$$83. \quad \left\{ \begin{array}{l} \Omega_H = \left[3 \frac{(3-2\nu)(1+n-2\nu)}{(3-2\nu)(1+3n-2\nu)+6} \right]^{\frac{3-2\nu}{2}} \\ \text{if} \\ \nu > \frac{1+n}{2} \text{ and } \nu > 1.5 \end{array} \right.$$

i. e., Ω_H (83) will be the most suitable modulus of the powder for the given gun and conditions of firing in all cases that usually arise.

Although expression 83 for the most suitable modulus of a powder can not be considered as exact on account of the approximate character of formula 72, nevertheless the fact established by it of the existence of a most suitable modulus for all given cases of powder and gun is supported by the generally known fact that for a given pressure permissible in a given gun the initial velocity increases on employing slower-burning powders, but only to a known limit. The weight of the charge in such cases also increases.

For some kinds of powder, however, it is useless to employ grains of those least dimensions, which correspond to the most favorable modulus, as by this there results either a great difference in initial velocities or else the gases inflame upon opening the breech, or, what is very dangerous, an oscillatory movement of the gases in the bore of the gun may develop.

73. *Limits of employment of binomial formula (72), page 206.*

The binomial formula for velocities affords a maximum value for $\Omega=1$ and for a certain high pressure P_B . The value of the negative term in brackets in expression (72) for velocities becomes for this, as is seen from (78), equal to one-third. Such a value of the negative term (equal and corresponding to $\Omega=1$) must be considered as the upper limit of the use of binomial formula, but as they cease to express with necessary accuracy changes of velocity when near the maximum, for a least dimension of grain (a), slightly differing from a_1 (78), the use of the binomial formula must be limited to a modulus <1 . Sarrau proposes to take for this $\Omega=\frac{9}{11}$. The value of the negative term (72) then becomes

$$84. \quad D \left(\frac{2fA\kappa}{a} \right)^{\frac{2\nu-1}{3-2\nu}} \frac{2A\lambda}{a} \omega \frac{G^{\frac{2\nu-1}{3-2\nu}} \frac{1}{L_0-l_0}}{\frac{4-\nu}{c^{\frac{1}{3-2\nu}}}} = \frac{1}{3} \left(\frac{9}{11} \right)^{\frac{2}{3-2\nu}}$$

As the lower limit of use of the formulæ in question we shall assume the value of the modulus (83), to which the value of the negative member in (72) corresponds.

$$85 \quad D\left(\frac{2f\Lambda\kappa}{x}\right)^{\frac{2\nu-1}{3-2\nu}} \frac{2\Lambda\lambda}{a} \frac{\omega}{G} \frac{1}{c' \frac{4\nu}{3-2\nu}} \frac{2(\nu-1)}{3-2\nu} (L_0 - l_0) = \\ = \frac{(3-2\nu)(1+n-2\nu)}{(3-2\nu)(1+3n-2\nu)+6}$$

In the latter the weight of charge ω is determined from (81) for a given pressure P_B and other elements of firing.

Consequently, in calculating velocities, it is necessary to determine first of all, either the value of the negative term in (72) or else the value of the modulus (80). If the value of the former lies without the limits (84) and (85), and if the latter lies without the limits $\Omega = \Omega_1$ and Ω_H (83), then it happens that in the given case the formulæ considered are not applicable; if in this case $\Omega < \Omega_H$, more rapidly burning powder should be used, i. e., a finer grained powder, and if $\Omega > \Omega_1$, we must proceed as indicated in section 77.

74. General monomial formula of Sarrau for initial velocities.

In order to deduce this formula we must consider expression 79 for velocities as a function of one variable, i. e., as a function of the modulus of the powder $\left(\Omega = \frac{a_1}{a}\right)$. In accordance with what was stated in 69, page 206, we may substitute for any function a monomial formula in some power of the variable, employing for this expressions 75. In the given case

$$f(x) = \Omega^{\frac{1}{3-2\nu}} \left[1 - \frac{1}{3} \Omega^{\frac{2}{3-2\nu}} \right] = N' \Omega^{n'} \\ N' = \frac{1 - \frac{1}{3} \Omega^{\frac{2}{3-2\nu}}}{\Omega^{n'} \frac{1}{3-2\nu}}; \quad n' = \frac{1}{3-2\nu} \frac{1 - \Omega^{\frac{2}{3-2\nu}}}{1 - \frac{1}{3} \Omega^{\frac{2}{3-2\nu}}}$$

In consequence of this the monomial expression for velocities will be

$$V_0 = BN' \left(\frac{2f\Lambda\kappa}{a_1} \right)^{\frac{1}{3-2\nu}} \frac{1-\nu}{\delta^{\frac{1-\nu}{3-2\nu}}} \times \\ \times \frac{\omega^{\frac{\nu}{3-2\nu} + \nu} (L_0 - l_0)^\nu}{c' \frac{2}{3-2\nu} \frac{1-\nu}{3-2\nu} \frac{4\nu-4(1-\nu)}{3-2\nu} l_0^{2\nu-2} \frac{2(1-\nu)}{3-2\nu}} \Omega^{n'}.$$

Substituting here instead of a_1 and Ω their values (78) and (80) we obtain

$$86 \quad V_0 = B_{\Omega} \frac{(f\kappa)^{\frac{1+n'(2\nu-1)}{2}}}{\lambda^{\frac{1-n'(3-2\nu)}{2}}} \left(\frac{2\Lambda}{a}\right)^{n'} \delta^{\frac{1-\nu}{3-2\nu} - \gamma} \times \\ \times \frac{1}{\omega^{\frac{2(3-2\nu)}{2} + \frac{2\nu-1}{2} - n' + \gamma}} \left(L_0 - l_0\right)^{\gamma - \frac{1-\nu}{3-2\nu} + (1-\nu)n'} \\ \times \frac{2\left[\frac{1-\nu}{3-2\nu} - \gamma + \nu n'\right]}{c'} \frac{1}{G} \frac{1}{2} - \frac{n'}{2} \frac{4\gamma - \frac{4(1-\nu)}{3-2\nu} 2\gamma - \frac{2(1-\nu)}{3-2\nu}}{d' l_0},$$

in which

$$B_{\Omega} = BN' (3D)^{n'} \frac{3-2\nu}{2} - \frac{1}{2}$$

75. *Determination of the exponents in the formulæ for velocities and pressures—different forms of formulæ.*

On examination of the monomial formula 86 for velocities, and 76 for pressures we see that the results of firing are proportional to certain powers of different elements influencing results of firing. The exponents of these powers will be known when we determine the quantities ν , γ , n' and n . Of these quantities ν may be determined as indicated in paragraphs 51 and 52, pages 147-149, but in general terms we shall assume it as unknown.

Let us assume that we possess results of two firings from a given gun with the same powder but with different weights of charges ω and ω' , all other conditions being equal; then

$$v' = v' \left(\frac{\omega'}{\omega} \right)^p \\ P'_B = P_B \left(\frac{\omega'}{\omega} \right)^{p_1}$$

whence we find,

$$p = \frac{\log \frac{v'}{v}}{\log \frac{\omega'}{\omega}}$$

and

$$p_1 = \frac{\log \frac{P'_B}{P_B}}{\log \frac{\omega'}{\omega}}$$

In order to determine the values of the exponents (p , p_1) accurately, it is necessary that the values of the initial velocities or pressures differ considerably from each other. In fact, expressing the ratio of the velocities by x and the corresponding weights of charges by ζ we obtain,

$$x = \zeta^p$$

whence

$$p = \frac{\log x}{\log \zeta}$$

and

$$\frac{dp}{p} = \frac{\log e}{\log x} \frac{dx}{x}$$

or

$$\frac{dp}{p} = \frac{\log e}{\log x} \frac{dx}{x}$$

It follows, then, that for the same fundamental error $\left(\frac{\Delta x}{x}\right)$, in estimating the final result (in the given case, the initial velocity), the value p will be determined the more accurately, i. e., the fundamental error $\frac{\Delta p}{p}$ will be less, the greater the absolute value of $\log x$; in other words, the greater the difference between the determining quantities.

The values found for p and p_1 should be, as stated above, equal to the corresponding exponents in formulæ 86 and 76. Proceeding in the same manner with the other elements entering into these formulæ we find, on the basis of observed velocities,

$$\frac{1}{2(3-2\nu)} + \frac{2\nu-1}{2} n' + \gamma = p, \text{ (only weight of charge changed)}$$

$$\gamma - \frac{1-\nu}{3-2\nu} + (1+\nu) n' = q, \text{ (only length of path changed)}$$

$$\frac{1}{2} + \frac{n'}{2} = \nu, \text{ (only weight of projectile changed)}$$

$$2\gamma - \frac{2(1-\nu)}{3-2\nu} = s, \text{ (only length of powder chamber changed)}$$

and on the basis of observed pressures,

$$\frac{3}{3-2\nu} + n = p_1 \text{ (only weight of charge changed)}$$

$$\frac{1}{3-2\nu} - n = \nu_1 \text{ (only weight of projectile changed)}$$

$$\frac{2}{3-2\nu} = s_1 \text{ (only length of powder chamber changed)}$$

In order to determine the exponent sought for in each system, it is necessary to have at least four similar equations. If the possibility presents itself of constituting equations for a greater number of elements than those entering into the formulæ 86 and 76, then the determination of the values ν , γ , n' and n must be made by the application of the method of least squares. After finding these quantities it is not difficult to calculate all the exponents both in 86 and 76 and in 72. On the basis of these calculations we select for the values of ν , γ , n and n' , such series of decimal fractions close to them that the formula 72 for velocities and 76 for pressures may express as nearly as possible the results of firing powder of a given kind from different guns.

On the basis of what was stated in sections 51 and 52, pages 147-149, and also from the results of firings from different guns, Sarrau assumed the following exponents in the formulæ for velocities and pressures:

(a) For smoking powders (black* and chocolate),

$$\nu = \frac{1}{2}, n = \frac{1}{4}, \gamma = \frac{3}{8}$$

* See Sarrau's *Nouvelles Recherches sur les effets de la poudre dans les armes*, 1876.

(b) For pyroxyline powders,*

$$\nu = \frac{3}{4}, \quad n = \frac{1}{12}, \quad \gamma = \frac{1}{3}$$

whence, formulæ for velocities, pressures, and moduli become:

A. For smoking powders

$$\left\{ \begin{array}{l} P_B = \Xi \frac{2fA\kappa}{n} \left(\frac{1}{\delta} \right)^{\frac{1}{2}} - \frac{\varpi_4^2 G^{\frac{1}{2}}}{c'^2 d'^2 l_0} \\ V_0 = B \left(\frac{2fA\kappa}{a} \right)^{\frac{1}{2}} \left(\frac{1}{\delta} \right)^{\frac{1}{2}} \frac{\omega^{\frac{1}{2}} (L_0 - l_0)^{\frac{1}{2}}}{c'^{\frac{1}{2}} G^{\frac{1}{2}} d'^{\frac{1}{2}} l_0^{\frac{1}{2}}} \left[1 - \frac{2A\lambda}{a} \frac{G^{\frac{1}{2}} (L_0 - l_0)^{\frac{1}{2}}}{c'} \right] \\ V_0 = B \Omega \frac{(f\kappa)^{1/2}}{\lambda \frac{1-2n'}{2}} \left(\frac{2A}{a} \right)^{n'} \left(\frac{1}{\delta} \right)^{1/3} \frac{\varpi^{\frac{5}{3}} (L_0 - L)^{\frac{1}{3} + \frac{n'}{2}}}{c'^{n' - \frac{1}{3}} G^{\frac{1}{2} - \frac{n'}{2}} d'^{\frac{1}{2}} l_0^{\frac{1}{2}}} \\ \Omega = 3 \left(D - \frac{2A\lambda}{a} \frac{G^{\frac{1}{2}} (L_0 - l_0)^{\frac{1}{2}}}{c'} \right) \\ n' = \frac{3}{2} \frac{1 - \Omega}{3 - \Omega} \\ N' = \frac{1 - \frac{1}{3}\Omega}{\Omega n' - \frac{1}{2}} \\ B_{\Omega} = BN' (3D)^{n' - \frac{1}{2}} \\ B = B_2 \pi^{\frac{1}{2}} \frac{1}{4} \left(\frac{g}{\phi} \right)^3 \\ D = D_2 \left(\frac{4}{\pi} \right)^{2/3} \left(\frac{\phi}{g} \right)^{2/3} \\ \Xi = \Xi_2 \left(\frac{4}{\pi} \right)^{4/3} \left(\frac{4\phi}{g} \right)^{2/3} \\ \Xi n = \Xi N \end{array} \right.$$

The limits of use of the binomial formulæ;

upper, $\Omega < \frac{9}{11} = 0.81818$; value of negative term, < 0.273 .

lower, $\Omega_H > \frac{3}{8} = 0.375$; value of negative term, > 0.125 .

* See Jacob, Capitaine de l'Artillerie de la Marine: "Étude sur les effets de la poudre dans un canon de 16-cent. a rallonges mobiles," 1894.

B. For pyroxyline powders:

$$\begin{aligned}
 P_B &= \Xi_n \left(\frac{2fA\kappa}{a} \right)^{4/3} \left(\frac{1}{\delta} \right)^{2/3} \frac{\omega^{25/12} G^{7/12}}{c'^{8/3} d'^{8/3} l_0^{4/3}}, \\
 V_0 &= B \left(\frac{2fA\kappa}{a} \right)^{2/3} \left(\frac{1}{\delta} \right)^{1/6} \frac{\omega^{5/6} (L_0 - l_0)^{1/3}}{c'^{2/3} G^{1/6} d'^{2/3} l_0^{1/3}} \left[1 - \right. \\
 &\quad \left. - D \left(\frac{2fA\kappa}{a} \right)^{1/3} \frac{2A\lambda}{a} \frac{\omega^{1/3} G^{2/3} (L_0 - l_0)^{1/3}}{c'^2} \right], \\
 V_0 &= B \Omega^{\frac{2+n'}{4}} \left(\frac{2A}{a} \right)^{n'} \left(\frac{1}{\delta} \right)^{1/6} \frac{\omega^{2/3+1/4n'} (L_0 - l_0)^{1/6+1/4n'}}{c'^{\frac{3}{2}n'-1/3} G^{1/2-1/2n'} d'^{2/3} l_0^{1/3}}, \\
 [88] \dots \Omega &= \left[3 D \left(\frac{2fA\kappa}{a} \right)^{1/3} \frac{2A\lambda}{a} \frac{\omega^{1/3} G^{2/3} (L_0 - l_0)^{1/3}}{c'^2} \right]^{3/4}, \\
 n' &= 2 \frac{1 - \Omega^{3/4}}{3 - \Omega^{3/4}}, \\
 N' &= \frac{1 - \frac{1}{3} \Omega^{3/4}}{\Omega^{n'-\frac{1}{3}}}, \\
 B_\Omega &= B N' (3D)^{3/4n'} - \frac{1}{2}, \\
 B &= B_2 4^{1/3} \left(\frac{g}{\varphi} \right)^{1/6}, \\
 D &= D_2 \frac{4}{\pi} \left(\frac{\varphi}{g} \right)^{1/6}, \\
 \Xi &= \Xi_2 \left(\frac{4}{\pi} \right)^{8/3} \left(\frac{4\varphi}{g} \right)^{2/3}, \\
 \Xi_n &= \Xi N.
 \end{aligned}$$

The limits of use of binomial formula:

upper, $\Omega \leq 9/11 = 0.81818$; value of negative term, ≤ 0.255 ,

lower, $\Omega \geq 0.3774$.

MONOMIAL FORMULÆ FOR VELOCITIES.

In view of the convenience that they afford in calculation, similar monomial formulæ are presented below for velocities. They correspond to an assumed classification of powder—quick, medium, slow, and very slow. Knowing approximately the modulus of a powder in the given form of gun, we may use one of the formulæ presented below without resorting to the calculation of the actual modulus, for the change in its value between certain limits does not influence sensibly the magnitude of the initial velocity, as is evident from the formulæ presented below:

A. For smoking powders.

(a) "Quick" powders:

$$\Omega = \frac{9}{11} = 0.818; n' = \frac{1}{8}$$

$$89 \quad V_0 = B_{\Omega} \frac{(f\kappa)^{1/2}}{\lambda^{3/8}} \left(\frac{2A}{a}\right)^{1/8} \left(\frac{1}{\delta}\right)^{1/8} \frac{\omega^{5/8} (L_0 - l_0)^{3/16} c^{1/8}}{G^{7/16} d'^{1/2} l_0^{1/4}}$$

(b) "Medium" powders:

$$\Omega = \frac{3}{4}; n' = \frac{1}{6}$$

$$90 \quad V_0 = B_{\Omega} \frac{(f\kappa)^{1/2}}{\lambda^{1/3}} \left(\frac{2A}{a}\right)^{1/6} \left(\frac{1}{\delta}\right)^{1/8} \frac{\omega^{5/8} (L_0 - l_0)^{5/24} c^{1/12}}{G^{5/12} d'^{1/2} l_0^{1/4}}$$

(c) "Slow" powders:

$$\Omega = \frac{3}{5}; n' = \frac{1}{4}$$

$$91 \quad (V_0 = B_{\Omega} \frac{(f\kappa)^{1/2}}{\lambda^{1/4}} \left(\frac{2A}{a}\right)^{1/4} \left(\frac{1}{\delta}\right)^{1/8} \frac{\omega^{5/8} (L_0 - l_0)^{1/4}}{G^{3/8} d'^{1/2} l_0^{1/4}})$$

(d) "Very slow" powders, corresponding to the most suitable modulus:

$$\Omega_H = \frac{3}{8}; n' = \frac{5}{14}$$

$$92 \quad V_0 = B_{\Omega} \frac{(f\kappa)^{1/2}}{\lambda^{1/7}} \left(\frac{2A}{a}\right)^{9/14} \left(\frac{1}{\delta}\right)^{1/8} \frac{\omega^{5/8} (L_0 - l_0)^{17/56}}{c^{3/28} G^{9/28} d'^{1/2} l_0^{1/4}}$$

B. For pyroxyline powders.

(a) "Quick" powders:

$$\Omega = 0.8054; n' = \frac{2}{9}$$

$$93 \quad V_0 = B_{\Omega} \frac{(f\kappa)^{5/9}}{\lambda^{1/3}} \left(\frac{2A}{a}\right)^{2/9} \left(\frac{1}{\delta}\right)^{1/6} \frac{\omega^{13/18} (L_0 - l_0)^{2/9}}{G^{7/18} d'^{2/3} l_0^{1/3}}$$

(b) "Medium" powders.

$$\Omega = 0.759; n' = \frac{4}{15}$$

$$(94) \quad V_0 = B_{\Omega} \frac{(f\kappa)^{17/30}}{\lambda^{3/10}} \left(\frac{2A}{a}\right)^{1/15} \left(\frac{1}{\delta}\right)^{1/6} \frac{\omega^{11/15} (L_0 - l_0)^{7/30}}{c^{1/15} G^{11/30} d'^{2/3} l_0^{1/3}};$$

(c) "Slow" powders.

$$\Omega = 0.6817; n' = \frac{1}{3}$$

$$(95) \quad V_0 = B_{\Omega} \frac{(f\kappa)^{7/12}}{\lambda^{1/4}} \left(\frac{2A}{a}\right)^{1/3} \left(\frac{1}{\delta}\right)^{1/6} \frac{\omega^{3/4} (L_0 - l_0)^{1/4}}{c^{1/6} G^{1/3} d'^{2/3} l_0^{1/3}}$$

(d) "Very slow" powders corresponding to the most suitable modulus;

$$\Omega_H = 0.3774; n' = \frac{8}{15}$$

$$(96) \quad V_0 = B_{\Omega} \frac{(f\kappa)^{19/30}}{\lambda^{1/10}} \left(\frac{2A}{a}\right)^{8/15} \left(\frac{1}{\delta}\right)^{1/6} \frac{\omega^{4/5} (L_0 - l_0)^{3/10}}{c^{7/15} G^{7/30} d'^{2/3} l_0^{1/3}}$$

In all formulæ adduced (87-96) the constant factors B, D, Ξ_n and B_n are determined from results of firing from guns of different calibers. In these constant factors the elements $f, A, \kappa, \lambda, a, \delta$, characterizing the properties of the powder, may be also included, if such elements are unknown. In determining the constants B and D , it is necessary to have at least the results of two firings, and in determining the others, of one firing.

The calculation of pressures in the bore is conducted in all cases according to formulæ 87 and 88, according to the kind of powder used.

76. *The values of the constants in formulæ for velocities and pressures.*

The above deduced formulæ, 87 and 88, are for pressures and velocities. If we include in these formulæ as constants all elements depending on the kind of powder, they assume the form:

For smoking powders,

$$P_B = \Xi_n' \frac{\omega^{7/4} G^{1/4}}{c'^2 d'^2 l_0},$$

$$V_0 = B' \frac{\omega^{5/8} (L_0 - l_0)^{3/8}}{c'^{1/4} G^{1/4} d'^{1/2} l_0^{1/4}} \left[1 - D' \frac{G^{1/2} (L_0 - l_0)^{1/2}}{c'} \right];$$

and for pyroxyline powders,

$$P_B = \Xi_n' \frac{\omega^{25/12} G^{7/12}}{c'^{8/3} d'^{8/3} l_0^{4/3}},$$

$$V_0 = B' \frac{\omega^{5/6} (L_0 - l_0)^{1/3}}{c'^{2/3} G^{1/6} d'^{2/3} l_0^{1/3}} \left[1 - D' \frac{\omega^{1/3} G^{2/3} (L_0 - l_0)^{1/3}}{c'^2} \right]$$

The quantities Ξ_n', B' , and D' entering here depend not only on the properties of the powders, but on the structure of the gun. In order to estimate the values of these constants (for the units, the kilogram, decimeter, and second) we may limit ourselves to presenting the following data:

A. *For smoking powders.*

	Log Ξ_n' .	Log B' .	Log D' .
(a) Black:			
Quick.....	4.77	3.39	$\overline{2.11}$
Slow.....	4.57	3.30	$\overline{3.98}$
(b) Chocolate:			
Quick.....	4.51	3.24	$\overline{3.68}$
Slow.....	4.28	3.14	$\overline{3.58}$

B. *For pyro-collodion powders.*

Caliber of gun, in centimeters.	For quick powders.			For slow powders.		
	Log Ξ_n' .	Log B'.	Log D'.	Log Ξ_n' .	Log B'.	Log D'.
40	4.05	3.15	$\overline{3.20}$	3.95	3.00	$\overline{3.10}$
35	4.10	3.18	$\overline{3.25}$	4.00	3.13	$\overline{3.15}$
30	4.18	3.22	$\overline{3.34}$	4.08	3.17	$\overline{3.24}$
25	4.28	3.25	$\overline{3.44}$	4.18	3.20	$\overline{3.34}$
20	4.38	3.30	$\overline{3.54}$	4.28	3.25	$\overline{3.44}$
15	4.52	3.35	$\overline{3.66}$	4.42	3.30	$\overline{3.56}$
10	4.70	3.40	$\overline{3.84}$	4.60	3.35	$\overline{3.74}$

77. *Calculation of velocities when the modulus of the powder is greater than $\frac{9}{11}$.*

Calculation of velocities, when the modulus of the powder is greater than $\frac{9}{11}$ —i. e., when “extremely quick” or “very quick” burning powders are employed. In these cases, as stated above, sections 70 and 73, the binomial formulæ 72, 87, and 88 for velocities are inapplicable, and therefore we must proceed in the following manner:

If the modulus of the powder be included between the limits $1.1 > \Omega > 0.9$, we may calculate velocities with sufficient accuracy for practical purposes by the monomial formulæ 89 or 93, corresponding to the kind of powder, although they are established for moduli not higher than $\frac{9}{11}$.

If $\Omega > 1.1$, then in this case we must consider that the whole of the powder is burned at the moment that the projectile begins to move; and the initial velocities are calculated by formula 43, page 184, as is explained in section 61.

78. *Determination of duration of combustion of the powder grain in the open air, from the results of firing.*

In accordance with what was stated above on page 157, the duration of combustion sought is

$$\Pi_0 = \frac{a}{2A}, \frac{1}{P_0^\nu}.$$

where P_0 is the pressure in atmospheres and ν the exponent in formula 31, page 103, which may be determined on the basis of results of firing, as explained in section 75, page 215.

By employing the monomial formulæ 86, 87, and 88, we may determine $\frac{2A}{a}$ on the basis of results of firing two powders, if we know all the other elements except $\frac{2A}{a}$. In fact, denoting for the former powder all known factors by B_Ω^I and for the latter by B_Ω^{II} , we obtain

$$\left(\frac{2A}{a}\right)^I = \left(\frac{2A}{a}\right)^{II} \frac{\left(B_\Omega^I\right)^{\frac{1}{n_1}}}{\left(B_\Omega^{II}\right)^{\frac{1}{n_{II}}}}$$

If we know the value $\left(\frac{2A}{a}\right)^{\Pi}$ for the second powder, then we may ascertain the absolute value of $\left(\frac{2A}{a}\right)^{\text{I}}$, which gives the value Π_0 for this powder. If we do not know $\left(\frac{2A}{a}\right)^{\Pi}$, then results of firing will only determine the ratio of these quantities, i. e., the ratio of $\left(\frac{2A}{a}\right)^{\text{I}}$ to $\left(\frac{2A}{a}\right)^{\Pi}$.

79. Determination of the pressures and velocities of the projectile in different sections of the bore of the gun.

The formulæ 72, 76, and 86, deduced above, on pages 206–209 and 215, and also their results, 87–96, pages 217–219, afford with necessary accuracy only the initial velocities and maximum pressures in the bore of the gun and on the base of the shell. Nevertheless, in designing guns and projectiles it is indispensable to know also the velocities of the projectile and the pressures at various sections of the bore. The employment of analysis leads us to the solution of this question by the means of approximate tables, section 63, which are far from accurate; in fact, we may only employ them for the first moments of the movement of the projectile, up to the time of development of maximum pressure in the bore. In view of this, in order to determine pressures and velocities of the projectile at various sections of the bore of the gun, empirical* formulæ of different form are resorted to, or the assumption is made that by means of formulæ deduced for initial velocities we may calculate the velocities of the projectile along the whole length of the bore of the gun, and then obtain for these, in an established manner, expressions for pressures.† In either of these methods the bore of the gun is usually divided into two parts, the first up to the point of formation of maximum pressure, which is determined by tables, section 63, pages 195–197, for any given distance, and the second, the remaining portion, for which velocities and pressures are calculated by one formula or another. The employment of formulæ for initial velocities leads frequently, however, to impossible results; pressures reduce to 0 for a certain length of the actual path of the shell. In this way we become limited to the use of formulæ that are purely empirical.

As based on the form of formula 41, page 183, for pressure, when the powder is burned completely at the moment that the shell begins

*See, e. g., Colonel Zaboudski's work on "The pressure of gases of smokeless powder in the bore of the gun." *Morskoi Sbornik*, 1894.

†See, e. g., Longridge's *Interior Ballistics*, translated by Captain Nilus, 1891.

to move, we assume, for the case where the powder burns progressively, the following formula for pressure:

$$97 \quad P = \frac{f_1 \omega}{s l_1''} \left(\frac{l_1''}{l_1'' + l} \right)^{1 + \beta_1} \left[1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\xi} \right]$$

in which P is the pressure on the base of the projectile after traversing the path l ; f_1 is a quantity proportional to the force of the powder f ; ω is the weight of the charge; s is the area of the cross section of the bore (p. 176).

$$l_1'' = l_1 - \frac{\alpha_1 \omega}{s}$$

which, for $\alpha_1 = \alpha$, reduces to l_1' (p. 177).

α_1 is a quantity depending on the covolume α of the powder and upon the volume of the latter. Assuming that the powder burns during the whole time of movement of the shell, it follows that, in determining the pressure, we must subtract from the volume occupied by the gases in the bore of the gun not only the "covolume" of the products of the burned part of the powder, but also the volume of the unburned part of the powder, whence

$$\alpha_1 = \alpha \Psi + \frac{1 - \Psi}{\delta}$$

The value of the actual quantity Ψ for the given time is unknown, and it varies between 0 at the beginning of the motion of the shell and 1 at the end. Assuming, therefore, for α_1 the arithmetical mean between its corresponding extreme values, we may take it as

$$98 \quad \alpha_1 = \frac{1 + \alpha \delta}{2}$$

As a matter of fact this quantity α_1 may be either less or greater than its assumed mean value.

β_1 is a certain function of β , page 181, and depends upon the specific heats of the products of decomposition and the volume of the gases evolved by the powder.

ξ is an exponent depending on the conditions of firing. It may possess any desirable assumed value.

It is necessary, afterwards, to assume that both for $\xi = \infty$ as well as for $l = \infty$, the quantities f_1 , β_1 , and α_1 reduce correspondingly to f , β , and α .

In view of this, expression 97 for $\xi = +\infty$ reduces to 41, which corresponds to the case when the whole of the powder is burned up to the instant of movement of the shell. For $\xi = 0$, P is also equal to 0, which corresponds to the case when none of the powder is consumed.

On increasing l and also ξ , the value of the negative term in expression 97 diminishes and the law of change of pressures approximates to that for the adiabatic process, expression 41.

For $l = \infty$, the pressure P equals 0, as in the case of the adiabatic process. The n -th derivative of expression 97 for l may be written in the form

$$\frac{s l_1''}{f_1 \omega} \frac{d^n P}{dl^n} = (-1)^n \frac{1+\beta_1}{l_1''} \frac{2+\beta_1}{l_1''} \dots \frac{n+\beta_1}{l_1''} \left(\frac{l_1''}{l_1''+l} \right)^{n+1+\beta_1} \left[1 - \frac{1+\beta_1+\xi}{1+\beta_1} \frac{2+\beta_1+\xi}{2+\beta_1} \dots \frac{n+\beta_1+\xi}{n+\beta_1} \left(\frac{l_1''}{l_1''+l} \right)^{\xi} \right]$$

The first derivative of expression 97 reduces to 0 for

$$99 \quad l_2 = l_1'' \left\{ \left[\frac{1+\beta_1+\xi}{1+\beta_1} \right]^{\frac{1}{\xi}} - 1 \right\}$$

and as the second derivative of the same expression for $l = l_2$ has the value

$$-\frac{f_1 \omega}{s l_1''} \frac{(1+\beta_1)}{l_1''} \frac{\xi}{l_1''} \left(\frac{l_1''+l_2}{l_1''} \right)^{(3+\beta_1)}$$

which is always less than 0; therefore l_2 99 corresponds to the maximum pressure on the base of the projectile, i. e., $P_{l=l_2} = P_P$.

The value of l_2 is positive for all cases where $\xi > 0$ and varies from $l_1'' \left(\frac{1}{e^{1+\beta_1}} - 1 \right)$, when $\xi = 0$ to 0 when $\xi = \infty$. Moreover, $\frac{d^2 P}{dl^2}$ reduces to 0 for $l = l_3$, determined by the expression

$$100 \quad l_3 = l_1'' \left\{ \left[\frac{(1+\beta_1+\xi)(2+\beta_1+\xi)}{(1+\beta_1)(2+\beta_1)} \right]^{\frac{1}{\xi}} - 1 \right\}$$

which gives a positive value for l_3 for every positive value of ξ ; while for $\xi = \infty$, $l_3 = 0$. The third derivative $\frac{d^3 P}{dl^3}$ does not reduce to 0 for $l = l_3$; therefore, the curve 97 possesses a point of inflection the abscissa of which is l_3 100.

The difference

$$l_3 - l_2 = l_1'' \left(\frac{1+\beta_1+\xi}{1+\beta_1} \right)^{\frac{1}{\xi}} \left[\left(1 + \frac{\xi}{2+\beta_1} \right)^{\frac{1}{\xi}} - 1 \right]$$

is always greater than 0, and therefore

$$l_2 < l_3$$

i. e., the point of inflection of the pressure curve occurs in front of the point of maximum pressure on the base of the projectile.

Throughout the whole extent from $l = 0$ to l_3 the curve of pressures is convex to the axis of the bore, since throughout this extent $\frac{d^2 P}{dl^2}$ remains negative. For the remaining distance up to $l = \infty$, $\frac{d^2 P}{dl^2}$, is positive; therefore, this part of the curve is convex to the axis of the bore as in the case of the adiabatic process.

The above investigation of curve 97 shows that this function corresponds completely to existing conceptions in relation to the form of the pressure curve of the powder gases in the bore of the gun, of which it embraces all cases for the whole distance of movement of the projectile, from the beginning of its movement to its leaving the muzzle.*

The elementary work of powder gases will be

$$d\mathbf{T} = P s dl$$

Substituting here in place of P its value 97, and subsequently integrating the first member from 0 to \mathbf{T} and the second from 0 to l , we obtain

$$\mathbf{T} = f_1 \varphi \left\{ \frac{1}{\beta_1} \left[1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\beta_1} \right] - \frac{1}{\beta_1 + \xi} \left[1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\beta_1 + \xi} \right] \right\}$$

As the vis viva of the shell is less than the work of the powder gases (paragraph 61), we obtain after simple transformations,

$$101 \quad v^2 = \frac{2 g f_1 \omega}{\varphi G \beta_1 (\beta_1 + \xi)} \left\{ 1 - \frac{\beta_1 + \xi}{\xi} \left(\frac{l_1''}{l_1'' + l} \right)^{\beta_1} \left[1 - \frac{\beta_1}{\beta_1 + \xi} \left(\frac{l_1''}{l_1'' + l} \right)^{\xi} \right] \right\}$$

where φ is the coefficient (page 184), indicating the ratio of the complete work of the gases to the vis viva of the shell; it is included approximately between 1.20 and 1.02.

For $\xi = \infty$, formula 101 reduces to 43 (page 184), which corresponds to the case when the whole of the powder is consumed at the instant the projectile begins to move. For $\xi = 0$, v also equals 0.

The derivative of \mathbf{T} for β_1 is

$$\begin{aligned} \frac{d\mathbf{T}}{d\beta_1} = & - \frac{f_1 \varphi}{\beta_1^2} \left\{ 1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\beta_1} \left[1 + \beta_1 \log_e \frac{l_1'' + l}{l_1''} \right] \right\} + \\ & + \frac{f_1 \omega}{(\beta_1 + \xi)^2} \left\{ 1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\beta_1 + \xi} \left[1 + (\beta_1 + \xi) \log_e \frac{l_1'' + l}{l_1''} \right] \right\}. \end{aligned}$$

This expression is <0 for $+\infty > \beta_1 > 0$ since, in this case the first term of the derivative is always <0 , and the second is always >0 ; but the absolute value of the first number is greater than that of the second.

* This formula was proposed by us in 1894; subsequently, in 1899, there appeared a very interesting article in French by Major Valier (see Comptes Rendus, Vol. CXXVIII, p. 1305, and Vol. CXXIX, p. 258), which proposed for the whole length of flight a very simple empirical formula, but it was not so "elastic," and therefore not so applicable to the different cases of firing from guns.

For $\beta = +\infty$, $\frac{d\mathbf{T}}{d\beta_1} = 0$. On account of this \mathbf{T} , and consequently V , attain a maximum for $\beta_1 = 0$.

$$102 \quad \left\{ \begin{array}{l} \mathbf{T}_{\beta_1=0} = f_1 \omega \left\{ \log_e \frac{l_1'' + l}{l_1''} - \frac{1}{\xi} \left[1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\frac{\xi}{\xi}} \right] \right\}, \\ V^2_{\beta_1=0} = \frac{2g f_1 \omega}{\varphi G} \left\{ \log_e \frac{l_1'' + l}{l_1''} - \frac{1}{\xi} \left[1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\frac{\xi}{\xi}} \right] \right\}, \end{array} \right.$$

and for $\beta_1 = +\infty$, they attain a minimum. (0).

The first term of expression 102 corresponds to the work of expansion of gases at constant temperature, which they attain for $\xi = +\infty$.

The derivative of \mathbf{T} for l equals the second member of expression 97 multiplied by s , and the derivative of \mathbf{T} for l_1'' will be

$$\frac{d\mathbf{T}}{dl_1''} = -\frac{f_1 \omega l}{l_1''^2} \left(\frac{l_1''}{l_1'' + l} \right)^{1+\beta_1} \left\{ 1 - \left(\frac{l_1''}{l_1'' + l} \right)^{\frac{\xi}{\xi}} \right\}$$

An examination of these derivatives shows: (1) That $\frac{d\mathbf{T}}{dl}$ reduces to 0 for $l = 0$ and for $l = +\infty$. The first corresponds, as should be expected, to minimum ($\mathbf{T} = 0$) and the second to maximum work, for which we obtain

$$\mathbf{T}_{\max} = \frac{f_1 \omega}{\beta_1 \left[1 + \frac{\beta_1}{\xi} \right]}$$

which, for a value of ξ greater than β_1 , almost equals the potential of the powder (paragraph 61); (2) That $\frac{d\mathbf{T}}{dl_1''}$ is always < 0 , changing from $-\infty$ for $l_1'' = 0$ to 0 for $l_1'' = +\infty$.

In view of this the value of the work of the gases is reduced by increasing l_1'' and conversely. This deduction is true, however, only in the case that β_1 and ξ do not depend on l_1'' ; as a matter of fact, as we shall see later, there exists a relation between them, and therefore, taking the derivative of \mathbf{T} by l_1'' , ξ and β_1 must be considered as functions of l_1'' , in which case $\frac{d\mathbf{T}}{dl_1''}$ may reduce to 0 for certain values of l_1'' and \mathbf{T} will, in this case, be either a maximum or a minimum, which must be borne in mind on changing the value of α_1 .

By employing 74, 95, and 98, we may, for given values of V_0 , P_B and φ , calculate the velocities and pressures in various sections of the bore of the gun, proceeding in the following manner:

Assume, as points of departure, values for α_1 and f_1 ; the first should be chosen as small as possible, the second as large as possible; then, for the pressure P_B we determine the maximum pressure on the base of the projectile P_P (74), assuming in this formula $\Theta = \frac{3}{2}$ for smoking powders and $\Theta = \frac{1}{2}$ for smokeless powders. As this pressure is only developed at a distance l_2 (99), then substituting the value of the latter in formula (97) in the place of l we obtain

$$P_P = \frac{f_1 \omega}{s l_1''} \left[\frac{1 + \beta_1}{1 + \beta_1 + \xi} \right]^{\frac{1 + \beta_1}{\xi}} \frac{\xi}{1 + \beta_1 + \xi}$$

whence, substituting,

$$(1 + \beta_1) y = \xi \quad (103)$$

it follows that

$$\frac{f_1 \omega}{P_P s l_1''} = \frac{1 + y}{y} (1 + y) \frac{1}{y} = f(y) \quad (104)$$

As β_1 is not less than 0, then (103) y changes together with ξ from 0 to $+\infty$.

The left member of expression (104) for given values of f_1 , ω , s , and l_1'' , is inversely proportional to the pressure P_P , and as the latter changes from 0 to P_1 (page 177), this (left) member may, generally speaking, vary from $+\infty$ to $+1$. The right member of the same expression, or $f(y)$, changes also within these limits; namely, $f(y) = +\infty$ for $y = \xi = 0$ and $f(y) = +1$, for $y = \xi = +\infty$. In consequence of this, there corresponds to each possible case (P_P , s , l_1'' , f_1 , ω), a certain definite value $f(y)$. The logarithms of its values for different values of y are given in Table XLIX. In making calculations we must limit our search in the table to the nearest value less than $\log f(y)$, and calculate the corresponding corrected value P_P by formula 104.

Substituting for ξ in formula 101 its value in 103 we obtain,

$$\begin{aligned} \frac{\varphi G v^2}{2 g f_1 \omega} = & \frac{(1 + \beta_1) y}{\beta_1 [\beta_1 + (1 + \beta_1) y]} \left\{ 1 - \frac{\beta_1 + (1 + \beta_1) y}{(1 + \beta_1) y} \left(\frac{l_1''}{l_1'' + l} \right)^{\beta_1} \left[1 - \right. \right. \\ & \left. \left. - \frac{\beta_1}{\beta_1 + (1 + \beta_1) y} \left(\frac{l_1''}{l_1'' + l} \right)^{(1 + \beta_1) y} \right] \right\} \end{aligned} \quad (105)$$

TABLE XLIX.—*Values of log f(y).*

y	$\log f(y)$	y	$\log f(y)$	y	$\log f(y)$	y	$\log f(y)$
$+\infty$	0	2.7000	0.34728	1.1200	0.56850	0.6600	.73407
1,000.0000	0.00343	2.6000	.35529	1.1000	.57376	.6500	.73915
500.0000	.00627	2.5000	.36376	1.0800	.57914	.6400	.74435
200.0000	.01369	2.4000	.37272	1.0600	.58436	.6300	.74950
100.0000	.02436	2.3000	.38222	1.0400	.59037	.6200	.75507
75.0000	.03083	2.2500	.38720	1.0200	.59611	.6100	.76057
50.0000	.04275	2.2000	.39234	1.0000	.60206	.6000	.76617
40.0000	.05104	2.1500	.39718	.9800	.60816	.5900	.77191
30.0000	.06395	2.1000	.40312	.9600	.61443	.5800	.77775
25.0000	.07363	2.0500	.40879	.9400	.62084	.5700	.78371
20.0000	.08730	2.0000	.41464	.9200	.62745	.5600	.78949
15.0000	.10830	1.9500	.42072	.9000	.63423	.5500	.79602
10.0000	.14553	1.9000	.42702	.8800	.64123	.5400	.80269
9.5000	.15096	1.8500	.43342	.8600	.64839	.5300	.80890
9.0000	.15687	1.8000	.44031	.8400	.65580	.5200	.81553
8.5000	.16333	1.7500	.44634	.8200	.66342	.5100	.82235
8.0000	.17043	1.7000	.45465	.8000	.67127	.5000	.82930
7.5000	.17828	1.6500	.46229	.7900	.67528	.4500	.86676
7.0000	.18700	1.6000	.47021	.7800	.67938	.4000	.90940
6.5000	.19677	1.5500	.47849	.7700	.68352	.3500	.95863
6.0000	.20780	1.5000	.48714	.7600	.68774	.3000	1.01662
5.5000	.22035	1.4500	.49619	.7500	.69203	.2500	1.08661
5.0000	.23481	1.4000	.50566	.7400	.69639	.2000	1.04139
4.5000	.25167	1.3500	.51561	.7300	.70083	.1500	1.28928
4.0000	.27165	1.3000	.52604	.7206	.70533	.1000	1.45529
3.5000	.29577	1.2500	.53701	.7100	.70991	.0500	1.74602
3.2000	.31287	1.2000	.54859	.7000	.71456	.0100	2.43632
3.0000	.32563	1.1800	.55341	.6900	.71930	.0010	3.43043
2.9000	.33247	1.1600	.55831	.6800	.72414	.0001	4.40004
2.8000	.33968	1.1400	.56344	.6700	.72907	0	$+\infty$

Substituting in it instead of v and l , the values of the initial velocity and the full length of the path traversed by the projectile, we find by a series of simple trials such a value for β_1 (include in this number $\beta_1=0$) as will satisfy the expression under consideration. Sometimes, however, it may happen that for $\beta_1=0$, the second member of formula 105 will be less than the first member. In this case we must increase P_p and in some cases, f_1 ; sometimes it is also useful to change the quantity α : Afterwards we must again select a value of y that will satisfy (104) and then, through the employment of (105), find β_1 , as was explained above. In view of the possibility of varying the value of φ between certain limits, these calculations do not generally present any special difficulties, especially when Table L is employed, in which are given for various values of the ratio $\frac{l}{l_1'}$, the logarithms of the expression $\left(\frac{l_1''}{l_1''+l}\right)^\xi$, for which the exponent varies in the table only from 1 to 9 by units. By employing these figures, we may without difficulty write the expression $\log \left(\frac{l_1''}{l_1''+l}\right)^\xi$ for any exponent expressed in tenths and hundredths.

Determining β_1 by the method described, we also find ξ by formula 103, and thus determine all elements necessary for calculating the pressures and velocities of the projectile in various sections of the bore of the gun.

Results of these calculations by the empirical formulæ 97 and 101 thus established are fully sufficient for the practical purposes for which they are actually intended. The calculations show that for considerable changes of the values f_1 , α_1 , and β_1 , the final results obtained agree together quite closely.

We may observe that the adduced formulæ are only applicable to possible conditions of the problem, which are determined by the following considerations:

(1) The maximum pressure P_P on the base of the projectile should be greater than the mean pressure P_M , whose action on the projectile throughout the whole length of the bore would develop a vis viva of projectile Ψ_P equal to that developed by the expansion of the powder gases; at the same time, P_P should not be greater than P_1 , which would be developed in the chamber if the whole charge was consumed at the instant that the shell begins to move—i. e.,

$$\frac{f \omega}{s l_1'} = \frac{f \Delta}{1 - \alpha \Delta} > P_P > P_M = \frac{G v_0^2}{2 g s L} = \frac{\Phi_P}{s L}$$

The value $\frac{P_M}{P_B}$ is called the coefficient of pressure of the powder gases, and some assume it as a measure of the development of the energy of the powder through expansion of the gases in the bore of the gun. The nearer this coefficient is to unity,* the greater the energy of the projectile in a given gun for the same weight of charge and maximum pressure.

TABLE L.—Values of $\log \left(\frac{l_1''}{l_1'' + l} \right)^\xi$ and of $\log \log_e \frac{l_1'' + l}{l_1''}$.

Value of the ratio $\frac{l}{l_1''}$	Values of $\log \left(\frac{l_1''}{l_1'' + l} \right)^\xi$, for the exponents ξ .									Values of $\log \log_e \frac{l_1'' + l}{l_1''}$
	1.	2.	3.	4.	5.	6.	7.	8.	9.	
0.001	1.99957	1.99914	1.99871	1.99828	1.99785	1.99742	1.99699	1.99656	1.99613	1.99569
2	913	826	739	652	565	478	391	304	217	3.30174
3	870	740	610	480	350	220	090	98960	98830	47616
4	827	654	481	308	135	98962	98789	616	443	60027
5	783	566	349	132	98915	698	481	264	047	69868
0.010	568	136	98704	98272	97840	97408	96976	96544	96112	99770
20	140	98280	97420	96560	95700	94840	93980	93120	92260	2.29672
30	98716	97432	96148	94864	93580	92296	91012	89728	88444	47079
40	297	96594	94891	93188	91485	89782	88079	86376	84673	59343
50	97881	95762	93643	91524	89405	87286	85167	83048	80929	68835
0.100	95861	91722	87583	83444	79305	75166	71027	66888	62749	97912
150	93930	87860	81790	75720	69650	63580	57510	51440	45370	1.14541

* See Capt. Heydenreich, Die Lehre von Schuss und die Schusstafeln, 1898.

TABLE L.—Values of $\log \left(\frac{l''}{l''+l} \right)^\xi$ and of $\log \log_e \frac{l''+l}{l''}$ —Continued.

Value of the ratio $\frac{l}{l''}$	Values of $\log \left(\frac{l''}{l''+l} \right)^\xi$, for the exponents ξ .									Values of $\log \log_e \frac{l''+l}{l''}$
	1.	2.	3.	4.	5.	6.	7.	8.	9.	
200	1.92082	1.84164	1.76246	1.68328	1.60410	1.52492	1.44574	1.36656	1.28738	1.26084
250	90309	80618	70927	61236	51545	41854	32163	22472	12781	34859
300	88606	77212	65818	54424	43030	31636	20242	08848	2.97454	41890
0.350	86967	73934	60901	47868	34835	21802	08769	2.95736	82703	47726
400	85387	70774	56161	41548	29635	12322	2.97709	83096	68483	52696
450	83863	67726	51589	35452	19315	03178	87041	70904	54767	57004
500	82391	64782	47173	29564	11955	2.94346	76737	59128	41519	60795
550	80937	61934	42901	23868	04835	85802	66769	44736	28703	64173
0.600	79588	59176	38764	18352	97940	77528	57116	36704	16292	67211
650	78252	56504	34756	13008	91260	69512	47764	26016	04268	69964
700	76955	53910	30865	07820	84775	61730	38685	15640	3.92595	72480
750	75696	51392	27088	02784	78480	54176	29872	05568	81264	74790
800	74473	48946	23419	2.97892	72360	46832	21304	3.95776	70257	76922
850	73283	46566	19849	93132	66415	39698	12981	86264	59547	78901
900	72125	44250	16375	88500	60625	32750	04875	77000	49125	80743
950	70997	41994	12991	83988	54985	25982	3.96979	67976	38973	82466
1.000	69897	39794	09691	79588	49485	19382	89279	59176	29073	84083
1.100	67778	35556	03334	71112	38890	06668	74446	42224	10002	87037
1.200	65758	31516	2.97274	63032	28790	3.94548	60306	26064	4.91822	89678
1.300	63827	27654	91481	55308	19135	82962	46789	10616	74443	92060
1.400	61979	23958	85937	47916	09895	71874	33853	4.95832	57811	94224
1.500	60206	20412	80618	40824	01030	61236	21442	81648	41854	96204
1.600	58503	17006	75509	34012	3.92515	51018	09521	68024	26527	98024
1.700	56864	13728	70592	27456	84320	41184	4.98048	54912	11776	99706
1.800	55284	10568	65852	21136	76420	31704	86988	42272	5.97556	0.01268
1.900	53760	07520	61280	15040	68800	22560	76320	30080	88840	02724
2.000	52288	04576	56864	09152	61440	13728	66016	18304	70592	04085
2.500	45593	2.91186	36779	3.82372	27965	4.73558	19151	5.64744	10337	09787
3.000	39794	79588	19382	59176	4.98970	38764	5.78558	18352	6.58146	14186
3.500	34679	69358	04037	38716	73395	08072	42753	6.77432	12111	17727
4.000	30103	60206	3.90309	20412	50515	5.80618	10721	40824	7.70927	20668
4.500	25964	51928	77892	03856	29820	55784	6.81748	07712	38676	23166
5.000	22185	44370	66555	4.88740	10925	33110	55298	7.77480	8.99665	25328
6.000	15490	30980	46470	61960	5.77450	6.92940	08430	23920	39410	28913
7.000	09691	19382	29073	38764	48455	58146	7.67837	8.77528	9.87219	31795
8.000	04576	09152	13728	18304	22880	27456	32032	36608	41184	34188
9.000	00000	00000	00000	00000	00000	00000	00000	00000	00000	36222
10.000	2.95861	3.91722	4.87583	5.83444	6.79305	7.75166	8.71027	9.66888	10.62749	37983
11.000	92082	84164	76246	68328	60410	52492	44574	36656	28738	39531
12.000	88606	77212	65818	54424	43030	31636	20242	08848	11.97454	40909
13.000	85387	70774	56161	41548	26935	12322	9.97709	10.83096	68483	42145
14.000	82391	64782	47173	29564	11955	8.94346	76737	59128	41519	43266
15.000	79588	59176	38764	18352	7.97940	77528	57116	36704	16292	44290
16.000	76955	53910	30865	07820	84775	61730	38685	15640	12.92595	45228
17.000	74473	48946	23419	6.97892	72365	46838	21311	11.95784	70257	46096
18.000	72125	44250	16375	88500	60625	32750	04875	77000	49125	46901
19.000	69897	39794	09691	79588	49485	19382	10.89279	59176	29073	47651
20.000	67778	35556	03334	71112	38890	06668	74446	42224	10002	48353

The vis viva of the projectile Φ_P should be less than the work of the powder gases through adiabatic expansion on the assumption that the whole charge is burned the instant the shell begins to move, i. e.,

$$\frac{Gv^2}{2g} = \Phi_P < \frac{f\omega}{\beta} \left[1 - \left(\frac{l_1'}{l_1' + L} \right)^\beta \right]$$

and generally

$$\Phi_P < f\omega \log_e \frac{l_1' + L}{l_1'}$$

The time of movement of the projectile in the bore of the gun is determined as indicated on page 184, taking the velocities developed through expression 101.

80. The investigation of formulæ for initial velocities and pressures on the base of the projectile as applied to a gun of a given caliber, length, and limit of strength.

The above-considered general conceptions relating to changes in velocity and pressure on change of different elements of firing do not afford the necessary instruction in relation to the principles which must be utilized in designing guns. From among the many elements upon which results of firing depend (i. e., the value of the initial velocity and the vis viva of the projectile), the following must be considered as fundamental: The caliber of the gun (more correctly, the reduced caliber of the gun, c'); the length of the bore L_0 ; the limit of strength, i. e., the maximum pressure permissible at the base of the bore P_B . The elements c' , L_0 , and P_B we shall assume as given. All other elements may be varied arbitrarily, more or less. The problem consists in determining such relations among these latter elements as would afford most favorable results, i. e., maximum initial velocity or vis viva of the projectile for least weight of charge.

From among the number of elements at our disposal we shall determine the weight of the charge ω on the condition that the pressure on the base of the bore P_B is not greater than that permitted by the gun design. To this end it is necessary to determine the charge according to equation 81, page 212.

For every limiting value of the elements entering into this expression we obtain a limiting value for the weight of charge. Substituting this in the equations for initial velocities 72 and 86, we obtain:

$$\begin{aligned}
 V_0 = & \frac{B}{\Xi \chi_1} \left(\frac{2fAx}{a} \right)^{\frac{1-2\chi_1}{3-2\nu}} \frac{\delta^{\frac{\chi_1}{3-2\nu}} P_B (L_0-l_0)^{\nu}}{c'^{\frac{2}{3-2\nu}} [1-\nu(3-2\nu)-2\chi_1]} \times \\
 & \times \frac{l_0^{\frac{2}{3-2\nu}} [\chi_1-\nu(3-2\nu)+1-\nu]}{G^{\frac{1-\nu}{3-2\nu} + \frac{1-n(3-2\nu)}{3-2\nu}} \frac{4}{d'^{\frac{1}{3-2\nu}} [\nu(3-2\nu)-(1-\nu)-\chi_1]}} \left[1 - \right. \\
 & - \frac{D}{\Xi \chi_2} \left(\frac{2fAx}{a} \right)^{\frac{2\nu-1-2\chi_2}{3-2\nu}} \times \\
 & \times \left. \frac{2A\lambda}{a} \frac{\delta^{\frac{\chi_2}{3-2\nu}} P_B (L_0-l_0)^{\frac{2(1-\nu)}{3-2\nu}} l_0^{\frac{2\chi_2}{3-2\nu}} d'^{\frac{4\chi_2}{3-2\nu}}}{\frac{4(\nu-\chi_2)}{c'^{\frac{1}{3-2\nu}}} G^{\frac{1-n(3-2\nu)}{3-2\nu}} \chi_2^{\frac{1}{3-2\nu}}} \right], \\
 106 \quad & \text{in which } \chi_1 = \frac{\nu+\nu(3-2\nu)}{3+n(3-2\nu)} \text{ " } \chi_2 = \frac{2\nu-1}{3+n(3-2\nu)}; \\
 V_0 = & \frac{B \Omega (f\kappa)}{\Xi \chi_2} \frac{1+n'(2\nu-1)}{\lambda^{\frac{1-n'(3-2\nu)}{2}}} - \frac{\chi}{3-2\nu} n' - \frac{\chi}{3-2\nu} \times \\
 & \times \frac{2[1-\nu-\nu(3-2\nu)]+\chi}{\delta^{\frac{2(3-2\nu)}{2(3-2\nu)}}} \times \\
 & \times \frac{2}{c'^{\frac{2}{3-2\nu}} [1-\nu-(\nu-\nu n')(3-2\nu)-\chi]} \times \\
 & \times \frac{P_B \frac{\chi}{2} (L_0-l_0)^{\nu} - \frac{1-\nu}{3-2\nu} + (1-\nu)n' l_0^{\frac{\chi+2[1-\nu-\nu(3-2\nu)]}{3-2\nu}}}{d'^{\frac{2}{3-2\nu}} [2\{\nu(3-2\nu)-(1-\nu)\}-\chi] G^{\frac{(1-n')(3-2\nu)+\chi[1-n(3-2\nu)]}{2(3-2\nu)}}}, \\
 & \text{in which } \chi = \frac{1+2\nu(3-2\nu)+(2\nu-1)(3-2\nu)n'}{3+n(3-2\nu)}.
 \end{aligned}$$

Through the use of this expression we may ascertain the influence of the following elements on the magnitude of initial velocity: l_0 , d' , G , L_0 , L_0-l_0 , P_B ; and also c' , assuming in the latter case that the lineal dimensions of the bore of the gun and of the shell are proportional to c' .

(a) *General remarks upon the influence of different elements in the binomial formulæ 106 for initial velocities.*—Consideration of this formula shows that the influence of the separate elements upon the magnitude of initial velocity may be expressed by functions of the following form:

$$(\alpha) \quad V_0 = B_1 x^p [1 - D_1 x^q]$$

If the considered element, x , exerts a direct influence, independent of the others, or

$$(\beta) \quad V_0 = B' x^p (f(x))^r [1 - D' x^q (f(x))^s],$$

if one of the elements is present as a function of another, e. g., if for a given length of gun L_0 , we may consider the length of the paths traversed by the shell $L = L_0 - l_0$ as a function of the length of the powder chamber l_0 .

In these formulae B_1 , B' , D_1 , and D' , are constant and are independent of the value of the given element x ; p , q , r , and s are exponents; they depend upon the properties of the powder and may be any quantities at all (positive or negative).

In the first case, formula (α) shows, page 209, that the initial velocity is a maximum for $x = x_0 = \frac{1}{D_1} \left(\frac{p}{p+q} \right)^{1/q}$ and reduces to 0 for $x = x_1 = \left(\frac{1}{D} \right)^{1/q}$, if p and q possess the same sign.

For such values of x (x_0 , x_1) the modulus of the powder falls in all cases between limits established (sec. 73, p. 213), for the use of the binomial formula.

When one of the exponents $q = 0$, the velocity p increases or decreases with the increase of the element considered according to the sign of p .

Cases where one of the elements constitutes a function of another, formula (β) must be considered separately, as the influence of these elements depends on the form of $f(x)$.

(b) *Influence of the length l_0 of the powder chamber and its most favorable value ($l_0)_n$.* Let us assume that all elements, including here the general length of the bore of the gun L_0 , are constant, and that only the length of the powder chamber changes. Correspondingly to this the length of the path followed by the shell ($L_0 - l_0$) also changes, as well as the weight of the charge ω , determined by formula 81. Expressions 106 for initial velocities show, among other things, that for a certain ratio between the values l_0 and L_0 , velocities attain a maximum. Such a value for the length of the powder chamber will be theoretically the most suitable. In order to obtain this, it is necessary to take the derivative of expression 106 for l_0 (or, what is the same thing), expressions

$$V_0 = B_1 (L_0 - l_0)^{\gamma} \frac{2}{l_0^{3-2\nu}} [x_1 - \gamma (3-2\nu) + (1-\nu)] \left[1 - \right. \\ \left. - D_1 (L_0 - l_0)^{\frac{2(1-\nu)}{3-2\nu}} \frac{2}{l_0^{3-2\nu}} x_2 \right] \\ V_0 = B_1' (L_0 - l_0)^{\gamma - \frac{1-\nu}{3-2\nu} + (1-\nu)n'} \frac{x_2 - 2 [\gamma (3-2\nu) - (1-\nu)]}{l_0^{3-2\nu}}$$

in which B_1 , D_1 , and B_1' are factors independent of the length of the

chamber and the length of the bore), and then equate them to 0. Observing that, as was said in paragraphs 70 and 73,

$$D_1 (L_0 - l_0) \frac{2(1-\nu)}{3-2\nu} \frac{2\chi_2}{l_0^{3-2\nu}} = \frac{1}{3} \Omega \frac{2}{3-2\nu}$$

the equation sought for will be

$$\begin{aligned} & \left[\frac{1}{3} \left(\gamma + \frac{2(1-\nu)}{3-2\nu} \right) \Omega \frac{2}{3-2\nu} - \gamma \right] (l_0)_H + \left[2 \frac{\chi_1 - \gamma (3-2\nu) + 1 - \nu}{3-2\nu} - \right. \\ & \left. - \frac{2}{3} \frac{\chi_2 + \chi_1 - \gamma (3-2\nu) + 1 - \nu}{3-2\nu} \Omega \frac{2}{3-2\nu} \right] (L_0 - (l_0)_H) = 0 \end{aligned}$$

and

$$\frac{\chi - 2[\gamma(3-2\nu)] - (1-\nu)}{3-2\nu} (L_0 - (l_0)_H) - \left[\gamma - \frac{1-\nu}{3-2\nu} + (1-\nu)n' \right] (l_0)_H = 0$$

whence we find two expressions for $(l_0)_H$:

$$\begin{aligned} (l_0)_H = L_0 \frac{6[\chi_1 - \gamma(3-2\nu) + 1 - \nu] - 2[\chi_2 + \chi_1 - \gamma(3-2\nu) + (1-\nu)] \Omega \frac{2}{3-2\nu}}{6\chi_1 + 6(1-\nu) - 3\gamma(3-2\nu) - [\chi_1 + \chi_2] - \gamma(3-2\nu) + 4(1-\nu)] \Omega \frac{2}{3-2\nu}} \end{aligned}$$

and

$$(l_0)_H = L_0 \frac{\chi - 2[\gamma(3-2\nu) - (1-\nu)]}{\chi - [\gamma(3-2\nu) - (1-\nu)] + (1-\nu)(3-2\nu)n'}$$

Both of these expressions for the most suitable theoretical length of the powder chamber show that it depends on the length of the whole bore of the gun; on the properties of the powder (γ , ν) and on the relative quickness of action of the powder in relation to the given gun (Ω and n'); but that it does not depend either upon the diameter of the powder chamber, the weight of shell or charge, the pressure, or upon many other elements influencing in one way or another the value of the initial velocity. Substituting instead of n' , its value page 214, we find that both these expressions afford one and the same value $(l_0)_H$ for the same moduli of the powder.

In the case of smoking powders ($\nu = \frac{1}{2}$; $\gamma = \frac{3}{5}$; $n = \frac{1}{4}$), and in the case of pyroxyline powders ($\nu = \frac{3}{4}$, $\gamma = \frac{1}{4}$, $n = \frac{1}{12}$), whence the expression as written takes the form

A. For smoking powders,

$$(l_0)_H = 6 L_0 \frac{3 - \Omega}{81 - 55 \Omega}$$

$$(l_0)_H = \frac{6 L_0}{13 + 28 n'}$$

B. For smokeless powders,

$$(l_0)_H = L_0 \frac{45-31 \Omega^{4/3}}{120-81 \Omega^{4/3}}$$

$$(l_0)_H = \frac{4 L_0}{8} \frac{7+12 n'}{26+41 n'}$$

Tables LI and LII present the fundamental values $\left(\frac{(l_0)_H}{L_0}\right)$ of the most suitable lengths of powder chambers for smoking and pyroxylin powders of different moduli.

TABLE LI.—*Most suitable relative lengths of powder chambers for smoking powders.*

Kind of powder.	Modulus of powder Ω .	Corresponding value of n' .	Most suitable corresponding length of powder chamber.
	1	0	$\frac{6}{13}=0.46$
Quick.....	$\frac{1}{11}=0.820$	$\frac{1}{6}$	$\frac{1}{11}= .37$
Mean.....	$\frac{1}{2}= .750$	$\frac{1}{8}$	$\frac{1}{8}= .34$
Slow.....	$\frac{6}{10}= .600$	$\frac{1}{4}$	$\frac{1}{5}= .30$
Very slow.....	$\frac{1}{2}= .375$	$\frac{5}{14}$	$\frac{2}{3}= .26$
	0	$\frac{1}{2}$	$\frac{2}{3}= .22$

TABLE LII.—*Most suitable corresponding length of powder chamber for pyroxylin powders.*

Kind of powder.	Modulus of powder Ω .	Corresponding value of n' .	Most suitable corresponding length of powder chamber.
	1	0	$\frac{1}{3}=0.360$
Quick.....	0.8054	$\frac{2}{3}$	$\frac{2}{3}= .370$
Mean.....	.7590	$\frac{1}{4}$	$\frac{1}{2}= .370$
Slow.....	.6817	$\frac{1}{2}$	$\frac{1}{2}= .370$
Very slow.....	.3774	$\frac{1}{5}$	$\frac{1}{3}= .370$
	0	$\frac{1}{2}$	$\frac{1}{2}= .375$

The first of these tables (for smoking powders) shows at a glance that the "slower" the powder used in a gun, the shorter must be the length of the powder chamber. In the case of pyroxylin powders (Table LI) the length of the most suitable chamber remains almost constant for powders of every degree of quickness, and there is even observed, contrary to that which constitutes the essential point for smoking powders, a certain, very small, it is true, yet sensible increase in this length for increase of slowness of the powder. This, perhaps, is due to the fact that the assumptions made in deducing the formula occasion a large error in the case of smokeless powders. In general the most suitable length for pyroxylin powders is almost the same as that for "quick" smoking powders.

We may observe that the weights of charge ω , for a given pressure P_B , do not depend (formula 81), page 212, on the modulus of the powder.

Let us consider how the fundamental values of initial velocity and weight of charge change with change in length of powder chamber between the limits 0.10 and 0.4 of the full length of the bore, assuming as unity that initial velocity which is developed for a length of chamber which is 10 per cent of the length of the bore.

To this end we employ formulæ 81, page 212, for weights of charge and the monomial formulæ 106, page 232, for initial velocities. These expressions assume the form:

A. For smoking powders,

$$107 \left\{ \begin{aligned} \omega &= \Xi_n^{-\frac{4}{7}} \left(\frac{a}{2fA\kappa} \right)^{\frac{4}{7}} \delta^{\frac{2}{7}} P_B^{\frac{4}{7}} \frac{(c'd')^{\frac{8}{7}} l_0^{\frac{5}{7}}}{G^{\frac{1}{7}}} \\ V_0 &= \frac{B_\Omega}{\Xi_n^{\frac{1}{7}} \lambda^{\frac{1}{2}}} \frac{(f\kappa)^{\frac{1}{2}}}{1-2n'} \left(\frac{2A}{a} \right)^{n'-\frac{5}{14}} \delta^{-\frac{2}{7}} P_B^{\frac{5}{14}} \times \\ &\quad \times \frac{(L_0-l_0)^{\frac{1}{8}+\frac{n'}{2}} l_0^{\frac{3}{28}} d'^{\frac{3}{14}} c'^{\frac{27}{28}-n'}}{G^{\frac{33-28n'}{56}}} \end{aligned} \right.$$

B. For pyroxylin powders,

$$108 \left\{ \begin{aligned} \omega &= \Xi_n^{-\frac{12}{15}} \left(\frac{a}{2fA\kappa} \right)^{\frac{16}{25}} \delta^{\frac{8}{25}} P_B^{\frac{12}{25}} \frac{(c'd')^{\frac{32}{25}} l_0^{\frac{16}{25}}}{G^{\frac{7}{25}}}, \\ V_0 &= \frac{B_\Omega}{\Xi_n^{\frac{1}{25}} \lambda^{\frac{1}{4}}} \frac{(f\kappa)^{\frac{11}{150}+\frac{9}{100}n'}}{8+3n'} \left(\frac{2A}{a} \right)^{\frac{63n'-32}{75}} \delta^{\frac{7+12n'}{150}} P_B^{\frac{8+3n'}{25}} \times \\ &\quad \times \frac{(L_0-l_0)^{\frac{1}{6}+\frac{1}{4}n'} l_0^{\frac{7+12n'}{75}} d'^{\frac{14+24n'}{75}} c'^{\frac{178-177n'}{150}}}{G^{\frac{206-129n'}{300}}} \end{aligned} \right.$$

Tables LIII and LIV present the relative values of velocities and charges for different lengths of powder chambers for smoking and pyroxylin powders.

TABLE LIII.—*Relative initial velocities and weights of charge for smoking powders.*

Kind of powder.	Relative length $\left(\frac{l_0}{L_0}\right)$ of powder chamber.						
	0.10	0.15	$\frac{2}{3}$	$\frac{2}{3}$	0.3	$\frac{1}{3}$	$\frac{1}{3}$
RELATIVE INITIAL VELOCITIES.							
Quick	1	1.033	1.060	1.068	1.073	1.076	1.076
Mean	1	1.032	1.054	1.064	1.067	1.069	1.068
Slow	1	1.030	1.050	1.055	1.056	1.055	1.053
Very slow	1	1.026	1.042	1.044	1.042	1.038	1.034
RELATIVE WEIGHTS OF CHARGE.							
For all powders	1	1.261	1.578	1.729	1.874	2.011	2.091

TABLE LIV.—*Relative initial velocities and weights of charge of pyroxylin powders.*

Kind of powder.	Relative length $\left(\frac{l_0}{L_0}\right)$ of powder chamber.						
	0.10	0.15	0.20	0.25	0.30	0.35	$(l_0)_H = 0.37^*$
RELATIVE INITIAL VELOCITIES.							
Quick	1	1.040	1.065	1.081	1.089	1.092	1.093
Mean	1	1.043	1.068	1.086	1.097	1.098	1.099
Slow	1	1.046	1.075	1.093	1.103	1.107	1.108
Very slow	1	1.057	1.092	1.115	1.130	1.134	1.135
RELATIVE WEIGHTS OF CHARGE.							
For all powders	1	1.296	1.554	1.798	2.039	2.230	† 2.323

* The figures for velocities in this table are calculated for the most favorable length for each kind of powder (see Table L).

† The weight of the charge is calculated for the most favorable length of chamber with a very slow powder (see Table LI).

The figures in both tables show at a glance that although there exists a most favorable length of powder chamber for each powder, yet it is not necessary in designing guns to fix upon the most favorable length of chamber. We may, in fact, limit the length of the chamber to between 20 and 25 per cent of the length of the bore of the gun, for which the loss in velocity in comparison with the velocity from the most suitable length of chamber does not exceed, at the outside, for smoking and pyroxyline powders, 2 per cent. By doing this we effect a gain in the weight of charge of 50 per cent. In this way we may assume in practice 20 to 25 per cent of the length of the bore of the gun as the most favorable length of powder chamber, and this does not depend either on the kind of powder used or upon the pressure which it is supposed to develop on firing. The justice of this deduction is fully borne out by many experiments of firing smoking powders. Comparatively little data exists in relation to smokeless powders, but in any case there is no reason for expecting considerable improvement in ballistic results afforded by them in existing guns from changing powder chambers.

(c) *The influence of diameter of powder chamber*, for smoking powders, as is seen from formula 107, does not depend on the modulus of the powder; while in the case of smokeless pyroxyline powders, formula 108, it does depend upon the modulus. Tables LV and LVI present the variations of relative velocities and weights of charges on changing the diameter of the chamber beginning with d' equal to the reduced caliber c' (p. 176.)

TABLE LV.—*Relative initial velocities and charges for chambers of different diameters for smoking powders.*

	Relative diameters of chamber $\frac{d'}{c'}$.						
	1.	1.05.	1.10.	1.15.	1.20.	1.25.	1.30.
For all kinds of smoking powders:							
Relative initial velocities	1	1.011	1.021	1.030	1.040	1.049	1.058
Relative weights of charge.....	1	1.057	1.115	1.173	1.229	1.290	1.350

TABLE LVI.—*Relative initial velocities and charges for chambers for pyroxyline powders.*

Kind of powder.	Relative diameter of chamber $\frac{d'}{c'}$.						
	1.	1.05.	1.10.	1.15.	1.20.	1.25.	1.30.
RELATIVE INITIAL VELOCITIES.							
Quick	1	1.013	1.025	1.037	1.048	1.059	1.070
Mean	1	1.013	1.026	1.039	1.051	1.063	1.074
Slow.....	1	1.014	1.028	1.042	1.055	1.068	1.080
Very slow	1	1.018	1.035	1.051	1.067	1.083	1.098
RELATIVE WEIGHTS OF CHARGE.							
For all powders	1	1.065	1.130	1.196	1.263	1.331	1.398

The figures in these tables show that change in diameter of chamber throughout quite a wide range has less significance in increasing the velocities in the case of smoking powders than it has for smokeless powders. Similarly, it is evident that in this way the same increase of velocity is obtained, as is obtained by lengthening the powder chamber, but for a considerably less increase in the weight of charge.

(d) *Influence of the weight of projectile.*—Generally speaking, increasing the weight of projectile diminishes the initial velocity, but what is important to us is not the initial velocity but the vis viva of the projectile. Therefore, in determining the influence of weight of projectile, we must investigate the change of its vis viva, assuming at the same time that all other conditions are equal.

When all the powder is consumed up to the moment that the projectile begins to move, formulæ 41, page 183, show that the vis viva of the projectile and the pressure do not depend on its weight and therefore, for the same pressures and other elements, the velocities

are inversely proportional to the square roots of the weights of projectiles (43).

On proceeding to more slowly burning powders, for which the velocities are determined by formula 106, the relationship of velocity and weights of charge, and consequently, the change in vis viva, become more complex. In order to determine this we square the formula for velocities and multiply it by $\frac{G}{2g}$. Denoting $\frac{G V^2}{2g}$ by Φ and assuming that all other elements except weights of charges do not change, we obtain,

$$\Phi = B_{III} G^{\frac{1-2[1-n(3-2\nu)]\chi_1}{3-2\nu}} \left[1 - D_{III} G^{\frac{1-[1-n(3-2\nu)]\chi_2}{3-2\nu}} \right]^2$$

$$\Phi = B'_{III} G^{n' - \frac{[1-n(3-2\nu)]\chi}{3-2\nu}}$$

We shall investigate first of all the expression obtained from the binomial formula for velocities. Its first derivative for G reduces to 0 for

$$G_1 = \left\{ \frac{1-2[1-n(3-2\nu)]\chi_1}{3-2[1-n(3-2\nu)]\chi_1 + \chi_2} \cdot \frac{1}{D_{III}} \right\}^{\frac{3-2\nu}{1-[1-n(3-2\nu)]\chi_2}}$$

and for

$$G_2 = \left(\frac{1}{D_{III}} \right)^{\frac{3-2\nu}{1-[1-n(3-2\nu)]\chi_2}}$$

The second derivative of the same expression $\frac{d^2\Phi}{dG^2}$ is negative for $G=G_1$, and positive for $G=G_2$. Consequently, the first of these roots corresponds to the maximum vis viva of the projectile and the second to the minimum, in which case the least vis viva, as is not difficult to see, reduces to 0. We also note that $G_1 < G_2$ for all powders employed, and besides this, for $G=G_1$, the modulus of the powder will be

$$\Omega_1 = \left[3 \frac{1-2[1-n(3-2\nu)]\chi_1}{3-2[1-n(3-2\nu)]\chi_1 + \chi_2} \right]^{\frac{3-2\nu}{2}},$$

i. e., $\Omega=0.72$ for smoking powders, and $\Omega=0.545$ for pyroxyline powders.

In this way all the conditions of firing for which the vis viva of the projectile diminishes on increasing its weight (commencing with $G_1 = \left(\frac{9}{37} \frac{1}{D_{III}} \right)^2$, for smoking powders and with $G_1 = \left(\frac{15}{101} \frac{1}{D_{III}} \right)^{\frac{15}{13}}$ for pyroxyline powders) are included between limits of use of the binominal formulæ for velocities.

In general, the vis viva of the projectile should increase on increasing the weight of charge (on account of the more complete combustion

of the powder) and strive toward a certain limit determined by expression 41. It reaches this limit when all the powder is consumed at the instant the projectile begins to move, and should remain the same on further increasing the weight of the latter.

The second of the resulting consequences—the impossibility of imparting to projectiles of weight G_2 any desired vis viva for all powders, relates to cases beyond the limits 87 and 88, pages 217–218, for the use of the binomial formulæ and, therefore, are never met with in practice under suitable circumstances.

Let us now turn to the monomial expression for vis viva. The exponent for G in this expression reduces to 0 for

$$n'_1 = \frac{[1-n(3-2\gamma)][1+2\gamma(3-2\gamma)]}{2(3-2\gamma)\{2-\gamma[1-n(3-2\gamma)]\}}.$$

We observe that we obtain such a value for n' from the general expression for n'_1 , page 214, if we take for Ω its value as found for it above or on page 239.

The value n'_1 equals $\frac{5}{28}$ for smoking powders and $=\frac{56}{128}$ for pyroxy-line powders.

In all cases where $n' < n'_1$, the exponent considered is < 0 and when $n' > n'_1$, it is > 0 .

This shows that for all “very quick,” “quick,” and “medium” powders, for moduli where $n' < n'_1$, the vis viva of the projectile diminishes on increasing the weight of the projectile; for all “medium” (beginning with $n' > n'_1$), “slow,” “very slow,” and “extremely slow” powders, the vis viva of the projectile increases to an unlimited extent on increasing the weight of the charge; finally, for “medium” powders for which $n' = n'_1$, the vis viva of the projectile does not depend on its weight.

If the weights of charges are not diminished on increasing the weights of the projectiles, then in this case, not only the vis viva but also the pressure increases toward the limits indicated above. Thus, in accordance with Noble's* experiments, there were obtained the following vires vivæ and pressures for various weights of projectiles and for the same charge of 10 English pounds weight:

Weights of projectiles.	Black charcoal powder, mark R. L. G. 2.		Chocolate powder.	
	Vis viva.	Pressure.	Vis viva.	Pressure.
Eng. lbs.	Eng. foot- tons.	Eng. tons per sq. in.	Eng. foot- tons.	Eng. tons per sq. in.
30	971.6	13.50	493.4	4.8
60	1,125.0	17.25	693.4
120	1,196.0	19.00	877.5
150	1,191.5	20.00	920.7
360	1,191.9	22.00	1,064.7	9.6

*See Captain Sir A. Noble's methods that have been adopted for measuring pressures in the bores of guns, 1894.

These experiments show that in the case of "quick" powder (R. L. G 2) the maximum vis viva was obtained by increasing the weight of the projectile four times, while on further increasing its weight there was practically no change in the vis viva; in the case of "slow" (chocolate) powder the limit was not reached even when the weight of projectile was increased 12 times in excess of the least weight of projectile employed in the experiment.

(c).—*Influence of weight of charge on the magnitude of the pressure in the bore of the gun.*—Expressions 76, 87, and 88, pages 209, 217, and 218, show that the pressures P_B increase to an unlimited extent, together with the weight of the projectile, both for smoking and for smokeless powders, and, therefore, are independent of the modulus of the powder. Such a consequence does not correspond, however, to the circumstances of the case. Actually, the projectile influences the increase of pressure only because its movement requires resistance to be overcome—i. e., it is necessary in order to move the shell, to apply a certain pressure to it which is developed by the portion of the charge consumed.

The greater the weight of the projectile the greater the resistance to movement, all other circumstances being equal, and consequently the greater the weight of powder that should burn before the shell is moved and the greater the pressure should be. The limit of increase of pressure will be, evidently, attained for such weight of projectile (see Noble's experiments cited above) for which all the powder burns before the shell is put in motion; this limit of pressure is evidently equal to that which would be obtained in a closed chamber for the same density of loading. The expression for the limiting pressure becomes consequently 18, 19, or 20, pages 77 and 78, which do not depend on the weight of the projectile. In this manner we perceive that the influence of the weight of charge in formulæ 76, 87, and 88 is exaggerated in extreme cases of the employment of these formulæ.

(f) *Influence of length of bore of gun* for the same pressure, weight of charge, caliber of gun, and diameter of chamber. In relation to the length of powder chamber and weight of charge, we assume that the length of the chamber in all cases constitutes a certain definite part of the length of the bore and that the weight of charge is determined by formula 81, page 212. Let us assume, then, that

$$l_0 = \mu L_0$$

and

$$L = L_0 - l_0 = (1 - \mu)L_0,$$

where μ is a quantity less than unity.

In consequence of this formulæ 81 and 106 may be written as

$$\omega = \Xi_{nIV} L_0^{\frac{2}{3+n(3-2\nu)}},$$

$$V_0 = B_{IV} L_0^{\frac{2}{3-2\nu}[\chi_1 - \nu(3-2\nu) + 1 - \nu] + \nu} \left[1 - D_{IV} L_0^{\frac{2}{3-2\nu}[1 - \nu + \chi_2]} \right],$$

$$V_0 = B'_{IV} L_0^{\frac{\chi + (1-\nu)}{3-2\nu} - \nu + (1-\nu)n'}$$

All that has been said in par. (a) in relation to properties of a function of the form α , page 232, should be applied to the binomial formula for velocities.

The monomial expression for velocity shows that the latter increases to an indefinite extent together with L_0 , and the more rapidly, the more slowly that the powder burns, which in fact does not occur, and only results from the approximate accuracy of the formula. It may be foreseen, a priori, that in the case of unlimited increase of the length of the bore, if friction of the projectile as well as of the gun, be neglected, the velocity should tend toward a certain limit not greater than that obtained from expression 41 for the value T by substituting in it $L = \infty$.

In order to judge the absolute value of the influence of the length of the bore of the gun on the magnitude of the initial velocity, Tables LVII and LVIII are presented below, which are established from the monomial formulæ for velocities, which, as also the formula for ω , assume the forms:

A. For smoking powders:

$$\omega = \Xi_{nIV} L_0^{\frac{4}{7}}$$

$$V_0 = B'_{IV} L_0^{\frac{13}{56} + \frac{n'}{2}}$$

B. For pyroxylin powders:

$$\omega = \Xi_{nIV} L_0^{\frac{16}{25}}$$

$$V_0 = B'_{IV} L_0^{\frac{13}{50} + 0.41n'}$$

In these tables the values of velocities and weights of charges corresponding to a certain length, L_0 , are assumed as units.

Comparison of these tables with Tables LV and LVI, previously considered, shows that the most rational way of increasing initial velocities is to increase the length of the bore of the gun, as in this case the weights of the charges increase to a less degree than they increase from changing the other elements considered.

TABLE LVII.—*Relative initial velocities and charges for guns of different lengths, L'_0 , of bore, for smoking powders.*

Kinds of powder.	For a length of bore $L'_0=$					
	1.10 L_0	1.20 L_0	1.40 L_0	1.60 L_0	1.80 L_0	2 L_0
RELATIVE INITIAL VELOCITIES.						
Quick	1.028	1.055	1.104	1.149	1.189	1.227
Mean	1.030	1.059	1.112	1.160	1.204	1.244
Slow	1.035	1.067	1.128	1.183	1.234	1.281
Very slow	1.040	1.078	1.148	1.213	1.273	1.329
RELATIVE WEIGHTS OF CHARGES.						
For all powders	1.056	1.110	1.212	1.308	1.399	1.486

TABLE LVIII.—*Relative initial velocities and charges for guns of different lengths of bore, L'_0 , for pyroxylin powders.*

Kinds of powder.	For a length of bore $L'_0=$					
	1.10 L_0	1.20 L_0	1.40 L_0	1.60 L_0	1.80 L_0	2 L_0
RELATIVE INITIAL VELOCITIES.						
Quick	1.034	1.066	1.125	1.179	1.229	1.276
Mean	1.036	1.070	1.153	1.190	1.242	1.292
Slow	1.039	1.075	1.166	1.204	1.263	1.316
Very slow	1.047	1.091	1.203	1.252	1.325	1.394
RELATIVE WEIGHTS OF CHARGES.						
For all powders	1.083	1.166	1.327	1.484	1.639	1.790

(g) *The influence of the length of the path of the projectile, all other conditions being equal.*—In this case formula 81, page 212, remains constant for all guns of the same caliber and the same construction of chamber, and formulæ 107 and 108, page 236, on substituting in them $L_0 - l_0$ for L , assume the forms:

A. For smoking powders:

$$V_0 = B'_v L^{\frac{1}{3} + \frac{n'}{2}}$$

B. For pyroxylin powders:

$$V_0 = B'_v L^{\frac{1}{3} + \frac{1}{2}n'}$$

Results of calculations by these formulæ are presented in Tables LIX and LX.

TABLE LIX.—*Relative initial velocities for different lengths of path traversed by the projectile, for smoking powders.*

Kinds of powder.	For a length of path $L' =$					
	1.10 L	1.20 L	1.40 L	1.60 L	1.80 L	2 L
RELATIVE INITIAL VELOCITIES.						
Quick	1.018	1.033	1.065	1.092	1.117	1.139
Mean	1.020	1.039	1.073	1.103	1.130	1.155
Slow	1.024	1.047	1.088	1.125	1.158	1.189
Very slow	1.029	1.056	1.108	1.153	1.195	1.234

TABLE LX.—*Relative initial velocities for different lengths of path traversed by the projectile, for pyroxylin powders.*

Kinds of powder.	For length of path $L' =$					
	1.10 L	1.20 L	1.40 L	1.60 L	1.80 L	2 L
RELATIVE INITIAL VELOCITIES.						
Quick	1.021	1.041	1.078	1.110	1.140	1.164
Mean	1.022	1.042	1.082	1.116	1.147	1.176
Slow	1.024	1.045	1.088	1.125	1.156	1.189
Very slow	1.029	1.054	1.106	1.151	1.193	1.231

A comparison of these tables with LVII and LVIII shows, as might have been expected, that when the length of the bore of the gun is increased it is advantageous to increase the length of the powder chamber correspondingly, and not stop with increasing the bore alone.

(h) *The influence of the pressure P_B on the value of the initial velocity*, all other conditions remaining the same. In the given case formula 106 shows that for smoking powders the influence of pressure does not depend on the modulus of the powder, and therefore in the case of all kinds of these powders, both velocities and weights of charge increase together with increase of pressures. The degree of their increase is shown in Table LXI, from which it will be seen that they afford the same results as from increasing the length of the bore in the case of "slow" smoking powders. (See Table LIX.)

In the case of smokeless powders it is evident that, as is shown in formulæ 107, change of velocity in relation to pressure depends on the modulus of the powder, whence the value of the increase of pressure grows, although to an insignificant extent, with the slowness of the powder, as is shown in Table LXII.

TABLE LXI.—*Relative initial velocities and charges for different pressures for smoking powders.*

	For pressures $P''_B =$					
	1.10 P_B	1.20 P_B	1.40 P_B	1.60 P_B	1.80 P_B	2 P_B
For all kinds of powder:						
Relative initial velocities	1.035	1.067	1.128	1.183	1.234	1.281
Relative weights of charge	1.056	1.110	1.212	1.308	1.399	1.486

TABLE LXII.—*Relative initial velocities and charges for different pressures for pyroxylin powders.*

Kinds of powder.	For pressures $P''_B =$					
	1.10 P_B	1.20 P_B	1.40 P_B	1.60 P_B	1.80 P_B	2 P_B
RELATIVE INITIAL VELOCITIES.						
Quick.....	1.034	1.067	1.131	1.193	1.252	1.309
Mean.....	1.035	1.068	1.133	1.197	1.257	1.315
Slow.....	1.036	1.070	1.137	1.202	1.264	1.324
Very slow.....	1.038	1.075	1.148	1.220	1.286	1.352
RELATIVE WEIGHTS OF CHARGE.						
For all kinds.....	1.024	1.046	1.088	1.144	1.163	1.198

On comparing Tables LXI and LXII together, and also with the preceding ones, it is not difficult to see that in the case of pyroxylin powders, it is especially advantageous to increase the pressure at the base of the bore, both on account of the considerable increase in velocity and on account of the small increase in the weight of charge required for the same.

(i) *Influence of caliber in similar guns.* In paragraph 64, page 198, it was shown, by employing general formulæ, that the same velocities for the same maximum pressures on the base of the projectile were developed in similar guns similarly loaded. As the pressure at the base of the bore is that actually determined, and as in most cases where smoking powders are used, it is convenient to employ one and the same powder in guns of different calibers, it becomes useful to investigate the consequences that arise from formulæ 81, page 212, and 106 page 227, for similar guns, on firing the same powders from them. If the guns and projectiles are similar, we may assume that their corresponding lineal dimensions are proportional to the reduced calibers, and the weight of the projectiles to the cubes of the same, i. e.,

$$L_0 = \alpha c', \quad l_0 = \beta c', \quad d' = \gamma c' \quad \text{and} \quad G = \delta c'^3,$$

where α , β , γ , and δ are the coefficients of proportionality.

Substituting this in equations 81 and 106, and considering all elements under consideration, except reduced calibers, as constant, we obtain

$$\omega = \Xi_{nVII} c' \frac{7+3n(3-2\nu)}{3+n(3-2\nu)}$$

and

$$V_0 = B_{VII} c' \frac{7+3n(3-2\nu)}{3-2\nu} \chi_1 - 3\gamma + \frac{1-3\nu}{3-2\nu} \left[1 - D_{VII} c' \frac{7+3n(3-2\nu)}{3-2\nu} \chi_2 + \frac{5-6\nu}{3-2\nu} \right],$$

$$V_0 = B'_{VII} c' \frac{7+3n(3-2\nu)}{3-2\nu} \frac{\chi}{2} - \frac{3}{2(3-2\nu)} - 3\gamma + n' \frac{5-6\nu}{2}$$

All that is said in paragraph (a) is applicable to the binomial formula for velocities in what relates to the value of the function of the form (α), page 231.

In relation to the powders under consideration, the expressions written above for the charge and the monomial expression for velocities assume the forms:

A. For smoking powders,

$$\omega = \Xi_{n_{VII}} c'^{\frac{17}{7}}$$

$$V_0 = B'_{VII} c'^{n' - \frac{5}{14}}$$

B. For pyroxylin powders,

$$\omega = \Xi_{n_{VII}} c'^{\frac{59}{25}}$$

$$V_0 = B'_{VII} c'^{\frac{n'}{4} - \frac{241}{150}}$$

These formulæ show: (1) In the case of smoking powders employed in similar guns the velocities diminish on increasing the caliber in all cases where the modulus of the powder is less than the most favorable, i. e., when $n' < \frac{5}{14}$; while in the case of the most favorable modulus Ω , i. e., for $n' = \frac{5}{14}$, the velocities do not depend upon the caliber; (2) when pyroxylin powders are employed in similar guns the velocities always diminish on increasing the caliber, for $\frac{n'}{4}$ is always $< \frac{2}{5}\frac{1}{6}$.

(3) The weights of the charges are proportional, not to the cubes of the calibers, but to lesser powers thereof ($\frac{1}{4}$ for smoking powders, and $\frac{5}{2}\frac{2}{5}$ for pyroxylin powders).

(j) Although it was stated in previous investigations that Sarrau's formulæ do not afford exact solutions for all extreme cases of firing, nevertheless they afford fully satisfactory results in ordinary circumstances of firing, and, besides this, they afford useful data in relation to the influence on initial velocity of the most important elements, such as length of powder chamber, length of bore, modulus of powder, etc. All this makes Sarrau's formulæ the most trustworthy and the most practical of all existing formulæ based upon theoretical and empirical conceptions.

81. *An approximate method of determining the influence on velocity and pressure of small variations in the magnitudes of the different elements of firing—Allowances for dimensions of grains.*

We shall employ for this purpose the monomial expressions for velocities, 87 and 88, pages 217 and 218, and also formulæ there deduced for pressures on the base of the bore. By differentiating the Napierian logarithms of these formulæ we obtain:

A. For smoking powders:

$$\begin{aligned}\frac{dV_0}{V_0} &= \frac{1}{2} \frac{df}{f} - n' \frac{da'}{a'} - \frac{1}{8} \frac{d\delta}{\delta} + \frac{5}{8} \frac{d\omega}{\omega} + \left(\frac{1}{8} + \frac{n'}{2} \right) \frac{l_0}{L_0 - l_0} - \\ &\quad - \left(n' - \frac{1}{4} \right) \frac{dc'}{c'} - \left(\frac{1}{2} - \frac{n'}{2} \right) \frac{dG}{G} - \frac{1}{2} \frac{dd'}{d'} - \frac{1}{4} \frac{dl_0}{l_0} \\ \frac{dP_B}{P_B} &= \frac{df}{f} - \frac{da'}{a'} - \frac{1}{2} \frac{d\delta}{\delta} + \frac{7}{4} \frac{d\omega}{\omega} + \frac{1}{4} \frac{dG}{G} - 2 \frac{dc'}{c'} - \\ &\quad - 2 \frac{dd'}{d'} - \frac{dl_0}{l_0}\end{aligned}$$

where $a' = \frac{a}{2A}$ (see formula 36, p. 154).

B. For pyroxyline powders:

$$\begin{aligned}\frac{dV_0}{V_0} &= \frac{2+n'}{4} \frac{df}{f} - n' \frac{da}{a} - \frac{1}{6} \frac{d\delta}{\delta} + \left(\frac{2}{3} + \frac{1}{4} n' \right) \frac{d\omega}{\omega} + \\ &\quad + \left(\frac{1}{6} + \frac{1}{4} n' \right) \frac{d(L_0 - l_0)}{L_0 - l_0} - \left(\frac{3}{2} n' - \frac{1}{3} \right) \frac{dc'}{c'} - \\ &\quad - \left(\frac{1}{2} - \frac{n'}{2} \right) \frac{dG}{G} - \frac{2}{3} \frac{dd'}{d'} - \frac{1}{3} \frac{dl_0}{l_0} \\ \frac{dP_B}{P_B} &= \frac{4}{3} \frac{df}{f} - \frac{4}{3} \frac{da}{a} - \frac{2}{3} \frac{d\delta}{\delta} + \frac{25}{12} \frac{d\omega}{\omega} + \frac{7}{12} \frac{dG}{G} - \\ &\quad - \frac{8}{3} \frac{dc'}{c'} - \frac{8}{3} \frac{dd'}{d'} - \frac{4}{3} \frac{dl_0}{l_0}\end{aligned}$$

where a is the least dimension of the grain.

Assuming in these expressions the denominators as the normal values of the corresponding elements, and the numerators as small variations therefrom, and assuming also that such changes actually occur, we may state that the relative changes in velocities and pressures are equal to the algebraic sum of the corresponding deviations from normal values of the elements, multiplied by the corresponding exponents of the degrees with which these elements enter into the monomial expression for velocities and pressures.

As a matter of fact we know only the arithmetical mean squares or the probable errors in the elements considered, and therefore the variations in initial velocities and pressures arising therefrom are equal to the square roots of the sum of the squares of the quantities considered for the different elements.

As smoking powders do not burn in concentric layers, the true variations in the value a , or, what is the same thing, in the least dimensions of those elements into which the grains disintegrate on combustion, are unknown.

In the case of smokeless powders the deviations from the least dimensions of grain may be exactly known, and depend upon the conditions of manufacture.

In experimenting in the same gun with powders which are the same but taken from different lots, the deviations in velocities and pressures depend chiefly on the variations in the least dimensions of the grains. Therefore, in order that a probable variation in velocities should not exceed, e. g., ± 0.002 *, the corresponding value of the probable variation in least dimension of grain (thickness) should not exceed $\pm 0.002 \frac{1}{n'}$, which affords for "medium" pyroxyline powders (page 219), ± 0.0075 . In this manner the permissible variation for powder of 3-mm thickness amounts to ± 0.1 mm. This indicates the necessity of careful workmanship in preparing the ribbons of powder of the proper thickness.

Under the stated assumptions for thickness of grain and velocity, the probable deviation in pressures will be ± 0.01 ,—i. e., for a mean pressure of 2,500 atmospheres, the limiting value of ΔP_B will be about ± 112 atmospheres.

In the case of quicker powders n' is less, and therefore the allowance in relation to the least dimensions of grain may be less rigid, if we desire that the relative deviation in initial velocities for them should not exceed a given upper limit. In firing slower powders from guns the requirements as to dimensions should be more rigid.

* For velocities of 2,500 ft., $\Delta V = \pm 5$ ft.

Chapter VIII.

INFLUENCE OF DIFFERENT CIRCUMSTANCES ON THE RESULTS OF FIRING.

82. *General conceptions.*

The formulæ deduced in the previous chapter for initial velocities and pressures of powder gases do not indicate clearly the influence upon them of various circumstances connected with firing, such as the recoil of the gun and carriage; the influence of the rifling, etc. (See paragraph 60.) Besides this, the velocities and pressures determined by the above formulæ relate to certain mean conditions—i. e., to the mean dimensions of the grain of the powder, moisture, force of powder, dimensions of the bore, construction and condition of the same, temperature of the powder and gun, and certain formations of charge, etc. On varying all these data, velocities and pressures will also change. In the present chapter we take up the investigation of all these circumstances, and, when necessary, their influence on the results of firing.

83. *The vis viva of recoil of gun and carriage.*

In the given case we are limited only to the determination of the approximate value of the vis viva of the recoil of the gun, assuming that the pressure remains the same throughout the whole extent of the bore in rear of the base of the projectile; that all products of decomposition of the charge and its unburnt portions occurring in certain sections of the bore possess the same velocity parallel to the axis of the bore of the gun; that the whole system is free—i. e., there exists no resistance in the way of friction; that movement occurs in the direction of the axis of the gun, which is horizontal, and that the cross section of bore and chamber are the same. The system considered consists of the following parts: The projectile, charge, gun, and those parts of the carriage which partake of the recoil. The pressure of the powder gases acting on the various parts of the system is an internal force. As the system is at rest up to the moment of firing, then, in accordance with the law of conservation of the quantities of movement, the sum of the quantities of movement of the different parts of the system considered is equal to 0 for every instant. In

view of this, the movement of the gun should be in a direction opposite to that of the projectile and charge (see below). The center of gravity of the whole system should remain in the same position.

If we designate by Γ the weight of the recoiling gun and parts of the carriage, by λ_r the distance of their common center of gravity (on the assumption that it lies in the axis of the gun) from the base of the bore, by λ_ω^* and λ_g the distances of the centers of gravity of charge and projectile from the same plane, then the distance of the general center of gravity of the whole system will be, up to the moment of firing,

$$\lambda_c = \frac{\Gamma \lambda_r + \omega \lambda_\omega + G \lambda_g}{\Gamma + \omega + G}$$

and at the expiration of the time t , when the gun has recoiled through a distance x , the distance of the center of gravity from the displaced base of the bore will be $x + \lambda_c$.

Assume that at the end of the same time t the velocities of the projectile and gun are V and U , respectively, and let the velocity of the elementary sections of the charge of weight $d\omega$ be u , then the quantities of motion will be; the system recoiling with the gun, $\frac{\Gamma}{g}U$;

the projectile, $\frac{G}{g}V$; and the charge, $\int_0^\omega \frac{u}{g} d\omega$; whence

$$\Gamma U = G V + \int_0^\omega u d\omega$$

Although the particles of the charge adjacent to the base of the projectile will acquire the velocity V , and those adjacent to the base of the bore of the gun a velocity U , in an inverse direction to v , nevertheless, in view of the assumption made, we may assume that the velocities u of the different elements of the charge vary constantly from 0 to V —i. e., that u is an unknown and complex function, and therefore may be written as

$$\int_0^\omega u d\omega = u_c \omega$$

where u_c is the arithmetical mean of all the values u from 0 to V . Assuming, moreover, that this value u_c is proportional to the velocity V of the projectile, we obtain

$$\int_0^\omega u d\omega = \Theta_1 V \omega,$$

where Θ_1 is a certain positive quantity less than 1, since $u_c < V$.

* It equals one-half of the reduced length of the powder chamber.

In consequence of this, the above equation for quantities of motion assumes the form

$$\Gamma U = (G + \Theta_1 \omega) V$$

whence

$$U = \frac{G + \Theta_1 \omega}{\Gamma} V$$

and the vis viva of the recoil Φ_{gn} will be

$$\Phi_{gn} = \frac{\Gamma U^2}{2g} = \frac{(G + \Theta_1 \omega)^2 V^2}{2g \Gamma}$$

The ratio of this to the vis viva of the projectile will be

$$\frac{\Phi_{gn}}{\Phi_p} = \frac{G}{\Gamma} \left(1 + \Theta_1 \frac{\omega}{G} \right)^2,$$

or, on account of the small value of the second member in brackets,

$$\frac{\Phi_{gn}}{\Phi_p} = \frac{G}{\Gamma} \left(1 + 2 \Theta_1 \frac{\omega}{G} \right)$$

For the moment of flight of the projectile from the muzzle we may assume* $\Theta_1 = \frac{1}{2}$. As an actual fact, Θ_1 is a certain function of the time t and of the path traversed by the projectile. This value is determined experimentally from the recoil of a free gun. As $\frac{G}{\Gamma}$ equals from $\frac{1}{50}$ to $\frac{1}{200}$, and as $\frac{\omega}{G}$ varies approximately between $\frac{1}{10}$ and $\frac{1}{2}$, the vis viva of recoil of a free gun constitutes a small part (up to 3 per cent), of the vis viva of the projectile.

84. The vis viva of the products of decomposition of the charge.

In order to deduce an expression for the vis viva of the products of decomposition, we assume that the gun does not recoil and that when the time t has elapsed the velocity of the shell is v , the pressure on the unit area of its base is P , and on the unit area of the base of the bore is P'_B , and that the path traversed by the projectile is l . Dividing the whole extent between the base of the projectile and of the bore of the gun into elementary sections, we may assume that both the densities and the mean velocities of the particles included in them vary on passing from one layer to another, in accordance with a certain unknown law. In view of this, instead of determining the actual vis viva of the products of decomposition, we may calculate similarly to the method explained in 60, page 176, the work of the forces producing the same.

In fact, the force moving the products will be $s(P'_B - P)$, and its work in the extent between O and L , which is equal to the vis viva sought for, ϕ_{Pr} , will be

$$\phi_{Pr} = s \int_0^L (P'_B - P) dl$$

*Sebert et Hugoniot "Études des effets de la poudre, etc.," p. 47.

On the basis of formula (74), page 208,

$$P'_B = P \left(1 + \Theta_2 \frac{\omega}{G} \right)$$

where Θ_2 is a certain constant quantity, generally speaking, unequal to the coefficient Θ in formula 74.

In consequence of this

$$\phi_{Pr} = \Theta_2 \frac{\omega s}{G} \int_0^L P \, dl$$

As the approximate vis viva of the projectile ϕ_P for the same time will be

$$\phi_P = s \int_0^L P \, dl$$

then

$$\frac{\phi_{Pr}}{\phi_P} = \Theta_2 \frac{\omega}{G}$$

The value Θ_2 may be found only by experiment. Assuming that the density of the products of decomposition throughout the whole extent in rear of the projectile is constant, Θ_2 equals $\frac{1}{3}$ * for smoking powders.

Sebert and Hugoniot† arrived at the conclusion that $\Theta_2 = \frac{1}{2}$. Count St. Robert‡, assuming that the density of the products in rear of the projectile was constant, and bearing in mind the recoil of the gun, found a value for the desired function of something less than one-third.

In view of this we may assume in general

$$\Theta_2 = \mu \Theta$$

where Θ has the value $\frac{2}{3}$, previously deduced, for smoking powders, and the value $\frac{1}{2}$ for pyroxyline powders, and where μ may vary from 0.2 to one-third.

On these assumptions the vis viva of the products of decomposition of the charge constitutes in the case of smoking powders from 3 to 19 per cent of the vis viva developed by the projectile for the same charge, and constitutes from 1 to 8 per cent of the same value in the case of smokeless powders.

85. The influence of rifling and general equations of progressive and rotary motion of the projectile.

The influence of the rifling includes the following: (1) When the projectile moves in the bore of the gun, the driving edges of the studs on the projectile take against the corresponding edges of the rifling,

* M. Helie "Traité de Balistique expérimentale," 2d ed. p. 41.

† See Sebert et Hugoniot "Étude sur les effets," etc., p. 63.

‡ See P. de Saint-Robert. "Mémoires Scientifiques," Vol. II, p. 199.

since the direction of the latter produced is inclined to the axis of the bore; (2) friction is developed by the pressure of the rifling against these studs or projections; and (3) when the metal of the band on the projectile is cut by the rifling there results not only a cutting, but an elastic yielding of the metal, so that pressure* is developed between the band and the walls of the gun. The amount of this pressure depends upon the metal of the band and of the gun, upon the dimensions of the walls of the latter and of the projectile, and upon the section of the compressed band. Generally speaking, this pressure arises from the reaction of the band; it is an unknown quantity and causes friction between the surfaces in contact.

Besides this, when the band cuts into the rifling to form projections on its surface, a certain amount of the work of the gases is expended in overcoming resultant resistances. The direct result of this is an increase of initial pressure or, more correctly, an increase in the rate of development of gas pressure on the base of the projectile, as the movement of the projectile in guns of present-day construction commences almost at zero pressure, while the resistance to cutting grows very rapidly at the beginning of motion.

The magnitude of the resistance of the rifling to the cutting of the rotating parts depends upon the metal and the structure of those parts, and also upon the structure of the rifling, but, generally speaking, this resistance is small.

A suitable investigation of this portion of the resistance is made in Part II of the present course.

From the combined action of the forces considered the projectile receives a rotary motion around its axis of figure, and the velocity of its progressive movement is, generally speaking, reduced.

For establishing conceptions of the influence of the cutting of rifling into projectile and the "reaction of the band" upon ballistic results of firing, we shall cite here certain results from the experiments of Sebert and Hugoniot.† On firing a charge of 3.2 kilograms of A_3s powder from a 10-centimeter gun they found that a projectile of 12-kilogram weight attained an initial velocity of 492 meters per second when the rifling cut into the rotating band of the projectile, and of 454 meters per second when the projectile was rotated in traversing the bore by means of projections previously cut upon it and fitting into the rifling. In the first case the maximum pressure on the base of the projectile was 2,117 kilograms per square centimeter, and in the second it was 1,235 kilograms per square centimeter. In consequence of the smaller resistance to the movement of the projectile in the case of

*See Artillery Journal, No. 7, 1898; paper by M. Rosenberg; "Materials for investigating the reactions of the rotating band upon the walls of the gun and upon projectiles."

† See their "Études sur les effets," etc., pages 52 and 63.

the already cut projections, the combustion of the powder in the bore of the gun proceeded very slowly and the projectile had a smaller velocity imparted to it, for which, however, the pressure was less than in the previous case. If we limit ourselves, in the case of projectiles upon which no studs are cut, to the same velocity as is obtained by the projectiles having studs already cut upon them, then we would have to reduce the weight of charge approximately by 3.05 kilograms, in which case the pressure would be about 1,690 kilograms—i. e., it would exceed the pressure obtained from the projectile upon which studs were cut by about 40 per cent. The useful work of the powder—i. e., the ratio of the vis viva of the projectile to the weight of the charge is about 5 per cent greater when the shell cuts into the rifling than when firing projectiles upon which studs are already cut. In other words, the cutting of the shell into the rifling increases the useful action of the powder; the pressure of the gases for the same initial velocity rises, while the weight of the charge diminishes.

The driving edges of the rifling may possess different sections. Their usual section—in a plane perpendicular to the axis of the bore—is a straight line, but it may be curvilinear, as in the case of the English rifling. The pressure between the studs and the rifling is always outward in a direction normal to the plane of contact of the studs, and is equally distributed along the driving edges of the studs, and therefore it reduces to one resultant along the center of each driving edge. If we denote the pressure on one projection by N and the number of riflings (for convenience let us suppose that this number is even) by n , then the whole pressure between the projections and the rifling will be nN . As the projectile moves, this pressure creates friction between the surface of the projections and the rifling. The amount of this equals $f n N$, where f is the coefficient of friction and is exerted in the direction of the tangent to the curve of the rifling. The pressure created through the reaction of the rotating band may, generally speaking, vary according to whether we consider the contact between the side of the rifling and the rotating band or between the bottom of the rifling and the rotating band; but, for the sake of simplicity, we assume it as uniform for the whole surface and to possess a certain mean value over the whole extent of bottom and side. Denoting it by Φ , the whole effect of the reaction is expressed by the quantity $n\Phi$, which creates a friction equal to $f' n \Phi$. The mean value of this force Φ may be determined experimentally (see Part II of the Course). Besides this, the pressure of the powder gases $P \frac{\pi c'^2}{4}$ acts on the base of the projectile, which may be considered as applied in the line of the axis of the gun. We may neglect the other forces acting on the projectile.

Applying, then, these forces $\left(nN, fnN, f'n\Phi, P\frac{\pi c'^2}{4} \right)$ to the projectile, the latter may be regarded as a free body and we may, therefore, employ equations of equilibrium in relation to it. The projection of the exterior forces and the forces of inertia in any three mutually perpendicular directions, and the moments of these forces with respect to the same axes, should reduce to 0. For simplicity, we shall resolve the forces in the three following perpendicular directions; X, the axis of the bore in the direction of movement of the projectile; Y, the perpendicular to the axis of the bore; Z, a direction opposite to that of the radius produced to the center of the driving edge of the stud or projection. The origin of coordinates is taken at such a point of the axis of the bore that the plane ZY passes through the vertex of the rifling curve, whose origin sometimes coincides with the plane ZY, but, for general purposes, we shall assume that the distance between the origin of coordinates—i. e., the distance between the vertex of the rifling curve and the origin of the curve in the gun—equals a . The conditions of equilibrium in this case will evidently be the same as for constant axes or coordinates.

For simplicity in final formulæ we may characterize the curvature of the driving edge by the angle δ , which is formed by the inner normal at the center of the driving edge of the projection with the inverse direction of the radius of the bore passing through the same center. Usually δ equals nearly 90° , i. e., the line of the driving edge almost coincides with the radius produced through its center. In the English system of rifling the value of δ is variable, and therefore a mean value must be assumed for it.

Generally speaking, the angle δ varies in the case of progressive rifling on passing from one section of the bore to another. The cause of this is due to the fact that in rifling the tool does not revolve around the radius produced to the center of the driving edge. In consequence of this, the *width* of the rifling is constant in the direction of curvature of the bore; but the *actual width* of the riflings—i. e., their dimensions in a direction perpendicular to the driving edge—diminishes toward the muzzle. The corresponding relations are deduced at the end of this paragraph.

Let us assume that in an interval of time t the shell has traveled along the axis of x a distance, l , so that its distance from the origin of coordinates is $x=a+l$; assume that during this time the projectile has rotated around its axis of figure in the direction of the curve of the rifling through a certain angle φ . Any other changes of position and rotation of the properly centered projectile in its movement along the bore are impossible.

The projection on the axis of X of the force of inertia of the progressive movement of the projectile on the axis of X will be

$$m \frac{d^2 x}{dt^2} = m \frac{d^2 l}{dt^2} = m \frac{dv}{dt}$$

where v is the velocity of the projectile.

The projections of this force of inertia on the other axes are equal to 0.

Denoting the moment of inertia of the body of the projectile with respect to its axis of figure, which coincides with the axis of X, by A, the moment of the force of inertia in relation to this axis will be

$$A \frac{d^2 \varphi}{dt^2} = \lambda m \left(\frac{c'}{2} \right)^2 \frac{d^2 \varphi}{dt^2}$$

where the coefficient λ depends upon the form and structure of the projectile. The following figures are usually assumed: $\lambda = 0.53$ for cast and armor-piercing projectiles of length of from 2.5 to 3.5 calibers; $\lambda = 0.55$ for the same projectiles of length from 3.5 to 4 calibers; and $\lambda = 0.60$ for the thin-walled mine shells.*

We find from the equation of the curve of rifling

$$y = \frac{c'}{2} \varphi = F(x)$$

$$\frac{c'}{2} \frac{d^2 \varphi}{dt^2} = F'(x) \frac{d^2 x}{dt^2} + F''(x) \left(\frac{dx}{dt} \right)^2$$

Observing that

$$F'(x) = \tan \alpha; \frac{dx}{dt} = v; \frac{d^2 x}{dt^2} = \frac{dv}{dt}$$

where α is the angle between the tangent to the curve of rifling and the prolongation of the bore, we obtain

$$A \frac{d^2 \varphi}{dt^2} = \lambda m \frac{c'}{2} \left[\tan \alpha \frac{dv}{dt} + v^2 F''(x) \right]$$

In order to determine the projections and the moments of the exterior forces with respect to the axes of coordinates, there is represented in Fig. 19 a perspective view of the bore of the gun, without showing the walls of the latter, with a portion of the progressive rifling, the dimensions of which are increased for convenience in inspection. We shall consider in this sketch only one pair of projections or riflings, with the driving edges and the bottoms of the riflings adjoining them and situated at the extremities of the same diameter, AA'.

In this figure the lines XX, YY, and ZZ are the axes of coordinates; the arrow drawn on XX indicates the direction of progressive movement of the projectile; UAU is the projection of the driving edge of

*See N. Zaboudski's Exterior Ballistics.

the rifling produced through the center A of one of the studs or projections on the projectile; AM is the tangent to the curve UAU at the point A ; $U\Phi$ is the bottom of the rifling adjacent to the driving edge; AL is the inner normal to the curve UAU at the point A , along

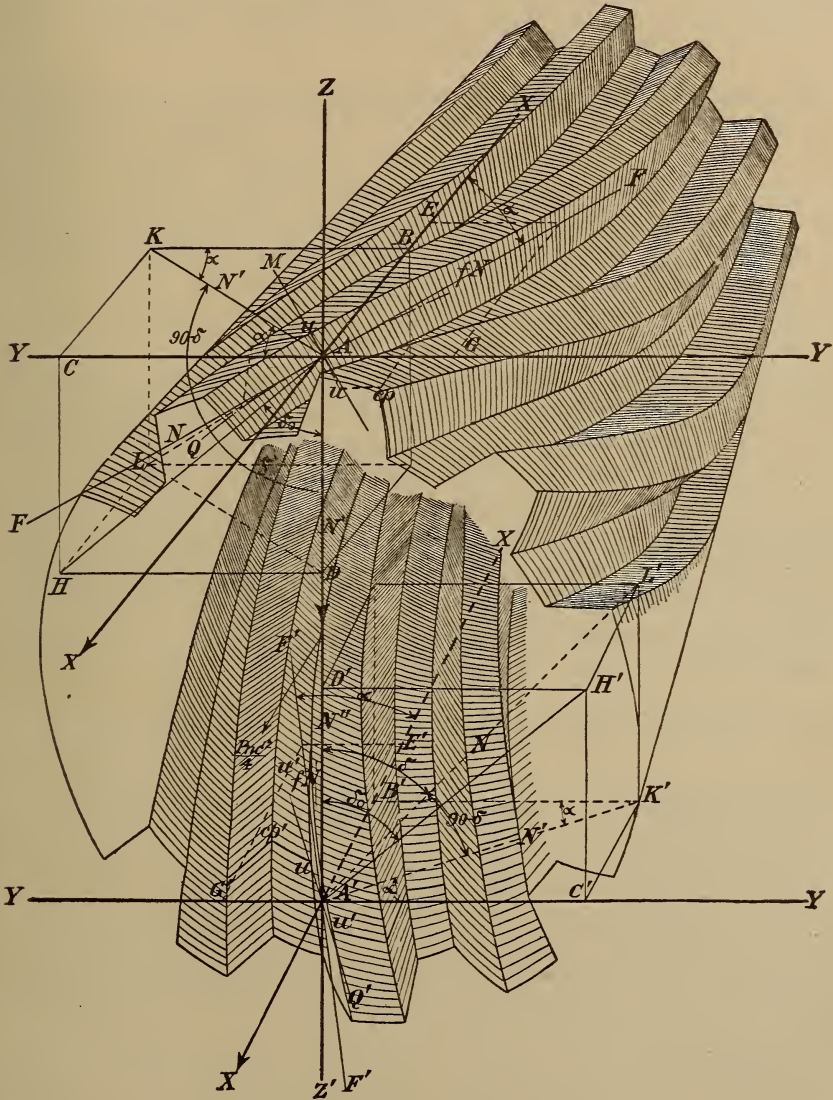


FIG. 19.

which the normal pressure N is directed; AH is perpendicular to AM we shall denote the angle HAD by δ_0 —it lies in the plane of the cross section of the bore of the gun, YZ , and is a constant quantity throughout the whole extent of the bore of the gun for all riflings;

AQ is the curve of rifling; AF is the tangent thereto—along this line is directed the force of friction fN —the force $f'\Phi$ is parallel to the same tangent and directed similarly to the force fN (for simplicity the force $f'\Phi$ is not shown); the force actuating the projectile $P \frac{\pi c'^2}{4}$ is applied to the center of the projectile in the axis of the bore of the gun.

The tangent AF lies in the plane XY, perpendicular to the line ZZ and AL, and, therefore, to the plane ZADL; in consequence of this the line AF is also perpendicular to the line AK. The angle XAF, formed by the tangent to the rifling curve and the line XX, is denoted by α . Since the sides AF and AK, AX and AY are perpendicular to one another, $\angle CAK = \alpha$. The angle LAD is the angle formed by the normal and the inverse to the direction of the radius. It is denoted by δ . Finally, $\angle LAZ = 180^\circ - \delta$. The projections of the force N on the axes X, Y, and Z are AB, AC, AD; and the projections of the force fN are AE and AG; the projections of the force $f'\Phi$ are in directions parallel to the projections of the force fN . Considering the projections of the same forces on the land lying opposite, whose driving edge is U'A'U', and whose bottom is U'\Phi, and observing that the whole pressure N is directed in rear of the plane YZ, we see that certain projections of forces, namely, A'C', A'B', A'G', and also the corresponding projections of the force $f'\Phi$, have opposite directions in comparison with the same projections for the first land; but the absolute value of the projections of each force on both sides are evidently equal to one on another. In consequence of this,

	N,	fN ,	$f'\Phi$,	$P \frac{\pi c'^2}{4}$,
Projections on the axes of coordinates, of the forces acting on two opposite studs at the extremities of the same diameter.	$\left\{ \begin{array}{l} \text{X: } -2N \sin \delta \sin \alpha, -2fN \cos \alpha, -2f'\Phi \cos \alpha, \frac{2}{n} P \frac{\pi c'^2}{4}; \\ \text{Y: } 0 \qquad \qquad \qquad 0 \qquad \qquad \qquad 0 \qquad \qquad 0 \\ \text{Z: } 0 \qquad \qquad \qquad 0 \qquad \qquad \qquad 0 \qquad \qquad 0 \end{array} \right.$			
Moments of the projections on the axes of coordinates of the forces acting on the same two studs.	$\left\{ \begin{array}{l} \text{X: } c'N \sin \delta \cos \alpha, -c'fN \sin \alpha, -c'f'\Phi \sin \alpha, 0; \\ \text{Y: } 0 \qquad \qquad \qquad 0 \qquad \qquad \qquad 0 \\ \text{Z: } 0 \qquad \qquad \qquad 0 \qquad \qquad \qquad 0 \end{array} \right.$			

In the latter, instead of the actual point of application of the forces, the distance $\frac{c'}{2}$ is taken, but the difference between these two quantities is insignificant.

From this it is evident that the projections and moments of the external forces with respect to the axes of Y and Z are each actually equal to 0.

Taking the sum of the projections of the external forces and the sum of their moments with respect to the axis of X; multiplying this quantity by the number $\frac{n}{2}$, the number of pairs of riflings, and equating them afterwards with the corresponding expressions for the forces of inertia, we obtain

$$109 \quad \left\{ \begin{aligned} m \frac{dv}{dt} &= P \frac{\pi c'^2}{4} - nN [\sin \delta \sin \alpha + f \cos \alpha] - nj' \Phi \cos \alpha, \\ m\lambda \left[\tan \alpha \frac{dv}{dt} + v^2 F''(x) \right] &= nN [\sin \delta \cos \alpha - f \sin \alpha] - \\ &\quad - nj' \Phi \sin \alpha = 2 \frac{A}{c'} \frac{d^2 \varphi}{dt^2}. \end{aligned} \right.$$

If there were no riflings, the former of these equations would reduce to the form

$$m \frac{dv}{dt} = P \frac{\pi c'^2}{4}$$

and the second would not exist at all.

Therefore, the resistance R opposed by the rifling to the progressive movement of the projectile will be

$$110 \quad R = n \frac{N (\sin \delta \tan \alpha + f) + j' \Phi}{\sqrt{1 + \tan^2 \alpha}}$$

In order to see more completely the influence of the rifling we shall eliminate nN from equation 109;

$$\left\{ \begin{aligned} m + m\lambda \tan \alpha \frac{\sin \delta \sin \alpha + f \cos \alpha}{\sin \delta \cos \alpha - f \sin \alpha} \frac{dv}{dt} &= P \frac{\pi c'^2}{4} - \frac{nj' \Phi \sin \delta}{\sin \delta \cos \alpha - f \sin \alpha} \\ - m\lambda v^2 F''(x) \frac{\sin \delta \sin \alpha + f \cos \alpha}{\sin \delta \cos \alpha - f \sin \alpha} & \end{aligned} \right.$$

Whence it follows that the influence of the riflings is, in the first place, as if the mass of the projectile were increased by the quantity

$$m\lambda \tan \alpha \frac{\sin \delta \sin \alpha + f \cos \alpha}{\sin \delta \cos \alpha - f \sin \alpha}$$

and in the second place as if the moving forces were diminished by

$$\frac{m\lambda v^2 F''(x) (\sin \delta \sin \alpha + f \cos \alpha) + nj' \Phi \sin \delta}{\sin \delta \cos \alpha - f \sin \alpha}$$

while, for rifling of constant curvature $F''(x) = 0$; and for progressive rifling $-F''(x) > 0$, since the curve of rifling is convex to the axis of the bore throughout its whole length.

In this way the influence of the rifling, in the case that its curvature is constant, may easily be found by the data α , δ , f , f' , and Φ . When it is progressive, however, it is necessary to know, besides this, the pressure of the gases and the velocity of the projectile at each section of the bore.

We shall limit ourselves to considering rifling of constant curvature and projectiles with studs already made upon them, i. e., assuming $\Phi=0$. Taking $\delta=90^\circ$, $\alpha=8^\circ$, $f=0.2$, $\lambda=0.53$, we find that the influence of the rifling of a given constant curvature ($\alpha=8^\circ$) is equivalent to an increase in the mass of the projectile by $0.028 m$, and as, e. g., in formulæ 89, 93, 87, and 88 velocities for quick, smoking powders and smokeless powders are respectively inversely proportional to $G^{\frac{1}{16}}$ and $G^{\frac{1}{18}}$, and pressures are directly proportional to $G^{\frac{1}{4}}$ and $G^{\frac{1}{12}}$, it follows that the influence of constant rifling whose curvature is 8° is insignificant, since the velocities, in the case of smoking and smokeless powders, diminish by less than $0.012 v$, while pressures are increased by not more than $0.008 P_B$ and $0.018 P_B$. If we should subsequently desire to reduce pressures correspondingly, velocities would be reduced by 0.003 , and therefore the whole influence of the rifling would consist in diminishing velocity by 0.015 per cent; the expenditure of work of the powder gases due to the increase of the rifling constitutes about $0.03 \phi_P$. Such is the result of investigation on the assumption that the character of combustion of the same powder is the same in both a rifled and a smooth-bore cannon. This, however, is not the case in practice, as is indicated by the formulæ of Sebert and Hugoniot, given above on page 253.

Eliminating from both equations (109) $m \frac{dv}{dt}$, we obtain the following expression for the full normal pressure on one projection (stud):

$$111. \quad N = \frac{1}{n} \frac{\lambda \left(P \frac{\pi c'^2}{4} \tan \alpha + m v^2 F''(x) \right) \sqrt{1 + \tan^2 \alpha} + (1 - \lambda) n f' \Phi \tan \alpha}{\sin \delta [1 + \lambda \tan^2 \alpha] - (1 - \lambda) f \tan \alpha}$$

Bearing in mind that $\tan \alpha$ is usually not > 0.14 , f is not > 0.3 , and $\lambda < 1$, we may simplify the last expression for N and also the expression for R by removing from the denominators the terms containing products of the quantities $\tan \alpha$, f and λ .

In consequence of this

$$112. \quad \begin{cases} N = \frac{1}{n} \frac{\lambda \left[P \frac{\pi c'^2}{4} \tan \alpha + m v^2 F''(x) \right] + n f' \Phi \tan \alpha}{\sin \delta} \\ R = n [N (\sin \delta \tan \alpha + f) + f' \Phi] \end{cases}$$

The pressure N' , which endeavors to cut studs on the projectile, equals (fig. 19) the projection AK , of the pressure N on the plane XY , namely,

$$N' = N \sin \delta$$

The pressure N'' , which centers the shell and in general endeavors to crush the studs, equals the projection AD of the pressure N on the axis Z , namely,

$$N'' = N \cos \delta$$

In both these expressions we must replace N by one of the values given for it above.

We observe that in the rifling employed by us δ almost equals 90° , and therefore, in this case, $N' = N$ and $N'' = 0$.

Assuming that projections (studs) are already cut on the projectile, i. e., that $\Phi = 0$, the second equation (109) shows that the moment with respect to the axis of X of the projection of the external forces on the axis of Y equals 0 for a certain value $\delta = \delta'$, namely, for

$$\sin \delta_1 = f \tan \alpha$$

which gives for positive values of δ_1 , less than 180° , two values, δ_1 and $180 - \delta_1$. For every value of δ satisfying the inequality

$$180^\circ - \delta_1 > \delta > \delta_1$$

the moment in the expression is positive, and therefore the projectile acquires the necessary rotating motion in its movement along the bore (in the direction of the curvature of the rifling), if the rotating parts are sufficiently strong.

For any value of δ satisfying the inverse inequality

$$180^\circ - \delta_1 < \delta < \delta_1$$

the moment of projection of the same forces will be negative; that is, the moment of projection of the force N on the axis of Y with reference to the axis of the projectile is less than the moment, with reference to the axis of the projectile, of the force fN on the axis of Y , and therefore the projectile will tend to turn in a direction opposite to the curve of rifling, which is impossible.

If $\delta = 90^\circ$, the moment of projection of the external forces may be reduced to 0 only for

$$\tan \alpha' = \frac{1}{f}$$

i. e., for $\alpha' = 60^\circ$, which is an inclination never given to rifling.

It is evident from expression 111 that the normal pressure N increases with the diminution of $\sin \delta$ and for

$$\sin \delta_2 = \frac{1 - \lambda}{1 + \lambda} \frac{f \tan \alpha}{\tan^2 \alpha}$$

it becomes infinitely great, along with the projections of N' and N'' .

This condition also affords two values for δ ; δ_2 and $180^\circ - \delta_2$. From equating them with δ_1 , it follows that $\sin \delta_1 > \sin \delta_2$, and therefore rotation of the projectile in this case is impossible. Similarly, the progressive movement of the projectile is impossible in this case, for the resistance R (110) to such movement is infinitely great.

It must be observed that the angles δ_1 and δ_2 are usually very small and their complements are large; thus, for $\alpha = 8^\circ$, $\delta_1 = 1^\circ - 8'$; $180^\circ - \delta' = 178^\circ - 52'$; while the angle δ_2 , as we saw above, is less than $1^\circ - 8'$. Riflings with such angles of inclination of the driving edge do not usually exist.

In the case of rifling for which $\delta = 90^\circ$, the normal pressure N may become infinitely great for a value of α , which satisfies the equation

$$\tan^2 \alpha = \frac{1-\lambda}{\lambda} f \tan \alpha + \frac{1}{\lambda} = 0$$

i.e., for

$$\tan \alpha' = f \frac{1-\lambda}{2\lambda} \left[1 \pm \sqrt{1 - \frac{4\lambda}{(1-\lambda)^2 f^2}} \right]$$

but this expression, for projectiles of usual construction, is always imaginary, and therefore for such rifling ($\delta = 90^\circ$) the pressure N can not be infinitely great.

As in actual practice, in designing proper rifling, the angle δ_0 between the normal AH to the driving edge of the projectile in the plane YZ and the inverse direction of the radius is given, we shall indicate the relationship between the angles δ and δ_0 . From the triangles ALK , AKC , and AHD (fig. 19) we have:

$$AK = KL \tan \delta = AC \sec \alpha; \quad DH = AC = AD \tan \delta_0;$$

whence, from the equality of the sides KL and AD , the desired relationship becomes

$$\tan \delta = \tan \delta_0 \sec \alpha.$$

This projection of the width of the rifling, q , on the plane XY (fig. 19), reckoning in circumference, is in the direction of the line AC , and the projection in the same plane of the actual width of the rifling, q_0 , is in the direction AK . Drawing an imaginary line from the extremities of the first projection on AC and parallel to AF to its intersection with AK , we find

$$q_0 = q \cos \alpha.$$

86. *Expressions for pressures on the driving edge and for resistances of the rifling, for various equations of their curves.*

(1) *Parabolic rifling with the vertex of the curve at the origin of coordinates.*

(a) *Parabola of the k-th degree.*—Let the equation of the curve of the rifling be

$$y = px^k = F(x),$$

then

$$\tan \alpha = kpx^{k-1}$$

and

$$F''(x) = k(k-1)px^{k-2}$$

where, according to what was said on page 255, $x = a + l$.

Substituting this in the complete expression for N (111) and R (110), we obtain:

$$113 \quad \begin{cases} N = \frac{kpx^{k-2}}{n} \frac{\lambda \left[P \frac{\pi c'^2}{4} x + m(k-1)v^2 \right] \sqrt{1+k^2 p^2 x^{2(k-1)}} + (1-\lambda)nf' \Phi x}{\sin \delta [1 + \lambda k^2 p^2 x^{2(k-1)}] - (1-\lambda)fkpx^{k-1}} \\ R = n \frac{N(kpx^{k-1} \sin \delta + f) + f' \Phi}{\sqrt{1+k^2 p^2 x^{2(k-1)}}} \end{cases}$$

The simplified expressions for N and R will be:

$$114 \quad \begin{cases} N = \frac{kpx^{k-2}}{n} \frac{\lambda \left[P \frac{\pi c'^2}{4} x + m(k-1)v^2 \right] + nf' \Phi x}{\sin \delta} \\ \text{and} \\ R = n(N[kpx^{k-1} \sin \delta + f] + f' \phi) \end{cases}$$

(b) *Parabola of the second degree.*—Assuming in the above equations $k=2$, we obtain the complete expression for N and R:

$$115 \quad \begin{cases} N = \frac{2P}{n} \frac{\lambda \left(P \frac{\pi c'^2}{4} x + mv^2 \right) \sqrt{1+4p^2 x^2} + (1-\lambda)nf' \Phi x}{\sin \delta [1+4\lambda p^2 x^2] - (1-\lambda)fp x} \\ R = n \frac{N(2px \sin \delta + f) + f' \Phi}{\sqrt{1+4p^2 x^2}} \end{cases}$$

Simplifying the expressions for N and R

$$116 \quad \begin{cases} N = \frac{2p}{n} \frac{\lambda \left(P \frac{\pi c'^2}{4} x + mv^2 \right) + nf' \Phi x}{\sin \delta} \\ \text{and} \\ R = n[N(2px \sin \delta + f) + f' \Phi]. \end{cases}$$

(2) *Rifling of constant curvature* is a special case of parabolic rifling.

Substituting $k=1$ we obtain

Complete expressions for N and R.

$$117 \quad \begin{cases} N = \frac{p}{n} \frac{\lambda P \frac{\pi c'^2}{4} \sqrt{1+p^2} + (1-\lambda)nf' \Phi x}{\sin \delta [1 + \lambda p^2] - (1-\lambda)fp} \\ R = n \frac{N(p \sin \delta + f) + f' \Phi}{\sqrt{1+p^2}} \end{cases}$$

Simplified expressions for N and R:

$$118 \quad \begin{cases} N = \frac{p}{n} \frac{\lambda P \frac{\pi c'^2}{4} + nf' \Phi x}{\sin \delta} \\ R = n(N[p \sin \delta + f] + f' \Phi). \end{cases}$$

In all the expressions given for N and R, for constant curvature of rifling, the value of p represents the tangent of the angle of inclination of the prolongation of the bore to the line of development of the rifling in a plane, and $x=l$.

(3) *Curved rifling*.—If we denote the radius of curvature of the rifling by ρ , and if the curve is tangent to the axis of X (prolongation of the bore), at the origin of coordinates, the equation of such curve of rifling will be

$$y = \rho - \sqrt{\rho^2 - x^2} = F(x)$$

and therefore

$$\text{tang } \alpha = \frac{x}{\sqrt{\rho^2 - x^2}} = F'(x)$$

and

$$F''(x) = \frac{\rho^2}{\sqrt{(\rho^2 - x^2)^3}}$$

where $x = a + l$.

Substituting this in the complete expressions for N and R we obtain

$$119. \quad \begin{cases} N = \frac{1}{n} \frac{\lambda \left(P \frac{\pi c'^2}{4} x + \frac{mv^2 \rho^2}{\rho^2 - x^2} \right) \rho + (1-\lambda) n f' \Phi x \sqrt{\rho^2 - x^2}}{\sin \delta [\rho^2 - x^2 (1-\lambda)] - (1-\lambda) f x \sqrt{\rho^2 - x^2}} \\ R = n \frac{N (x \sin \delta + f \sqrt{\rho^2 - x^2}) + f' \Phi \sqrt{\rho^2 - x^2}}{\rho} \end{cases}$$

We next substitute the values obtained for $\text{tang } \alpha$ and $F(x)$ in the simplified expressions for N and R, and, observing that we may neglect in them powers of x in comparison with ρ , as the latter is usually considerably larger than x , we obtain,

$$120. \quad \begin{cases} N = \frac{1}{n} \frac{\lambda \left(P \frac{\pi c'^2}{4} x + mv^2 \right) + n f' \Phi x}{\rho \sin \delta} \\ R = n \left[N \left(\frac{x}{\rho} \sin \delta + f \right) + f' \Phi \right] \end{cases}$$

87. *The vis viva of the rotary movement of the projectile.*

If the angle of rotation of the projectile in the time t is φ and the moment of inertia of the body of the projectile in relation to its axis of figure is A, the vis viva of the rotary movement of the projectile will be

$$A \frac{d\varphi}{dt} d\left(\frac{d\varphi}{dt}\right) = \frac{A}{2} d\left(\frac{d\varphi}{dt}\right)^2$$

The complete Φ_R equals the integral of the expression from 0 to $\varphi = \varphi_\Delta$ (we here denote by φ_Δ the angle of rotation of the rifling up to the muzzle), namely,

$$\Phi_R = \frac{A}{2} \left(\frac{d\varphi}{dt} \right)^2_{\varphi = \varphi_\Delta}$$

At the muzzle the angular velocity of rotation of the projectile for the given equation of the curve of rifling, page —, will be

$$\left(\frac{d\varphi}{dt}\right)_{\varphi=\varphi_{\Delta}} = \frac{2V}{c'} \tan \alpha_{\Delta}$$

where α_{Δ} is the angle of inclination of the rifling at the muzzle and V is the velocity of the projectile in its flight through the bore. Bearing in mind subsequently that $A = \lambda m \left(\frac{c'}{2}\right)^2$, the expression for Φ_R assumes the form

$$\Phi_R = \frac{\lambda m V^2}{2} \tan^2 \alpha_{\Delta}$$

or

$$\Phi_R = \lambda \tan^2 \alpha_{\Delta} \Phi_P$$

This shows that the work of the gases expended in imparting rotary movement to the projectile constitutes a very small part of the vis viva of the projectile. In fact, for $\lambda = 0.54$ and $\alpha = 8^\circ$ (which corresponds to one turn of the rifling at the muzzle in not less than 25 calibers),

$$\Phi_R = 0.011 \Phi_P.$$

88. Work of friction of the rotating band of the projectile against the rifling in the bore of the gun.

The elementary work $d\mathbf{T}_{Fr}$ of friction in the direction of axis of X will be

$$d\mathbf{T}_{Fr} = n (fN + f'\Phi) \cos \alpha dx$$

but as $\cos \alpha$ almost equals unity, we may assume the elementary work as equal to

$$d\mathbf{T}_{Fr} = n (fN + f'\Phi) dx$$

It is not difficult to see that the work of friction in the direction of the tangent will have the same expression. The work of friction from reaction of the rotating band is almost a constant quantity, and is considered in the Second Part of this course.

Substituting here instead of N the expression found for it in 113, 115, 117, and 119, we may write the equation for the elementary work of friction for parabolic, constant, or curved rifling. Substituting afterwards in them, for P and v , their corresponding values from 41 and 43, when all the powder is burned up to the instant of movement of the projectile, and from 97 and 101, when the powder burns progressively throughout the whole length of the bore, we may by integration obtain the complete work of friction. The expressions for progressive rifling become, however, very complex; therefore, we shall limit ourselves (1) to determining the work of friction for rifling of constant curvature on the assumption that the projectile has projections (studs) already cut upon it, and (2) to comparing the work of friction of constant and progressive rifling on the same assumption.

(a) *Work of friction of rifling of constant curvature.*—Expressions 117 and 118 for pressures N for the proposed construction of rifling and projectile are directly proportional to P , and therefore the elementary work of friction will be

$$dT_{Fr} = B f P \frac{\pi c'^2}{4} dl$$

where

$$B = \frac{\lambda p \sqrt{1+p^2}}{\sin \delta (1+\lambda p^2) - (1-\lambda)fp}$$

or

$$B = \frac{\lambda p}{\sin \delta}$$

according to the value we assume from expressions 111 or 112 for N .

The value $P \frac{\pi c'^2}{4} dl$ is none other than the elementary work of pressure on the base of the projectile, and therefore we may assume for simplicity, as a first approximation, that it is equal to the elementary vis viva of the projectile. In consequence of this:

$$T_{Fr} = f B \Phi_P$$

For $f = 0.2$, $\alpha = 8^\circ$, $\lambda = 0.54$, and $\delta = 90^\circ$, we obtain

$$T_{Fr} = 0.015 \Phi_P$$

i. e., the work of friction against the rifling in the case of projections (studs) already existing upon the projectile constitutes but a small part of the vis viva of the projectile. The whole influence of the rifling, which consists in the work of the powder gases in overcoming friction of the projections on the shell against the rifling, and in imparting a rotary movement to the projectile (87) does not exceed 3 per cent of the vis viva of the projectile—a quantity the same as that already obtained by another method (85, page 260).

(b) *Comparison of work of friction for constant and progressive riflings.*—As the work of friction is proportional to the pressure N , so, in comparing the work considered for various riflings, we assume that $\delta = 90^\circ$, and that the normal pressure N is equal to its mean value N_M determined on the assumption that the vis viva of the rotary movement of the projectile produced under condition of normal pressure, is required. In the given case the work of friction will be

$$T_{Fr} = f n N_M L.$$

It remains, consequently, to equate together the values obtained for N_M by the method indicated in the case of various riflings, assuming that the projectile is discharged from the gun, for each of them, with the required angular velocity.

The work of constant pressure $n N_M$ on the driving edge of the rifling will be $n N_M \frac{\Phi c'}{2}$.

Equating this to the vis viva of the rotary movement of the projectile for the same time, we obtain,

$$2\Phi_R = A \left(\frac{d\phi}{dt} \right)^2 = nc' N_M \phi$$

whence

$$N_M = \frac{A}{nc'} \left(\frac{d\phi}{dt} \right)^2 \frac{1}{\phi} = 2 \frac{\Phi_R}{nc'} \frac{1}{\phi}$$

It follows from this that, for a given vis viva of rotary movement of projectile, the mean normal pressure on the projection (lug) N_M , and consequently the work of friction (see above) is inversely proportional to the angle ϕ of rotation of the projectiles in their movement along the bore of the gun.

We shall denote (1) the angle of inclination of the rifling, i. e., the angle between the tangent to the curve of rifling and the axis of abscissæ, at the muzzle by α_M and at the origin of rifling by α_0 (for rifling of constant curvature $\alpha_M = \alpha_0$, and for progressive rifling $\alpha_M > \alpha_0$); (2) the coordinates corresponding to the travel of the projectile by $x = a + L$ (where L is the length of the rifled portion), and $y = y_L$; and at the origin of rifling by $-x = a$ and by $y = y_a$ (for rifling of constant curvature, $a = 0$); (3) the angles of rotation of the projectile for constant, parabolic, and curvilinear rifling, respectively, by ϕ cons, ϕ par, ϕ curv (the last two are each equal to the difference $\phi_M - \phi_0$).

In consequence of what has been stated we obtain

(a) for rifling of constant curvature

$$y = \frac{c'}{2} \phi = px,$$

consequently

$$\phi_{\text{cons}} = \frac{L}{c'} 2 \tan \alpha_M;$$

(b) for parabolic rifling

$$y = \frac{c'}{2} \phi = px^2;$$

$$\phi_M = \frac{2p(a+L)^2}{c'}, \quad \phi_0 = \frac{2pa^2}{c'};$$

$$\tan \alpha_M = 2p(a+L); \quad \tan \alpha_0 = 2pa;$$

whence

$$p = \frac{\tan \alpha_M - \tan \alpha_0}{2L}, \quad a = \frac{L \tan \alpha_0}{\tan \alpha_M - \tan \alpha_0}$$

$$a + L = \frac{L \tan \alpha_M}{\tan \alpha_M - \tan \alpha_0}.$$

consequently

$$\phi \text{ par} = \frac{L}{c'} (\tan \alpha_M + \tan \alpha_0) = \frac{2L}{c'} \frac{\sin(\alpha_M + \alpha_0)}{\cos(\alpha_M + \alpha_0) + \cos(\alpha_M - \alpha_0)}$$

(c) *For curved rifling.*—From the properties of the curve tangent to the axis of abscissæ at the origin of coordinates, we have

$$y = \frac{c'}{2} \varphi = \rho (1 - \cos \alpha),$$

$$x = \rho \sin \alpha,$$

$$L = \rho (\sin \alpha_M - \sin \alpha_0).$$

whence

$$\varphi_M = \frac{2\rho}{c'} (1 - \cos \alpha_M)$$

$$\varphi_0 = \frac{2\rho}{c'} (1 - \cos \alpha_0)$$

and

$$\varphi_{\text{curv.}} = \frac{2L \cos \alpha_0 - \cos \alpha_M}{c' \sin \alpha_M - \sin \alpha_0} = \frac{L}{c'} 2 \tan \frac{\alpha_M + \alpha_0}{2} = \frac{2L}{c'} \frac{\sin (\alpha_M + \alpha_0)}{\cos (\alpha_M - \alpha_0) + 1}$$

Comparing together the values obtained for φ cons., φ par., and φ curv. (assuming in the latter that α_0 is the same), we find that

$$\varphi \text{ cons.} > \varphi \text{ par.} > \varphi \text{ curv.}$$

and consequently the mean pressures on the projections (studs) and the works of friction for progressive (parabolic and curved) riflings are greater than for constant rifling, and the difference between them is the greater, the less the initial inclination of the rifling. In fact, if we assume $\alpha_0 = 0$ in the expressions for φ par. and φ curv., we see that, in the case of parabolic rifling, the work of friction is twice greater than it is for rifling of constant curvature; the difference between the latter and for curved rifling is still greater, for

$$\tan \alpha_M > 2 \tan \frac{\alpha_M}{2}$$

Whence it follows that in order to diminish the work of friction on the rifling, it is useful in the progressive system to increase the angle of their initial inclination.*

89. *Experiments of Captain Noble to determine the energy absorbed in friction of the projectile in the rifling of the gun.*

In order to determine the influence of various riflings on the ballistic results of firing, Captain Noble† conducted a series of experiments in 1891 on three 120-mm. (4.725 in.) cartridge guns, of which one was provided with riflings parallel to the axis of the bore—i. e., the pitch of this rifling was infinitely great; the second gun was provided with rifling of constant curvature of one turn in 35 calibers; while the third gun possessed rifling of progressive curvature, from one turn in 100 calibers at the origin of rifling to one turn in 35 cali-

*See G. Kaiser, *Construction der gezogenen Geschützrohre*, 1892, pp. 444 and 446.

†See Captain Noble, "Note on the energy absorbed by friction in the bores of rifled guns (*Proceedings of the Royal Society*, vol. 50).

bers at the muzzle, so that in the case of the two latter guns the projectiles should attain the same angular velocity. The section of the rifling in all the guns was the same and of the type employed in England with an inclined driving edge so that the normal to this edge in the plane of the cross section of the bore formed with the radius drawn through the center of the edge, an angle δ_0 , whose mean value was approximately $34^\circ 45'$. The projectiles employed in all these experiments were in the form of dense cylinders weighing 45 pounds English; the rotating band was of various sections. Firings were conducted with smoking powder P (pebble powder) with light-smoking powder prepared with ammonium-nitrate (amide powder), and with smokeless powder (cordite). Three shots were fired from each gun with each kind of powder and each kind of charge.

These experiments showed:

(1) The width of the rotating band and its section hardly exerted any influence on the velocities obtained from the guns experimented with.

(2) The highest vis viva was obtained on firing from the gun whose rifling was parallel with the axis, and the least from the progressive rifling. If the first be taken as a unit the loss resulting from rifling of constant curvature amounted to 1.5 per cent, while that for the rifling of progressive curvature was 4.25 per cent. These results fully confirm the above adduced theoretical investigation of the value of the amount of work expended in friction for various riflings.

(3) The mean value of the coefficient of friction for rifling of constant curvature was 0.203.

The pressures and velocities of the projectile at different sections of the bore were determined by preliminary experiments with the same guns. By employing data and formulæ like those deduced in the preceding paragraph, Noble calculated the values of the pressures nN on the driving edge of the rifling for constant and progressive curvature, both for solid and hollow projectiles.

Fig. 20 shows the curves for light-smoking powders for charges of about 10.5 pounds (about 12 pounds Russian).

The curves AA relate to rifling of constant curvature; BB to progressive rifling.

It follows from this data that the mean pressure on the driving edge of the rifling in the experiment considered amounted, for rifling of constant curvature, to 7.35 tons, while for progressive rifling it was as high as 11.06 tons, which also confirms above adduced theoretical considerations. (Par. 88.)

The influence of the kind of projectile on the magnitude of the pressure on the driving edge is considerable; the cause of this is the difference in moments of inertia.

It should be observed that, theoretically, in guns where the normal to the driving edge is perpendicular to the radius, the magnitude of the pressure on the driving edge, and consequently the work of

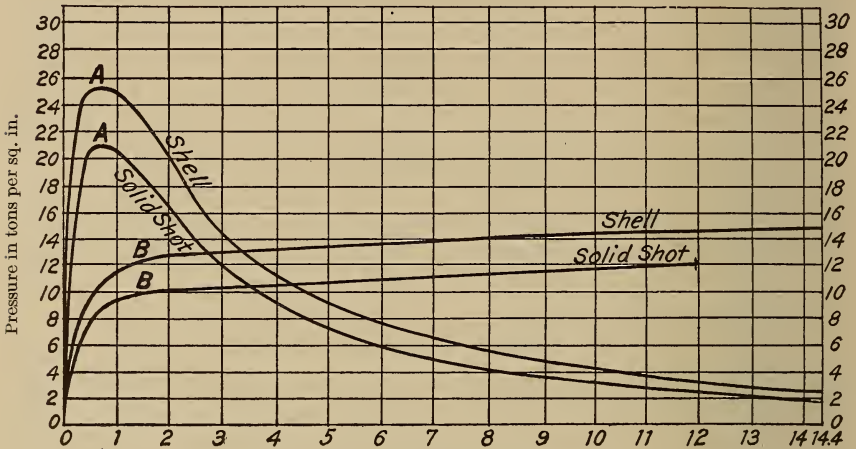


FIG. 20.—Path traveled by the projectile in feet.

friction should be less than in the case of an inclined driving edge. The corresponding experimental data in relation to this do not yet exist, however.

90. *The work of friction of the products of combustion of the powder.*

During their motion in the gun the friction of the gases is proportional, as is known, to their density and to the square of the velocity of their movement; but in order to simplify deductions we shall assume that this friction is proportional to the mean pressure, an assumption which may be made, as in the given case not only are the powder gases in motion, but also the unburned portion of the grains of the powder as well as the nonvolatile portions of the products of combustion, when smoking powders are employed.

We may assume that the mean pressure of the products of combustion P_M , at the given moment (see paragraph 69) is approximately equal to

$$P_M = \frac{P'_B + P'_P}{2} = P'_P \left(1 + \frac{\Theta}{2} \frac{\omega}{G} \right)$$

The surface of the bore upon which this pressure acts, equals $\pi c' (l_1 + l)$, and therefore the friction of the products of decomposition will be

$$\mu_1 \pi c' \left(1 + \frac{\Theta}{2} \frac{\omega}{G} \right) P'_P (l_1 + l),$$

where μ_1 is the coefficient of friction.

While the powder gases expand through dl , the change of the position of the center of gravity of the charge will be $\frac{dl}{2}$, and therefore the elementary work of friction of the products of decomposition will be

$$d T_{Pr} = \mu_1 \frac{\pi c'}{2} \left(1 + \frac{\Theta \omega}{2 G}\right) (l_1 + l) P'_p \, dl.$$

Substituting here, instead of P'_p , its value from (97), page 279, changing for simplification l_1 into l_1'' (by which we slightly diminish the work of friction), and integrating, we obtain

$$\begin{aligned} T_{Pr} = & 2\mu_1 \left(1 + \frac{\Theta \omega}{2 G}\right) \frac{f_1 \omega}{c'} \frac{l_1'' + L}{1 - \beta_1} \left(\frac{l_1''}{l_1'' + L}\right)^{\beta_1} \left\{1 - \right. \\ & \left. - \frac{\xi}{\beta_1 + \xi - 1} \left(\frac{l_1''}{l_1'' + L}\right)^{1 - \beta_1} \left[1 - \frac{1 - \beta_1}{\xi} \left(\frac{l_1''}{l_1'' + L}\right)^{\xi + \beta_1 - 1}\right]\right\} = \\ & = 2\mu_1 \left(1 + \frac{\Theta \omega}{2 G}\right) \frac{f_1 \omega}{c'} \frac{l_1'' + L}{1 - \beta_1} \left(\frac{l_1''}{l_1'' + L}\right)^{\beta_1} \left\{1 - \right. \\ & \left. - \frac{1 - \beta}{1 - (\beta_1 + \xi)} \left(\frac{l_1''}{l_1'' + L}\right)^{\xi} \left[1 - \frac{\xi}{1 - \beta_1} \left(\frac{l_1''}{l_1'' + L}\right)^{1 - (\beta_1 + \xi)}\right]\right\}. \end{aligned}$$

We do not know exactly the value of the coefficient of friction μ_1 ; for lack of more exact investigations we shall assume it as equal to 0.001.*

The deduced formulæ for work of friction of gases show that when μ_1 equals 0.001 the value of the work considered is insignificant.

91. *The work of gases in overcoming (a) the atmospheric pressure; (b) the resistance of the air, and (c) the weight of the projectile and its friction.*

(a) Denoting the pressure of the atmosphere on the unit area by Π , the work sought, for a length of path L will be

$$T_{pres} = \frac{\pi c'^2}{4} \Pi L$$

(b) Assuming that the resistance of the air in the given case is proportional to the square of the velocity of the projectile, the work sought for the length of path L will be

$$T_{R. A.} = \alpha \frac{\pi c'^2}{4} \frac{\Pi}{\Pi_0} \int_0^L \frac{L}{v^2} dv,$$

where α is the coefficient of proportionality, and according to Prof. N. A. Zaboudski's determination equals 0.029, assuming as units the

* In order to account for the rupture of the jacket of his wire-wound gun Longridge assumes that the friction of the products of combustion is great and that therefore the coefficient μ_1 equals 0.025. (See Longridge, "Interior Ballistics," translated by Captain Nilus, 1891, p. 190.) But, in fact, this value is exceedingly large. Assuming the friction of the gases as proportional to the square of velocity, the coefficient of friction μ_1' , by this hypothesis, for small velocities is considered equal to $\mu_1' = 0.00315$. (See H. Resal, "Traité de mécanique générale," Vol. II, p. 338.) On the same hypothesis, to explain the bursting of the jacket of the gun in question, Longridge assumes $\mu_1' = 0.116$, i. e., almost thirty-seven times greater in value than is ordinarily taken. In view of this, it would be necessary to diminish the value of μ_1 taken by Longridge thirty-seven times; the value assumed in the text is somewhat greater.

meter, kilogram, and second, $-II_0$, is the normal atmospheric pressure. Substituting here instead of v^2 its value 101, page 225, we obtain

$$T_{R.A.} = \alpha \frac{\pi c'^2}{4} \frac{II}{II_0} \frac{2 g f_1 \omega}{\varphi G} \frac{\xi}{\beta_1 + \xi} \left\{ L + \frac{1 - 2\beta_1 - \xi}{(1 - \beta_1)(1 - \beta_1 - \xi)} l_1'' - \right. \\ \left. - \frac{\beta_1 + \xi}{\xi} \frac{l_1'' + L}{1 - \beta_1} \left(\frac{l_1''}{l_1'' + L} \right)^\beta \left[1 - \frac{\beta_1}{\beta_1 + \xi} \frac{1 - \beta_1}{1 - \beta_1 - \xi} \left(\frac{l_1''}{l_1'' + L} \right)^\xi \right] \right\}$$

(c) Denoting the coefficient of friction corresponding to the weight of the projectile by μ , and the angle formed by the axis of the gun with the horizontal by ψ , the desired work for the weight of the projectile and its friction along the path of length L will be

$$T_F = G (\sin \Psi + \mu \cos \Psi) L$$

These three quantities of work are, generally speaking, very small.

92. Work of powder gases expended in expanding the walls of the gun on firing.

The dimensions of the gun are expanded both in diameter and length through the action of pressure of the powder gases on the surface of the bore. The work of the powder gases expended in expanding the walls of the gun is proportional to the change of the volume of the bore. In order to determine this work, it is necessary to know therefore the deformation of the bore of the gun as the result of firing. Assuming that the pressure of the powder gases is static, it is shown, in the lectures upon the resistance of the gun, that it produces, among other results, a tangential deformation in the direction of the tangent to the circle of cross section of the gun, and an axial deformation in the direction of the axis. The first of these deformations, estimated over the surface of the bore of the gun, gives an increase in diameter of the bore, and the second, which is constant in every section, gives a change of its length.

In the same course of lectures it was shown that the value of these deformations depends upon the magnitude of the pressure, upon dimensions of the layers, upon the coefficients of elasticity of metals, and upon fundamental conditions; i. e., whether the cylinder expands axially in the breech part, as would occur in the case of a hollow cylinder with a solid bottom, or whether it expands axially in the muzzle part, as would occur in case that such solid bottom did not exist; and, again, whether the gun is reinforced or not. These are styled "relative deformations."*

In order to solve the problem under consideration, we shall assume the simplest case where the metal of the different sections of the bore possesses one and the same coefficient of elasticity.

* See Maj. Gen. A. Brynk's "Design of Guns," Pt. I, 1901.

Assume:

- P_M = the mean pressure (on the unit surface for the considered length of gun) (see paragraph 90).
 r and r_{n+1} = the inner and outer radii of the gun.
 E_1 = the coefficient of elasticity of the metal of the gun.
 l = the length of the part of the gun under consideration.
 k_1'' = the corresponding tangential and } deformations of the surface of the
 λ'' = the axial } bore produced on firing.*

When the axial force does not exist (cylinders without bases), the tangential and axial deformations are the following:

$$k_1'' = \frac{2}{3} \frac{P_M}{E_1} \frac{2r_{n+1}^2 + r_1^2}{r_{n+1}^2 - r_1^2}$$

$$\lambda'' = -\frac{2}{3} \frac{P_M}{E_1} \frac{r_1^2}{r_{n+1}^2 - r_1^2}$$

When the axial force does not reduce to zero (cylinders with bases), the tangential and the axial deformations are the following:

$$k_1'' = \frac{P_M}{3E_1} \frac{4r_{n+1}^2 + r_1^2}{r_{n+1}^2 - r_1^2}$$

and

$$\lambda'' = \frac{1}{3} \frac{P_M}{E_1} \frac{r_1^2}{r_{n+1}^2 - r_1^2}$$

In consequence of what has already been said, the radius of the bore of the gun under action of a pressure P_M will be $r_1 (1 + k_1'')$ and its length will be $l (1 + \lambda'')$. The volume of deformation of the bore is $\pi r_1^2 (1 + k_1'')^2 l (1 + \lambda'')$. Consequently, the change of volume of the bore, from the action of the pressure P_M will be, neglecting terms of the second degree, in comparison with k_1'' and λ'' — $\pi r_1^2 l (2 k_1'' + \lambda'')$, and the corresponding work of the powder gases,

$$T_{\varepsilon\omega} = \pi r_1^2 l (2 k_1'' + \lambda'') P_M$$

Substituting instead of k_1'' and λ'' their values, as deduced above, we obtain the required expressions for the work of the powder gases expended on *expanding the walls* of the gun over a length l .

(1) When the axial force does not act,

$$T_{\varepsilon\omega} = \frac{2}{3} \pi r_1^2 l \frac{P_M^2}{E_1} \frac{4r_{n+1}^2 + r_1^2}{r_{n+1}^2 - r_1^2},$$

(2) When the axial force acts,

$$T_{\varepsilon\omega} = \frac{2}{3} \pi r_1^2 l \frac{P_M^2}{E_1} \frac{4r_{n+1}^2 + 1, 5r_1^2}{r_{n+1}^2 - r_1^2}.$$

The following must be observed in relation to the values of the pressure P_M ; in each section of the gun the pressure increases toward a maximum, and along with this the work of the gases spent in

* The axial deformation is constant in every section.

expanding the walls of the gun also increases. After attaining the maximum the pressure falls, the expansion of the walls of the gun is reduced, and the action of their forces of elasticity compresses the gases; consequently, the corresponding portion of the energy spent by the gases in expanding the walls of the gun is returned to the gases, so that in the end the expenditure of the store of energy of gases in expansion corresponds approximately to the pressure which exists in the gun when the shell leaves the muzzle. Some assume this pressure as constant throughout the whole extent of the bore and equal to a certain mean pressure as stated in paragraph 90.

As the diameters of the gun are not the same in various sections, it is necessary to divide the gun into sections with respect to its length, and to calculate the work of each one of them separately.

In relation to the influence of the dimensions of the gun on the magnitude of the work of expanding its walls, it is evident, from the above deduced expressions, that this work diminishes along with the increase of the outer diameter, and that, for $r_{n+1} = \infty$, it becomes the same in both cases, viz:

$$T_{r_{n+1}} = \frac{8}{3} \pi r_1^2 l \frac{P_M^2}{E_1}$$

93. *The influence of the loss of heat by the products of combustion of the powder on heating the walls of the gun—Degree of heating of small-arm barrels.*

On deducing formulæ for velocities of projectile and for pressures (Chapter VII) of the powder gases on the base of the bore, it was assumed that the walls of the gun were impenetrable to heat. As a matter of fact, they absorb a certain amount of heat, on account of their contact with the heated products of combustion that are evolved on burning the charge, and on account of this the initial-velocity of the projectile diminishes. In fact, it is evident from formulæ 76 and 86, pages 209 and 215, that the pressures on the base of the bore and the velocities of the projectile are proportional to certain powers of the force of the powder, namely:

$$P_B = \Xi'_n f^{\frac{2}{3-2\nu}}$$

and

$$V = B'_\Omega f^{\frac{1+n'(2\nu-1)}{2}}$$

where there are collected in Ξ'_n and B'_Ω the remaining factors that are independent of the force of the powder. On the other hand, expression 22, page 79, shows that the forces of powders are almost proportional to the quantity of heat evolved when the powder decomposes. As the walls absorb a part of this heat, we may represent the influence

of this cause by a corresponding diminution of the quantity of heat evolved on the combustion of the powder, or, what is the same thing, by a diminution of the force of the powder. Bearing this in mind, we take the Napierian logarithms of the expressions written. Differentiating them subsequently, we obtain

$$\frac{dP_B}{P_B} = \frac{2}{3-2\nu} \frac{df}{f} = \frac{2}{3-2\nu} \frac{dQ'_v}{Q'_v},$$

$$\frac{dV}{V} = \frac{1+n' (2\nu-1)}{2} \frac{df}{f} = \frac{1+n' (2\nu-1)}{2} \frac{dQ'_v}{Q'_v},$$

or, for all smoking powders,

$$\frac{dP_B}{P_B} = \frac{dQ'_v}{Q'_v},$$

$$\frac{dV}{V} = \frac{1}{2} \frac{dQ'_v}{Q'_v},$$

and for pyroxylin powders,

$$\frac{dP_B}{P_B} = \frac{4}{3} \frac{dQ'_v}{Q'_v},$$

$$\frac{dV}{V} = \frac{2+n'}{4} \frac{dQ'_v}{Q'_v},$$

where n' depends on the modulus of the powder (80), page 212, so that the coefficient in the latter expression varies in its relationship to the quickness of the powder, between the limits $5/9$ and $19/30$, which are greater than $1/2$. In this way it happens that the influence of the loss of heat from the heating of the walls of the gun upon diminishing the values of initial velocity is greater for pyroxylin powders than for smoking powders; at the same time, the pressures fall considerably more from this cause than the velocities fall.

In determining the numerical values of the influence of this effect on the magnitude of the pressures and velocities, we must, therefore, know the quantity of heat that is absorbed by the walls of the gun, on account of their contact with the heated products. In view of the extreme difficulty of solving this question in a theoretical manner, experiments were made, first of all, to determine, by direct experiment, the quantity of heat absorbed by the walls of the gun. By direction of Count Saint-Robert,* Colonel Balegno conducted the following experiment in Turin: Three steel small-arm rifle barrels, as much alike as possible, were taken and firings were conducted with them with a standard charge of 4.5 grams. Barrel No. 1 was

* Paul de Saint-Robert "Memoires Scientifiques," Vol. II, 1873, p. 212 et seq.

loaded in the usual manner; the bullet (ogival, weight, 33 grams) was placed directly against the charge. In barrel No. 2 the bullet was placed 0.02 meter from the muzzle. Barrel No. 3 was fired with a blank charge. Each of the barrels was wound in a certain manner with a number of turns of white woolen material, which was tightly wound around the barrel in the form of a ribbon. After firing, the temperature of the barrels was measured. In determining this, the barrels were filled with mercury at a known temperature and the channel closed at the muzzle with a gutta-percha cork, through which a thermometer was inserted. The indications were taken at the moment that the height of the mercury in the thermometer was at a maximum. The weight of the mercury filling the barrels was 3.350 kilograms. Specific heats were taken: for steel, as 0.118; for mercury, as 0.033. The results of these experiments and the calculations made on the basis of data afforded by them are presented in Table XLIII.

The figures in this table show:

(1) On firing with the bullet at the muzzle of the gun (barrel No. 2), the walls of the barrel were heated more than in blank firing (barrel No. 3), and more in the latter case (barrel No. 3) than under usual conditions of loading (barrel No. 1).

The cause of this* must be attributed to the fact that almost the whole length of barrel No. 2 was exposed to the action of the gases at the temperature of decomposition; while, in the case of barrel No. 1, this occurred only throughout the length of the powder chamber: i. e., for a length of about one-eighth of the bore. In blank firing, barrel No. 3 should be, for analogous reasons, heated less than barrel No. 2. The higher heating of barrel No. 3 in comparison with barrel No. 1 must be attributed to the greater duration of the heating of barrel No. 3 in consequence of the slower burning of the powder in it.

(2) The quantity of heat absorbed by the walls of the barrel for one shot, and therefore in each series for the first shot, is greater than for each successive shot.

(3) On increasing the number of shots in the series, the mean quantity of heat absorbed for one shot is considerably diminished.

(4) Limiting ourselves to the amount of heat found by experiment that is expended for one shot that goes to heat the walls of the barrel, and bearing in mind that in these experiments the barrel is forced to lose a part of its heat through radiation, etc., we are compelled to recognize the fact that the amount of heat determined by experiment to be lost is not less than 30 per cent of the whole heat evolved on the decomposition of the powder.

* Compare Alois Indra. "Analytische Theorie der Wärmeleitung in Geschützrohren," 1893, pp. 81-82.

TABLE LXIII.—Quantities of heat absorbed by small-arm barrels on firing.

Designation of elements.	Number of shots.							
	No. 1 ordinary loading.				No. 2 bullet at 0.02 meter from muzzle.			
Weight of barrel	2.050 kilograms.				2.064 kilograms.			
Number of shots	1	5	10	50	1	5	10	50
Temperature of barrel:								
On firingdegrees..	14.200	14.200	18.100	13.500	14.200	14.100	18.200	14.100
After firingdo....	16.700	23.100	33.400	54.000	17.800	31.800	43.200	89.000
Mean quantity of heat absorbed by the barrel for one shot.....	.881	.627	.539	.285	1.027	1.246	.885	.530
The same in large calories for 1 kilogram of powder ..	195.800	139.300	119.800	63.300	228.200	276.900	196.700	117.800
Vis viva of bullet.....	315 kgr. m.				102 kgr. m.			

Designation of elements.	Number of shots.			
	No. 3 without bullet.			
Weight of barrel	2.042 kilograms.			
Number of shots	1	5	10	50
Temperature of barrel:				
On firingdegrees..	14.200	14.200	18.400	15.000
After firingdo....	16.800	26.900	38.400	79.500
Mean quantity of heat absorbed by the barrel for one shot...	.914	.893	.708	.453
The same in large calories for 1 kilogram of powder	203.100	198.400	156.200	100.700
Vis viva of bullet	0			

Noble and Abel* determined also, through the employment of the calometric method, the quantity of heat absorbed by the walls of the steel, rifled breech-loading 12-pounder gun. Two series of shots were fired for this purpose. The first consisted of 9 shots with a charge of $1\frac{3}{4}$ English pounds of RLG. powder and with a projectile weighing 11 English pounds 12 ounces; the second, of 5 shots with a charge of $1\frac{1}{2}$ English pounds, and the same projectile. The necessary arrangements were made before the experiment for placing the gun, on the conclusion of a series of shots, in a chamber containing a definite quantity of water, the temperature of which consequently rose a certain extent. Knowing the weight of the water and of the gun, and also the heat capacity of the metal of the latter, it is possible to calculate, from the amount of heating of the water, the quantity of heat absorbed by the walls of the gun on firing. In this way it was found that in the first series (9 shots) the mean loss of heat for one round was 76.4 calories for 1 kilogram of powder, and in the second series (5 shots) it was 89.4 calories. In view of the fact that the second of these figures is greater than the first, it must be assumed that in this case, as also in the experiments of Count Saint-Robert, with the small-arm barrel, a larger part of the heat was lost in the first shots of each series. Bearing in mind subsequently that the mean quantity of heat

*See their works already referred to, p. 20, and Pashkevitch's translation.

lost corresponding to one shot from the small-arm barrels in the series of from 5 to 10 shots was from 40 per cent to 63 per cent less than the heat lost in firing a single shot (see Table XLIII), we may, according to Major Indra's assumption, assume that the same relations occurred between the quantities of heat lost in Noble and Abel's experiments. The corresponding calculation affords, for the quantity of heat lost for the first shot by the 12-pounder gun from the heating of its walls, beginning with the first series, as 124.4 calories; in the second series, as 125.2 calories—which afford, as a mean, 124.4 calories for 1 kilogram of powder, which constitutes about 17 per cent of the whole quantity of heat evolved by the combustion of the unit weight of this powder.

Corresponding determinations have not been made for guns of larger calibers, but, as the result of approximate estimation, Noble and Abel make the following assumptions: (1) That the quantity of heat lost by the unit weight of charge in heating the walls of the gun on firing is directly proportional to the magnitude of the surface of the bore of the gun, and inversely proportional to the weight of the charge; and (2) that the loss considered is proportional to time—i. e., to the interval of the firing—during which the walls of the gun exert a cooling influence on the products of decomposition. They found by calculations based on the results of their experiments with the 12-pounder gun that the loss of heat from heating the walls of the 10-inch gun should be 4 times less than for the 12-pounder gun—i. e., for the figures given above the loss of heat should be about equal to 31.2 calories for one kilogram of powder, or $4\frac{1}{4}$ per cent of the whole quantity of heat evolved on the burning of the charge.

Thus, the adduced experiments and considerations show that the following quantity of heat is lost in heating the walls of the gun, for one shot, for each kilogram of powder:

On firing from the rifled small-arm barrel, 195.8 calories, or 30 per cent of the whole quantity of heat.

On firing from the 12-pounder gun, 124.8 calories, or 17 per cent of the whole quantity of heat.

On firing from the 10-inch gun, 31.2 calories, or $4\frac{1}{4}$ per cent of the whole quantity of heat—i. e., when the caliber of the gun is increased, the quantity of the heat lost by the unit weight of the charge in heating the walls of the gun diminishes.

It now remains to consider more closely in what way the heat observed in the experiments is conducted to the walls of the gun; whether the heating of these walls is direct, in consequence of the contact of the products of combustion with the surface of the bore, which has occupied our attention in the given case, or whether the heating results from still other causes; in which case, should we deduct the corresponding quantity of heat from the amount found in the experiments above cited?

Approximate experiments made in the year 1894 by our Commission of the Marine Artillery fully confirmed the previously observed phenomenon of the lack of uniformity in the heating of the wall of the gun on firing. The experiment was as follows: A series of shots was fired from a given gun. Subsequently, there was introduced into the bore of the gun a rod bearing maximum thermometers, the latter being separated by partitions dividing the bore into different sections; a tompon was introduced into the muzzle and the breech was closed. The temperature of the bore of the gun was measured after firing. Table LXIV presents the results of firing pyrocollodion powders from the 37-mm. gun and brown prismatic powders from the 8-inch gun of 35 caliber length:

TABLE LXIV.—*Increase of the temperature of the gun in firing.*

Designation of the gun.	Conditions of firing.	Number of shots.	Increase of temperature on firing at a distance from the base of the bore of—			
			1 inch.	14 inches.	*27 inches.	
37 mm. single-firing gun	{ Pyrocollodion powder. Weight of charge 6½ zol; weight of projectile, 1 pound 22½ zol. Shots in each series were 15 seconds apart.	{ 10	° C.	° C.	° C.	
			4½	11½	10
			3½	13	13½
		{ 40	3½	25½	28½
8-in. gun of .35 caliber	{ Brown Okhta powder. Weight of charge, 126 pounds; weight of projectile 214½ pounds.	{ 16	2 inches.	6½ feet.	13 feet.	†19 feet.
			° C.	° C.	° C.	° C.
		{ 13	20	25	40	48½
			9	12½	37½	44½

*1½ inches from the muzzle.

†Near the muzzle.

It is evident from this table that there exists a marked difference in the degrees of heating of the different sections of the bore, notwithstanding the approximate nature of the experiment—namely, the chamber is heated considerably less than the muzzle part. The chief reason for this difference is the greater thickness of the walls of the gun near the breech. The difference in temperature of the walls of the gun is apparent even to the touch, and therefore it had been observed in all previous very varied firings, and was also apparent in case where the inner tube, almost uniform in thickness throughout its whole length, did not come at all into contact with the jacket in the guns employed (guns used for special experiments). This latter circumstance shows that the lack of uniformity in heating the walls in question is not alone due to contact with the heated products of decomposition.

In considering what circumstances might cause a heating of the walls of the gun, besides direct transfer of heat, we observe, first of all, that of the work of friction of the projectile over the rifling and upon the walls of the gun, and the work of friction of the

products of combustion on the walls of the gun, must be considered. The quantity of heat corresponding to this work (sections 88, 89, and 90) is not great, and in Count Saint-Robert's experiments it barely constituted 10 per cent of all the heat absorbed by the barrel.

Next, the following considerations undoubtedly exert an influence on the heating of the walls of the gun: (1) The subsidiary deformations which occur in the metal of the gun, both from the expansion of its walls and from vibration resulting from firing; (2) the subsidiary deformations in the layer of metal nearest to the surface of the bore, particularly in the inner tube, in consequence of the blow dealt it on firing; and (3) the internal work of friction between the molecules, in consequence of the vibration of the walls of the gun. Up to the present time, nothing has been determined as to how great the influence of these causes upon the heating of the walls of the gun may be. We shall only remark that all these causes occasion a considerable heating of the muzzle parts of the gun, which represent the least thickness of the walls of the gun in the case.

Considering especially the experiments of Count Saint-Robert, and also those of Noble and Abel, we can not but observe that these experiments determine actually not only the heat which passes into the walls of the gun on firing, but also the heat which was evolved in the supplementary chemical reactions between the nonvolatile products of combustion after the escape of the projectile from the muzzle. The supplemental evolutions of heat by the products of decomposition may take place at the very time of the observation of the increase of temperature of the water and mercury, as the capability of the products of combustion of smoking powders to oxidize, and therefore to evolve heat, is known from observations upon them (section 16).

It is not known, however, how great the quantity of heat evolved from these supplementary reactions may be.

From what has been said, it is evident how complex the question of the heating of the walls of the gun on firing is, and how little it has been investigated.*

DEGREE OF HEATING OF SMALL-ARM BARRELS.

In 1891-1898, experiments were conducted in France by various persons, to determine the temperature of heating of small arms upon repeated firings with smokeless powders.† On the basis of his theo-

* We may indicate the following attempts at the theoretical solution of this question: these attempts are, however, too approximate, and, therefore, far from being true, and moreover, Indra's theory is very complex; Sarrau. "Nouvelles Recherches sur les effets de la poudre dans armes," 1876, pp. 18 and 20. Longridge, "Interior Ballistics," translated by Captain Nilus in 1891, pp. 46-49; Alois Indra "Analytische Theorie der Wärmeleitung in Geschützrohren."

† "Mém. des Poudres et Salpêtres," article by Bertrand et Laroche. "Température développée dans armes par le tir"; Vol. IX, 1898, article by M. Billardon under the same title.

retical conceptions derived from these experiments, M. Billardon came to the conclusion that the temperature to which small barrels was raised did not exceed as a mean the values indicated in Table LXV.

TABLE LXV.—Increase of temperature of the French small arm, model 1886, in a given time t , firing n shots per minute.

Number of shots— n , per minute.	Increase of temperature after firing for an interval of time t .				
	5 min.	10 min.	20 min.	30 min.	Limits.
POWDER MARK BF.					
	° C.	° C.	° C.	° C.	° C.
1	29.5	47.0	70.8	83.8	94.1
2	52.4	87.3	124.5	139.2	150.6
5	118.0	180.0	229.6	243.2	248.3
10	202.0	281.8	323.7	331.1	332.7
POWDER MARK V.					
1	16.3	31.3	47.2	55.9	62.7
2	52.3	58.2	83.0	93.3	100.4
5	78.7	120.0	153.4	162.1	165.5
10	134.7	187.5	215.8	220.7	221.8

94. *The change of initial velocities of projectiles and pressures on the base of the bore in relation to external conditions of firing.*

In the above paragraphs there were considered the influence of changes of various principal elements of firing on the value of velocities and pressures, such as changes in weights of charges, projectiles, dimensions of bore of the gun, riflings, etc. In the present case we shall consider how velocities and pressures change in relation to certain exterior conditions of firing, such as the temperature of the charge, its diameter and length, the moisture of the powder, the state of the bore of the gun, etc.

Although many of these relations have not yet been rigidly established, nevertheless, it is useful to present general conclusions which have been deduced from experiments conducted up to the present time.

(a) *Influence of temperature of the powder in charges.*—Experiments show that the higher the temperature of the charge (independent of the temperature of the surrounding air) the greater the initial velocity and pressure, the change of this quantity through quite wide limits being almost proportional to the change in the temperature of the charges.

Thus:

(a) For smoking powders:

$$\Delta V = 0.00065V \cdot \Delta t; \Delta P_B = 0.004P_B \cdot \Delta t$$

(b) For pyroxyline powders:

$$\Delta V = 0.0011V \cdot \Delta t; \Delta P_B = 0.0036P_B \cdot \Delta t$$

where ΔV and ΔP_B represent the changes of velocities and pressures corresponding to a change of temperature of the charge by Δt^0 and

expressed in degrees centigrade; V and P_B are values obtained at normal temperature.

In view of this it was necessary, in order to establish comparisons, to compare results of firings together as reduced to a common general temperature. Our artillerists assume this temperature at 15°C . (army) and at 20°C . (navy).

(b) *Influence of moisture of the powder.*—When powders are kept, the quantity of moisture existing in them (for smokeless powders this refers to their general content of volatile substances, solvents, or moisture) will increase or diminish according to the circumstances of the keeping. The changes in velocities and pressures resulting from this are, even for broad differences in change of moisture, inversely proportional to the change, namely:

(a) For smoking powders:

$$\Delta V = -0.040 V \cdot \Delta q; \Delta P_B = -0.27 P_B \cdot \Delta q;$$

(b) For pyroxyline and pyrocollodion powders:*

$$\Delta V = -0.045 V \cdot \Delta q; \Delta P_B = -0.15 P_B \cdot \Delta q.$$

where ΔV and ΔP_B are corresponding changes in velocity and pressure for a change of moisture of Δq per cent.

These formulæ show that changes in the quantity of moisture or volatiles in the charges are especially significant in changing the values of pressures; therefore, it is very necessary to take all measures possible to prevent the drying of the powder on keeping and also the absorption of moisture by it.

(c) *The influence of the dimensions of charge.*—It is evident from the figures in Table LXVI that, for a powder chamber of given dimensions and a given weight of charge of the same kind of powder, the pressure of the powder gases is the greater and the more varied the shorter the charge happens to be (to a certain limit, however), in comparison with the full length of the powder chamber. The velocity of the projectile also increases with the diminution in length of the charge, but varies only slightly, however. Moreover, it is evident from the tables that the influence of the length of the charge under consideration occurs for all kinds of powders, but the numerical value of its amount differs. The cause of the increase of pressures with diminution in the length of the charge must evidently be sought for in the possibility of formation of wave pressures of the gases in the bore of the gun, in consequence of their unequal distribution throughout the length of the chamber (see Part II of this course).

* See the Journal of the Experimental Committee of the Okhta Powder Works, No. 15, 1887.

TABLE LXVI.—*Showing the influence of the length of charge on the initial velocity and pressure.*

Caliber and name of gun.	Weight of projectile.	Kind of powder.	Weight of charge.	Length of charge.	Length of powder chamber.	Initial velocity per sec.	Pressure on base of bore.
							<i>Atmos.</i>
17 cm., 25 calibers in length.*	52.1 kgm.	P. P. C/88 (black powder).	15 kgm.	About 14 in.	Unknown	523 meters.	2, 430
				About 18 in.do....	516.8 meters	2, 130
12 in., 40 calibers in length.	810 lbs...	Okhta choc. powd. manufact. 1891.	350 lbs.	53.9 in	77.8 in ...	2,092 ft.....	2, 445
				57.9 in		2,070 ft.....	2, 145
			378 lbs.	62 indo	2, 000
				62.9 in		2,168 ft.....	2, 905
6 in., 190 II.....	81½ lbs...	Kazan smokeless.	7¼ lbs...	65.6 in	25.1 indo	2, 345
				17 in		1,510 ft.....	2, 330
			do		1,512 ft.....	3, 000
15 cm., 30 calibers in length.†	40 kgm ..	Smokeless powd. for 7½ mm.	5 kgm ..	23 in	24.8 in ...	1,504 ft.....	1, 550
				555 mm.....		540 meters.	1, 580
				445 mm.....	584 mm...	543 meters.	1, 635

*G. Kaiser: "Construction der gezogenen Geschützrohre," 1892, p. 67.

†Experiments of the Krupp factory with smokeless powder, 1890, pp. 42, 43.

In view of what has been said, it is useful to make the length of the charge only as much shorter than the powder chamber as is necessary for convenience in loading and preparing the charge (in cartridge guns this length should be approximately from 3 to 7 cm. shorter than the chamber, depending on the form of the base of the projectile);—while the diameter of the charge should change in relation to its weight.

(d) *Influence of variations in the diameter of the bore of the gun* is to be remarked for all guns, but chiefly for small arms and guns of small caliber in general. Although the value c' of the reduced caliber enters into formulæ 86 and 76 for velocities and pressures, and therefore it would appear that the value of variations on the bore of the gun might be determined from these formulæ, yet, as a matter of fact, it is impossible to do this, since the variation in the given case directly influences the forcing of the projectile through the bore, and consequently to a greater or less degree the combustion of the powder, in consequence of which velocities will change to a greater extent than would follow from the formulæ adduced.

Thus, on firing smokeless powder from the French small arms, model 1886, the diameters of the bores of which, measured from the bottoms of the riflings, were 7.99 mm., 8.05 mm., and 8.15 mm., the corresponding velocities obtained were 613.3, 603, and 600 meters.

The diameter of the bore may change, not only at the base of the riflings, but over the rifling, which also affects the initial velocity and pressure, and, besides this, change in diameter may be irregular for the whole length of the bore, whence for determining the effect of

the combination of differences in the dimensions of the various parts of the bores of small arms, the following experiments were conducted with 3-twist rifled small-arm barrels at the Okhta Powder Works. Six different barrels were taken, and the volume of their corresponding bores was determined by the quantity of mercury poured into them when the cartridge was in place. The maximum quantity of mercury occurred for barrel No. 63, namely, 482.2 grams; for the others it was less, as shown in Table LXVII. Not less than two series of firings of 10 shots each were made for each barrel with the same kinds of powder and bullets in each series of experiments. The mean values of the velocities obtained for each barrel are shown in the numerators in the table given. These experiments, as might be expected, showed that the velocity was the greater the less the weight of mercury, i. e., the less the mean caliber of the barrel. We therefore have approximately

$$\Delta V_{25} = -3.5 \Delta \rho$$

where ΔV_{25} is the change in velocity in meters at a distance 25 meters from the muzzle of the gun and $\Delta \rho$ is the variation in weight (in grams) of the mercury filling the bore of the rifle.

TABLE LXVII.—*Influence of caliber of rifle barrel on value of initial velocity.*

Experi- ment.	Weight of charge and kind of powder.	Weight of bullet (grams).	Number of rifles.					
			63	38	45	50	284	293
			Deviation $\Delta \rho$ in weight of mercury (in grams).					
			0	-4.7	-1.3	-4.2	-5.0	-1.6
			Velocities at 25 meters from the muzzle.					
1	2.30 gr. powder B, $\frac{\text{No. 19}}{93 \text{ g.}}$ Okhta....	13.7	609	$\frac{624}{625}$	$\frac{617}{614}$	$\frac{630}{624}$
2do.....	13.7	607	$\frac{619}{623}$	$\frac{613}{612}$	$\frac{627}{622}$	$\frac{627}{625}$	$\frac{610}{613}$
3	2.55 gr. powder B, $\frac{\text{No. 38}}{91 \text{ g.}}$ Okhta....	13.75	592	$\frac{597}{597}$	$\frac{611}{616}$	$\frac{593}{598}$

The velocities calculated by this formula, assuming as normal those which are obtained with rifle No. 63, are shown in the denominators in the same table. A certain, comparatively insignificant, difference between the values of velocities found by experiment and by calculation may be frequently attributed to the fact that the difference in the volumes of the cartridge cases was not taken into account in the given case, which doubtless influenced the differences in the velocities. In every case this experiment showed at a glance that, in experimenting with powder in small arms, it is necessary to give attention to the variations in the dimensions of the bores of rifles, and that by following the method herein indicated the firings of different rifles may be compared together.

(e) *The influence of the wearing away of the bore of a gun on the magnitude of initial velocities and pressures.*—The more a gun is fired, especially when smoking powders are used, the more there may then occur a special kind of wearing away of the bore, in consequence of which the volume of the powder chamber is increased on the one hand, and on the other, the diameter of the bore over the rifling and from the bases of the rifling is increased, so that the forcing of the projectile, especially at the beginning of movement, is considerably diminished. Both of these causes taken together produce a diminution of initial velocities and pressures on firing the same charges and projectiles, in consequence of the less complete combustion of the powder.

This wearing away is produced differently in different guns, and, therefore, no regular relation can be established between diminutions of velocities and pressures and the number of shots fired from the gun.

The wearing away of guns also occurs in firing smokeless powders; but is much less when firing pyroxylin powders, while on firing nitroglycerin powders (e. g., cordite) the wearing away of the gun occurs in a considerably greater degree* than when smoking powders are used. Finally, all other conditions being equal, wearing away in guns where cartridge cases are employed is considerably more rapid than in the case of those guns where cartridges of powder are used.

Table LXVIII presents the results of firing the 65-mm. gun, the 47-mm. gun, and the 3-twist barrels (Russian 3-line rifle).

TABLE LXVIII.—*Influence of wearing away of bore on initial velocities and pressures.*

Caliber of gun.		Number of shots previously fired from gun.	Initial velocity.	Pressure at base of bore.
				<i>K. gr.</i>
65-mm. gun No. 2.....		New	706.3 m. per second ...	2,094
		After 2,245 shots	675.8 m. per second ...	1,635
47 mm. single-firing Hotchkiss.	No. 1	1,738 war and drill ..	1,945 feet per second ..	*1,920
	No. 6	908 war and drill ..	1,950 feet per second ..	*1,890
	No. 108	14 war and drill	1,970 feet per second ..	*2,030
	No. 872	250 war and drill	†593 m. per second.....	
3-line rifle barrels	No. 873	2,000 war and drill ..	†592 m. per second.....	
	No. 872	7,000 war and drill ..	553 m. per second.....	
	No. 869	22,000 war and drill ..	480 m. per second	

* Atmospheres.

† Almost normal velocities.

Firings from the 47-mm. gun were conducted with cubical chocolate powder from the Okhta factory (see Table I, p. 20), in charges of 1 pound 72 zolotniks, with projectiles of 3.65 pounds weight; firings from the 65-mm. gun, with pyroxylin smokeless powder of charge 0.875 kilogram and weight of projectile of 4 kilograms. The firings from the rifled small arms were conducted with standard charges of Okhta small-arm smokeless powder (Table IV, sample A, p. 25) and

* The cause of this is due to the fact that the temperature of decomposition of the latter powders is very high. (See par. 20, pp. 53-64.)

bullets. Table LXVIII shows that the influence of the wearing away of the bore of the gun on diminution of initial velocities becomes noticeable in the case of small-arms barrels and in guns of small calibers only after many shots have been fired.

(f) *The influence of the cartridge bag and fatty substances found in it on changing initial velocities and pressures.*—About 12 per cent volatile substances were found in silk cartridge bags, of which 6 per cent were fatty substances and the rest moisture.

In determining the effect on ballistic results, both of the cartridge-bag tissue and also the fatty substances found in it, the experimental committee of the Okhta Powder Works conducted a series of firings with the light 4-pounder gun with charges of smokeless powder made up without cartridge bags, in cartridge bags of previously washed fabric, and with cartridge bags of the usual fabric. In all cases firings were conducted with charges weighing 1 pound 36 zolotniks of powder of the same lot, the projectiles weighing $16\frac{3}{4}$ pounds. Experiments were conducted by firing charges of the first, second, and third categories, with 10 shots for each. The results are shown in Table LXIX. They permit us to make the following deductions for the considerations of firing above deduced: (a) The usual cartridge cloth diminishes initial velocity by from 10 to 15 feet; (b) the greasy substances, in quantities such as they occur in the fabrics tried, have no apparent effect on initial velocity, and (c) pressures at the base of the bore do not change from the presence of the cartridge fabric.

TABLE LXIX.—*Influence of cartridge bag on ballistic results of firing the light field gun.*

Name of charge.	Weight of charge.	Weight of projectile.	Initial velocity.	Pressure on base of bore.
		<i>Pounds.</i>	<i>Ft. per sec.</i>	<i>Atmos.</i>
Charges without cartridge bag	1 pound 36 zolot- niks,	$16\frac{3}{4}$	1,456	1,100
Charges with washed bag			1,447	1,090
Charges with unwashed bag			1,443	1,100

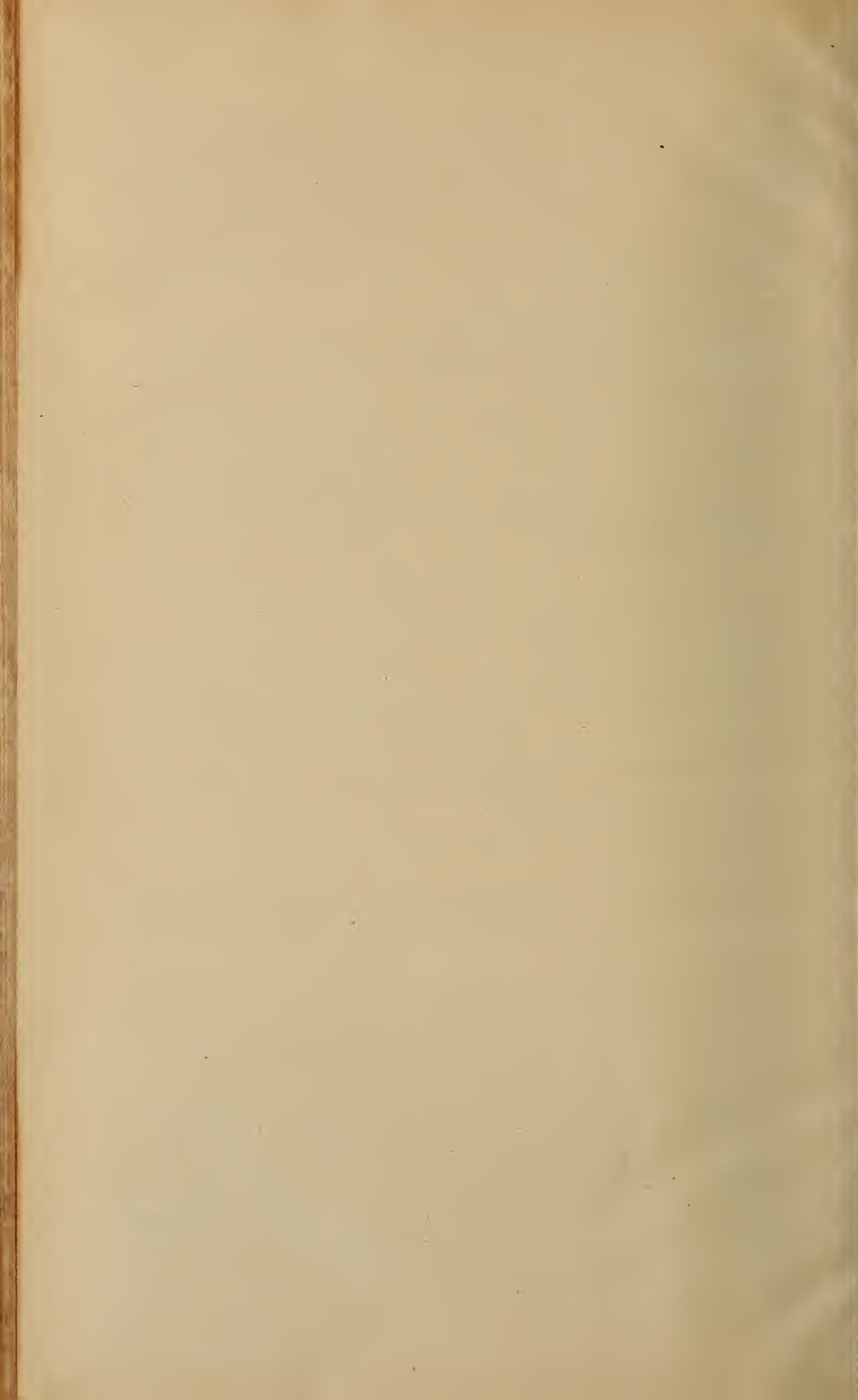
(g) *The influence of greasing the bore of the gun with vaseline on changing the initial velocity of the projectile.*—During experiments it was always observed that the velocity of the first shot differed from the velocity of the succeeding ones, which were conducted under exactly the same conditions, except that the surface of the bore was covered with lubricant at the first round, while for the later ones it was either covered with residue, if smoking powders were used, or was almost clean, when smokeless powders, such as pyroxylin powders, were employed. In the first case, the velocity was generally greater for the first shot, and in the second, less. In Noble's * experi-

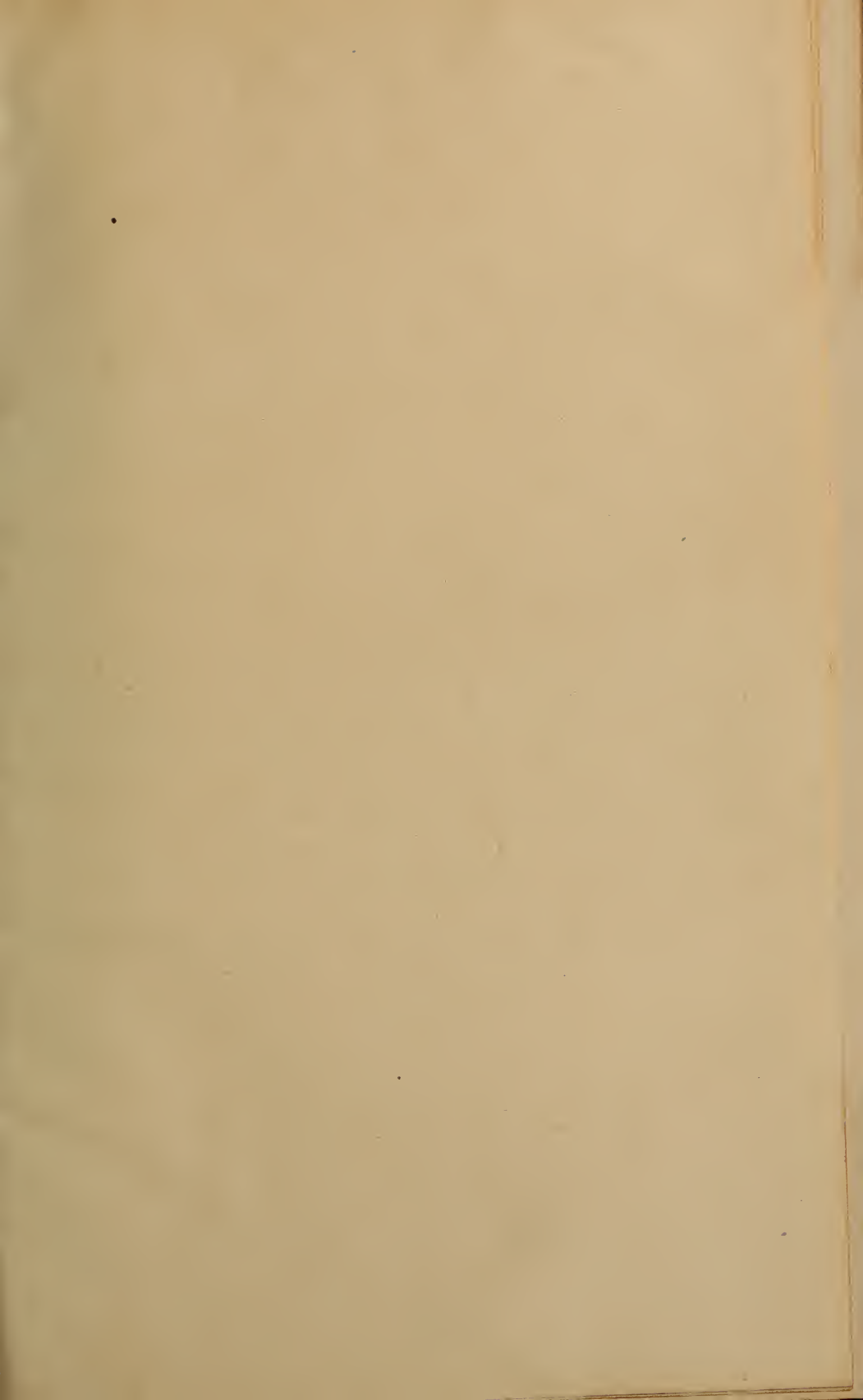
* See p. 334 of his article cited above.

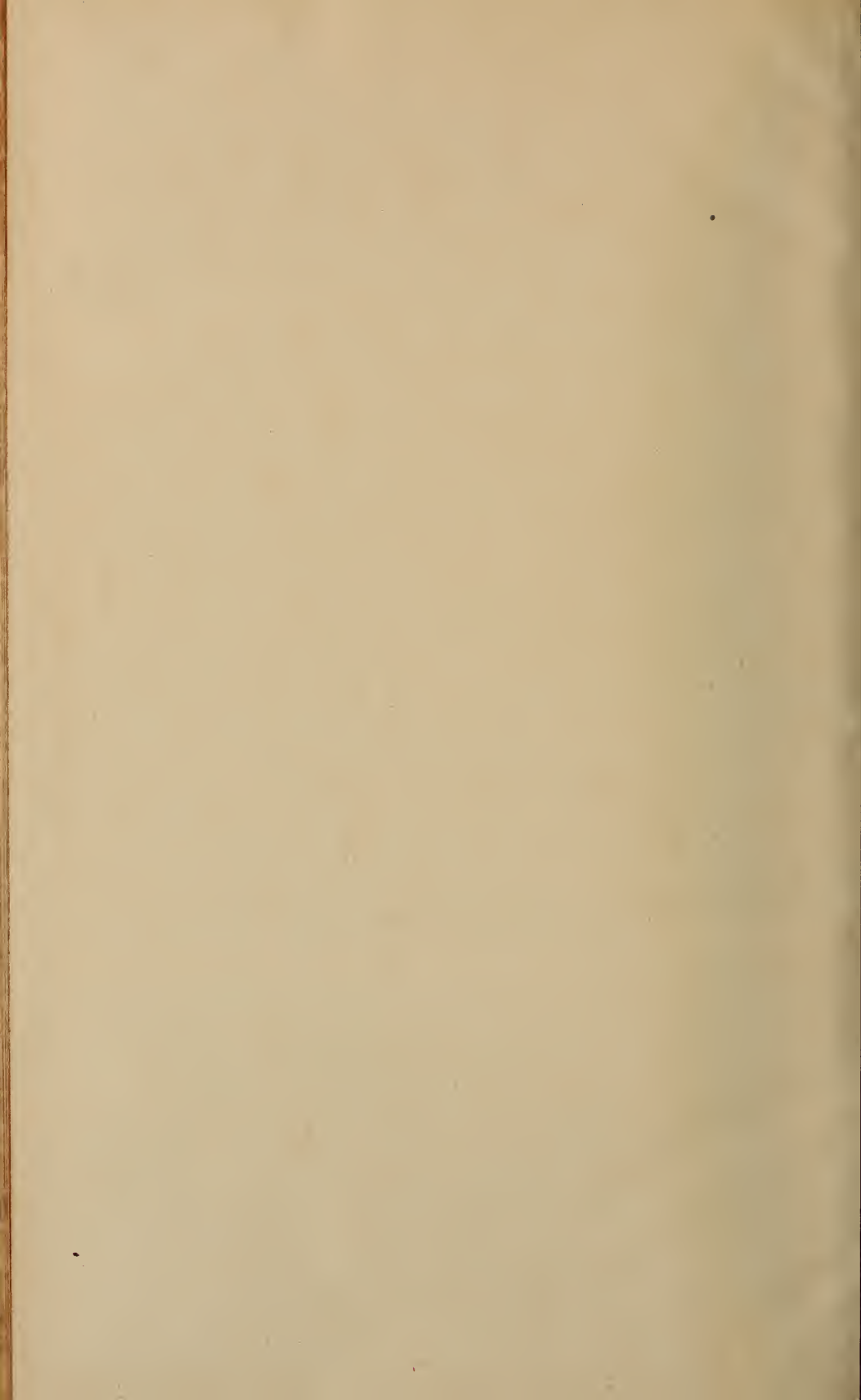
ments on firing smoking powder P (see Table VI, p. 34) from a 12-cm. gun with charges of 12 English pounds and projectiles weighing 45 English pounds, the following mean velocities were obtained; when the bore was carefully cleaned and greased (after each shot)—1,877.3 feet per second, and when the bore was covered with residue, 1,852 feet—i. e., approximately 2.73 per cent of the vis viva of the projectile was expended in overcoming friction created by the powder residue covering the bore of the gun.

In the experiments conducted by the Experimental Committee of the Okhta Powder Works, the following mean initial velocities were obtained from firing charges of 1 pound 65 zolotniks and projectiles weighing $16\frac{3}{4}$ pounds from the light gun: When the gun was not greased or lubricated at all and when it was lightly lubricated, 1,661 feet per second, and when the bore was abundantly lubricated before each shot, 1,625 feet per second; i. e., approximately about 4.3 per cent of the vis viva of the shell was lost in firing from a gun with an abundantly lubricated bore.

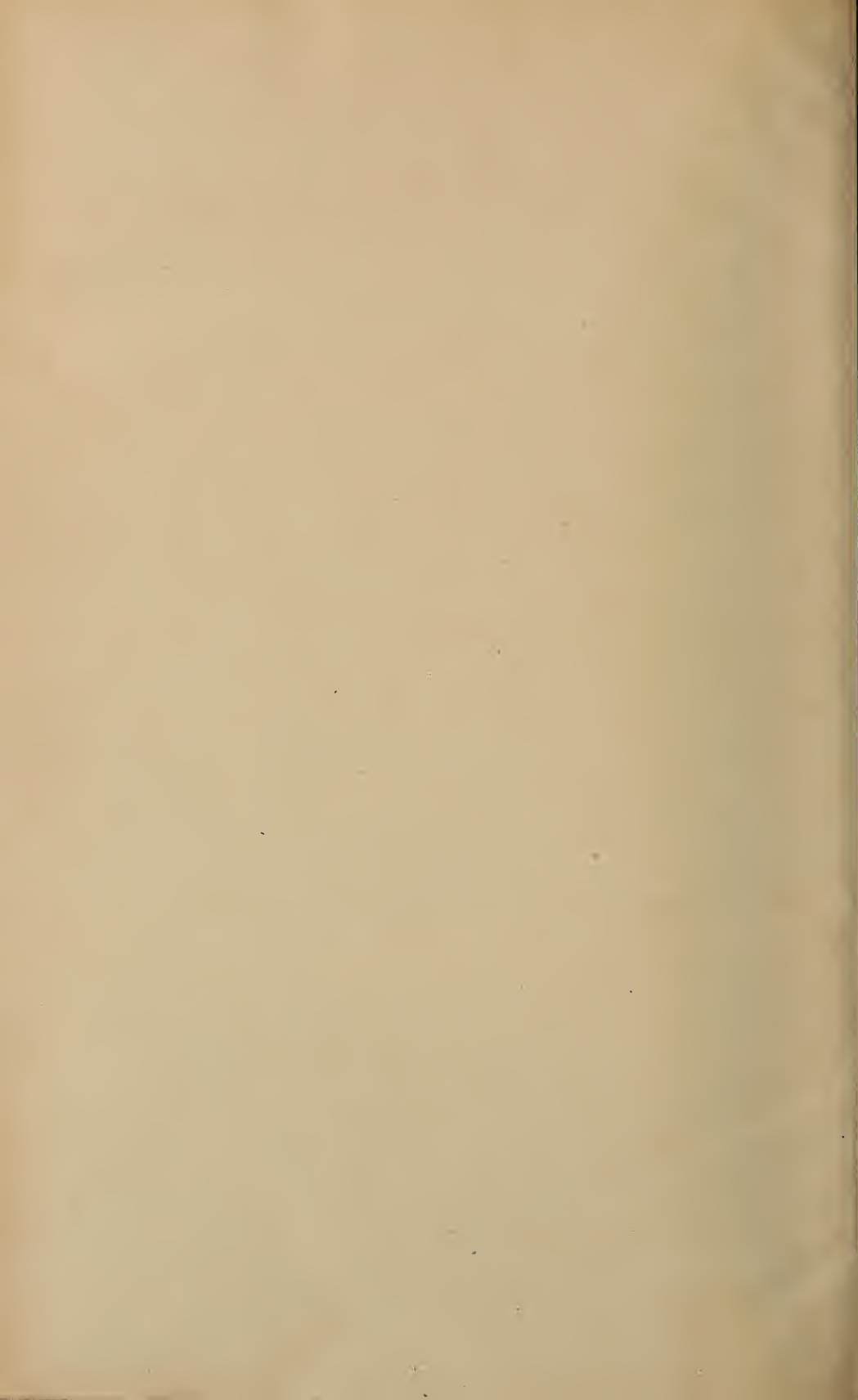




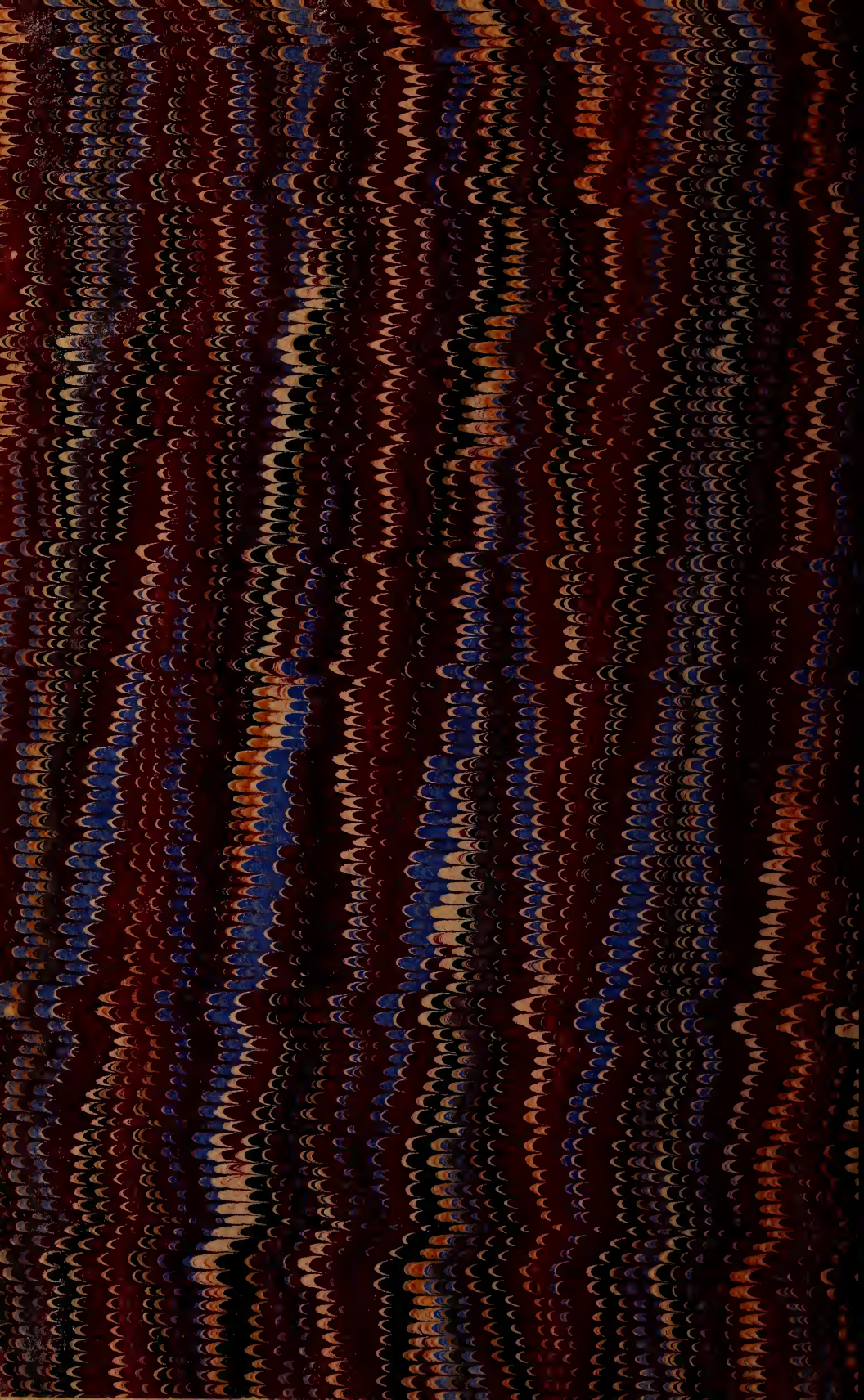


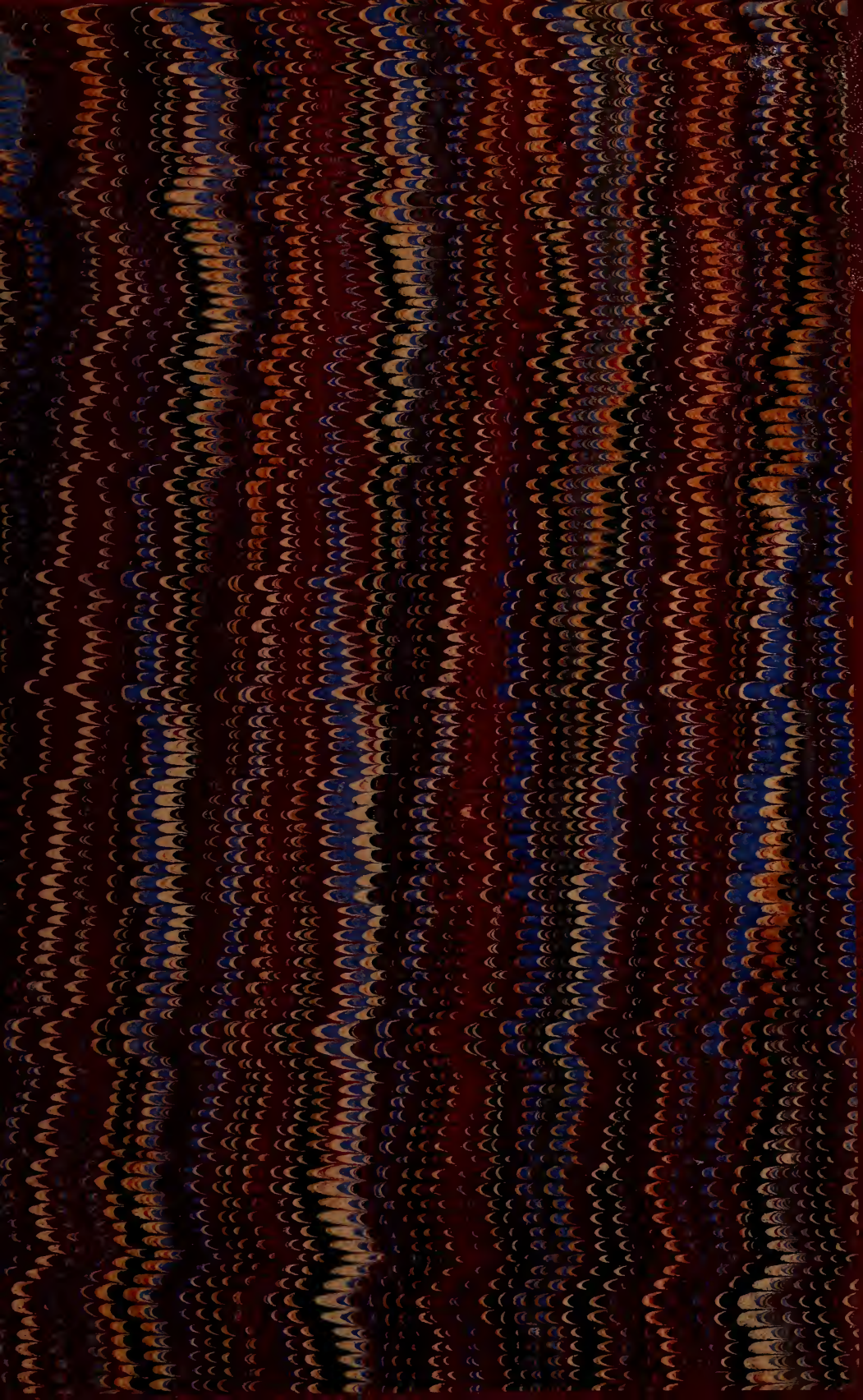












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